A Model Based on Linguistic 2-Tuples for Dealing with Multigranular Hierarchical Linguistic Contexts in Multi-Expert Decision-Making

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***Abstract*—In those problems that deal with multiple sources of linguistic information we can find problems defined in contexts where the linguistic assessments are assessed in linguistic term sets with different granularity of uncertainty and/or semantics (multigranular linguistic contexts). Different approaches have been developed to manage this type of contexts [1], [2], that unify the multigranular linguistic information in an unique linguistic term set for an easy management of the information. This normalization process can produce a loss of information and hence a lack of precision in the final results.**

In this paper, we shall present a type of multigranular linguistic contexts we shall call *linguistic hierarchies term sets*, such that, when we deal with multigranular linguistic information assessed in these structures we can unify the information assessed in them without loss of information. To do so, we shall use the 2-tuple linguistic representation model [3], [4]. Afterwards we shall develop a linguistic decision model dealing with multigranular linguistic contexts and apply it to a multi-expert decision-making problem.

***Index Terms*—Decision-making, linguistic hierarchies, linguistic preference modeling, linguistic variables, multigranular linguistic contexts.**

I. Introduction

PROBLEMS can present quantitative or qualitative aspects. When the aspects are qualitative, the use of the fuzzy linguistic approach [5], [39] is a good choice to model them, due to the fact that it represents the qualitative terms by means of linguistic variables instead of numerical values. A crucial task for dealing with linguistic information is to determine the *granularity of uncertainty*, i.e., the cardinality of the linguistic term set used to assess the linguistic variables. Depending on the uncertainty degree held by a source of information qualifying a phenomenon, the linguistic term set will have more or less terms [6], [1].

In this paper we deal with decision-making problems with linguistic preferences, focusing on those problems whose aspects are assessed by multiple sources of information, i.e., multi-expert decision-making (MEDM) problems. In these types of problems, it can be usual that each source of information has a different uncertainty degree over the alternatives. Then the linguistic information that manages the problem is assessed in different linguistic domains with different granularity and/or semantics. We shall denote this type of information as multigranular linguistic information.

On the other hand, decision-making problems that manage preferences from different experts follow a common resolution scheme [7] composed by two phases.

1) *Aggregation phase*: It combines the individual preferences to obtain a collective preference value for each alternative.
2) *Exploitation phase*: It orders the collective preference values according to a given criterion to obtain the best alternative/s.

In this paper, we deal with MEDM problems defined in multigranular linguistic contexts. In the literature, we can find different approaches to accomplish the aggregation phase of the above resolution scheme in these types of contexts [1], [2]. Those approaches carry out the aggregation phase in two processes.

- *Normalization process.*The multigranular linguistic information is expressed in an unique linguistic expression domain.
- *Combination process.*The unified linguistic information expressed in an unique linguistic term set is aggregated.

The main problem that presents the aforementioned approaches to carry out this aggregation process is the loss of information produced during the normalization process and hence a lack of precision in the final results.

The aim of this paper is to overcome the drawback of the loss of information in the normalization process. To do so, we shall present a set of multigranular linguistic contexts that we shall denote as *linguistic hierarchies term sets.* These contexts are designed under a set of *hierarchical linguistic basic rules,* in such a way that, if we deal with multigranular linguistic information assessed in a Linguistic Hierarchy, we shall carry out the normalization process without loss of information.

We shall take the 2-tuple linguistic representation model [3], [4] as representation base for the linguistic information of the problem. Therefore, we shall develop different functions that transform linguistic terms between different linguistic term sets of a Linguistic Hierarchy without loss of information. Afterwards we shall apply these functions to a decision model

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dealing with multigranular linguistic information and apply it to an MEDM problem.

This paper is structured as follows. In Section II, we make a brief review of the fuzzy linguistic approach and of the 2-tuple linguistic representation model, afterwards is presented an MEDM problem general scheme. In Section III, we shall define what is and how to build a linguistic hierarchy. In Section IV, we shall design transformation functions without loss of information between the different linguistic terms sets that belong to a linguistic hierarchy. In Section V, we shall solve an MEDM problem defined in a linguistic hierarchy, and finally we shall point out some concluding remarks.

II. Preliminaries

In this section, we briefly review the fuzzy linguistic approach and the 2-tuple fuzzy linguistic representation model. Afterwards, we shall present a general scheme for an MEDM problem.

A. Fuzzy Linguistic Approach

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [5], [39]. The fuzzy linguistic approach has been successfully applied to different areas, such as, *decision-making* [8]–[16], *information retrieval* [17], [18], *clinical diagnosis* [19], *marketing* [20], *risk in software development* [21], *technology transfer strategy selection* [22], *educational grading systems* [23], *scheduling* [24], *consensus* [25], [26], *materials selection* [27], *personnel management* [28], etc.

We have to choose the appropriate linguistic descriptors for the linguistic term set and their semantics. To do so, an important aspect to be analyzed is the "*granularity of uncertainty*," i.e., the level of discrimination among different degrees of uncertainty. Typical values of cardinality used in the linguistic models are odd ones, such as 7 or 9, where the mid term represents an assessment of "approximately 0.5," and the rest of the terms being placed symmetrically around it [6]. These classical cardinality values seem to satisfy the Miller's observation regarding the fact that human beings can reasonably manage to bear in mind seven or so items [29].

Once the cardinality of the linguistic term set has been established, the linguistic terms and its semantics must be provided. There exist different possibilities to accomplish this task [30], [17], [26], [31], [32]. One of them involves directly supplying the term set by considering all the terms distributed on a scale on which a total order is defined [31], [32]. For example, a set of seven terms S, could be

$$S = \{s_0 = N,\, s_1 = VL,\, s_2 = L,\, s_3 = M,\, s_4 = H,\, s_5 = VH,\, s_6 = P\}.$$

In these cases, it is usually required that there exist the following.

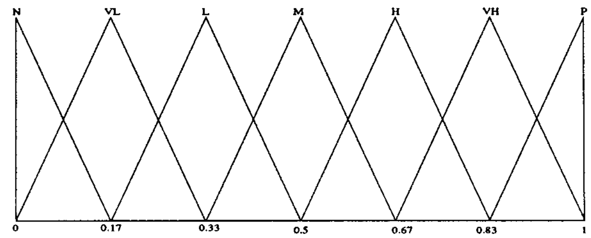

Fig. 1. Set of seven terms with its semantics.

1) A negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g - i$ ($g + 1$ is the cardinality).
2) $s_i \leq s_j \iff i \leq j$. A minimization and a maximization operator in the linguistic term set.

The semantics of the terms are represented by fuzzy numbers, defined in the [0,1] interval, described by membership functions. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [6]. The linguistic assessments given by the users are just approximate ones, then linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, [8]. This representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain [6]. A particular case of this representation is the triangular membership function, i.e., $b = d$, so we represent this type of membership function by a 3-tuple (a, b, c). For example, we may assign the following semantics to the previous set of seven terms, which is graphically shown in Fig. 1:

$$\begin{aligned} N &= (0,\, 0,\, 0.17) \quad VL = (0,\, 0.17,\, 0.33)\\ L &= (0.17,\, 0.33,\, 0.5) \quad M = (0.33,\, 0.5,\, 0.67)\\ H &= (0.5,\, 0.67,\, 0.83) \quad VH = (0.67,\, 0.83,\, 1)\\ P &= (0.83,\, 1,\, 1). \end{aligned}$$

Other authors use a nontrapezoidal representation, e.g., Gaussian functions [17].

B. The 2-Tuple Fuzzy Linguistic Representation Model

This model has been presented in [3], [4] where different advantages of this formalism are shown to represent the linguistic information over classical models, such as the following.

1) The linguistic domain can be treated as continuous, whilst in the classical models it is treated as discrete.
2) The linguistic computational model based on linguistic 2-tuples carries out processes of "computing with words" easily and without loss of information.
3) The results of the processes of "computing with words" are always expressed in the initial linguistic domain.

Due to these advantages, we shall use this linguistic representation model to accomplish our aim, to build transformation functions between different linguistic term sets without loss of information.

The 2-tuple fuzzy linguistic representation model represents the linguistic information by means of a 2-tuple, (s, α), where

s is a linguistic label and α is a numerical value that represents the value of the symbolic translation.

Definition 1: Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g+1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$ then α is called a *Symbolic Translation.*

Roughly speaking, the symbolic translation of a linguistic term, s_i, is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between a counting of information $\beta \in [0, g]$ obtained after a symbolic aggregation operation and the closest value in $\{0, \ldots, g\}$ that indicates the index of the closest linguistic term in $S(i = round(\beta))$.

From this concept, we shall develop a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-0.5, 0.5)$:

- s_i represents the linguistic label of the information;
- α_i is a numerical value expressing the value of the translation from the original result β to the closest index label, i, in the linguistic term set ($s_i \in S$), i.e., the symbolic translation.

This linguistic representation model defines a set of functions to make transformations between linguistic 2-tuples and numerical values:

Definition 2: Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta\colon [0, g] \longrightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \quad (1)$$

where round is the usual *rounding* operation, s_i has the closest index label to "β," and "α" is the value of the symbolic translation.

Example: Let us suppose a symbolic aggregation operation over labels assessed in $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ that obtains $\beta = 2.8$ as its result, then the representation of this counting of information by means of a 2-tuple will be

$$\Delta(2.8) = (s_3, -0.2).$$

Graphically, it is represented in Fig. 2.

Proposition 1: Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a function Δ^{-1}, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.

Proof: It is trivial, we consider the following function:

$$\Delta^{-1}\colon S \times [-0.5, 0.5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \quad (2)$$

Remark: From Definitions 1 and 2 and Proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation:

$$s_i \in S \Longrightarrow (s_i, 0).$$

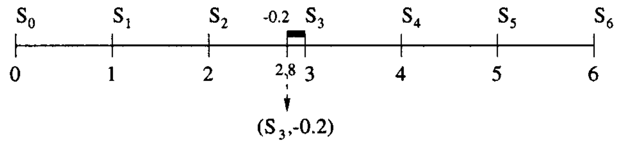

Fig. 2. Example of a symbolic translation computation.

TABLE I
GENERAL MEDM PROBLEM

Alternatives (a_i)	*Experts* e_1	e_2	...	e_k
a_1	y_{11}	y_{12}	...	y_{1k}
...	...	...	...	...
a_n	y_{n1}	y_{n2}	...	y_{nk}

In addition, together with this representation model, a linguistic computational approach is also defined, in which there exist the following.

1) 2-Tuple Comparison Operators.

The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, then

- if $k < l$ then (s_k, α_1) is smaller than (s_l, α_2)
- if $k = l$ then
  a) if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information
  b) if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2)
  c) if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2)

2) A 2-Tuple Negation Operator.

$$Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha))) \quad (3)$$

where $g+1$ is the cardinality of S, $S = \{s_0, \ldots, s_g\}$.

3) A wide range of 2-tuple aggregation operators has been developed extending classical aggregation operators, such as the LOWA operator, the weighted average operator, the OWA operator, etc. [3].

C. Multi-Expert Decision-Making Problem

Let $A = \{a_1, \ldots, a_n\}$ be a set of alternatives. Each one assessed by a set of experts $\{e_1, \ldots, e_k\}$. This scheme is shown in Table I.

There exists different literature on fuzzy MEDM problems [33], [34]. In the following, we focus in MEDM problems defined over multigranular linguistic term sets, i.e., problems where their preference values y_{ij} can be assessed in linguistic term sets S_j that can have different granularity of uncertainty and/or semantics.

III. LINGUISTIC HIERARCHIES

The linguistic hierarchies have been used in different areas as fuzzy rules based systems [35]–[37] and decision models [8]. In the same way that the hierarchical linguistic variables for the design of hierarchical systems of linguistic rules are introduced in [36], in the following we are going to introduce a hierarchical linguistic structure that allows us to improve the precision in the aggregation processes of multigranular linguistic information.

A. Linguistic Hierarchical Structure

A *linguistic hierarchy* is a set of levels, where each level is a linguistic term set with different granularity to the rest of levels of the hierarchy. Each level belonging to a linguistic hierarchy is denoted as

$$l(t, n(t))$$

being

1) t, a number that indicates the level of the hierarchy;
2) $n(t)$, the granularity of the linguistic term set of the level t.

Here, we must point out that in this paper we deal with linguistic terms whose membership functions are triangular-shaped, symmetrical and uniformly distributed in $[0, 1]$. In addition, the linguistic term sets have an odd value of granularity representing the central label the value of *indifference.*

The levels belonging to a linguistic hierarchy are ordered according to their granularity, i.e., for two consecutive levels t and $t+1$, $n(t+1) > n(t)$. This provides a linguistic refinement of the previous level.

From the above concepts, we shall define a linguistic hierarchy, LH, as the union of all levels t

$$LH = \bigcup_t l(t, n(t)).$$

In the following, we are going to develop a methodology to build linguistic hierarchies under a set of rules and conditions.

B. Building Linguistic Hierarchies

Here we show how to build a linguistic hierarchy. We must take into account that its hierarchical order is given by the increase of the granularity of the linguistic term sets in each level.

We start from a linguistic term set, S, over the universe of the discourse U in the level t

$$S = \{s_0, \dots, s_{n(t)-1}\}$$

being s_k, $(k = 0, \dots, n(t) - 1)$ a linguistic term of S.

To build a linguistic hierarchy, we extend the definition of S to a set of linguistic term sets, $S^{n(t)}$, each term set belongs to a level t of the hierarchy and has a granularity of uncertainty $n(t)$

$$S^{n(t)} = \left\{s_0^{n(t)}, \dots, s_{n(t)-1}^{n(t)}\right\}.$$

And afterwards, we develop a methodology which satisfies the following rules, that we call, *linguistic hierarchy basic rules.*

1) To preserve all *former modal points* of the membership functions of each linguistic term from one level to the following one.
2) To make *smooth transitions between successive levels.* The aim is to build a new linguistic term set, $S^{n(t+1)}$. A new linguistic term will be added between each pair of terms belonging to the term set of the previous level t. To carry out this insertion, we shall reduce the support of the linguistic labels in order to keep place for the new one located in the middle of them.

TABLE II
LINGUISTIC HIERARCHIES

	$L(t, n(t))$	$L(t, n(t))$
Level 1	$L(1, 3)$	$L(1, 7)$
Level 2	$L(2, 5)$	$L(2, 13)$
Level 3	$L(3, 9)$	

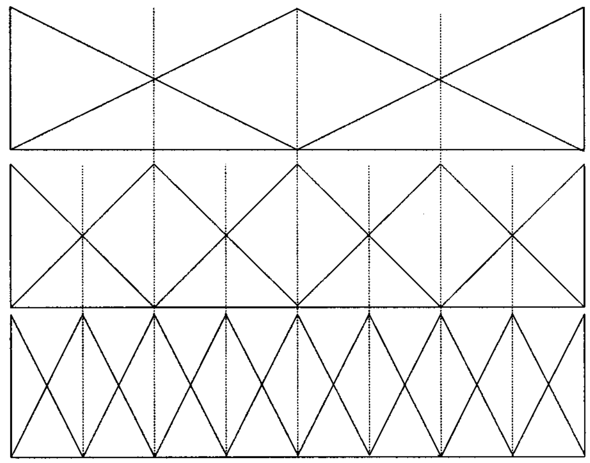

Fig. 3. Linguistic hierarchy of three, five, and nine labels.

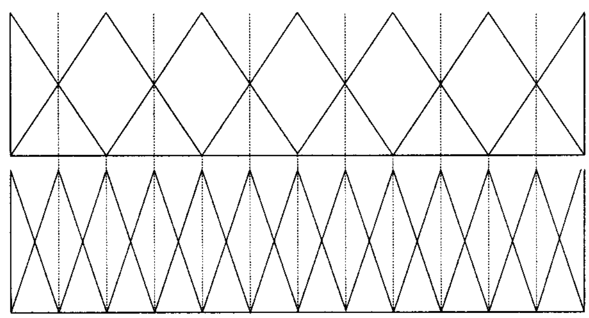

Fig. 4. Linguistic hierarchy of seven and 13 labels.

Table II shows the granularity needed in each linguistic term set of the level t depending on the value $n(t)$ defined in the first level (3 and 7 respectively). Generically, we can say that the linguistic term set of level $t+1$ is obtained from its predecessor as

$$L(t, n(t)) \rightarrow L(t+1, 2 \cdot n(t) - 1).$$

The graphical examples of the linguistic hierarchies presented in Table II are shown in Figs. 3 and 4, respectively.

Remark: When a problem is defined over a multigranular linguistic context where labels are assessed in linguistic term sets from different linguistic hierarchies, we can mix these labels according to the model presented in [2], but in this case loss of information can appear in the normalization process.

IV. Transformation Functions Among Levels of a Linguistic Hierarchy

We have seen that the main problem for aggregating multigranular linguistic information is the loss of information produced in the normalization process. To avoid this problem, we shall use *Linguistic Hierarchies term sets* as multigranular linguistic contexts, but also we need transformation functions among the linguistic terms of the linguistic hierarchy term sets that carry out these transformation processes without loss of information. To understand the *modus operandi* of these functions, we shall first define transformations between consecutive levels and following we shall generalize the transformation functions between any level of the hierarchy. These transformation functions will use the 2-tuple linguistic modeling.

Definition 4: Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple linguistic representation. The transformation function from a linguistic label in level t to a label in level $t+1$, satisfying the linguistic hierarchy basic rules, is defined as

$$TF_{t+1}^t \colon l(t, n(t)) \longrightarrow l(t+1, n(t+1))$$
$$TF_{t+1}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right)\cdot(n(t+1)-1)}{n(t)-1}\right). \quad (4)$$

Definition 5: Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \left\{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\right\}$, and let us consider the 2-tuple linguistic representation. The transformation function from a linguistic label in level t to a label in level $t-1$, satisfying the linguistic hierarchy basic rules, is defined as

$$TF_{t-1}^t \colon l(t, n(t)) \longrightarrow l(t-1, n(t-1))$$
$$TF_{t-1}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right)\cdot(n(t-1)-1)}{n(t)-1}\right). \quad (5)$$

Making a deep study of the definitions 4 and 5, we shall generalize these transformation functions to transform linguistic terms between any linguistic level in the linguistic hierarchy. This generalization can be carried out by means of the following recursive function.

Definition 6: Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$. The recursive transformation function between a linguistic label that belongs to level t and a label in level $t' = t + a$, with $a \in Z$, is defined as

$$TF_{t'}^t \colon l(t, n(t)) \longrightarrow l(t', n(t'))$$

If

$$|a| > 1$$

then

$$TF_{t'}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right) = TF_{t'}^{t+[(t-t')/(|t-t'|)]} \cdot \left(TF_{t+[(t-t')/(|t-t'|)]}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right)\right).$$

If

$$|a| = 1$$

then

$$TF_{t'}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right) = TF_{t+[(t-t')/(|t-t'|)]}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right). \quad (6)$$

This recursive transformation function can be easily defined in a non recursive way as follows:

$$TF_{t'}^t \colon l(t, n(t)) \longrightarrow l(t', n(t'))$$
$$TF_{t'}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right)\cdot(n(t')-1)}{n(t)-1}\right). \quad (7)$$

Proposition 2: The transformation function between linguistic terms in different levels of the linguistic hierarchy is bijective:

$$TF_t^{t'}\left(TF_{t'}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right)\right) = \left(s_i^{n(t)}, \alpha^{n(t)}\right).$$

Proof:

$$TF_{t'}^t\left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right)\cdot(n(t')-1)}{n(t)-1}\right)$$

therefore, see the equation shown at the bottom of the next page.

This result guarantees the transformations between levels of a linguistic hierarchy are carried out without loss of information.

Example: Here we show how the transformation functions act over the linguistic hierarchy, $LH = \bigcup_t l(1, 3)$, whose term sets are

$$l(1, 3) \quad \{s_0^3, s_1^3, s_2^3\}$$
$$l(2, 5) \quad \{s_0^5, s_1^5, s_2^5, s_3^5, s_4^5\}$$
$$l(3, 9) \quad \{s_0^9, s_1^9, s_2^9, s_3^9, s_4^9, s_5^9, s_6^9, s_7^9, s_8^9\}.$$

The transformations between terms of the different levels are carried out as

$$TF_1^3(s_5^9, 0) = \Delta^{-1}\left(\frac{\Delta(s_5^9, 0)\cdot(3-1)}{9-1}\right) = \Delta^{-1}(1, 25) = (s_1^3, 0.25)$$
$$TF_3^1(s_1^3, 0.25) = \Delta^{-1}\left(\frac{\Delta(s_1^3, 0.25)\cdot(8-1)}{3-1}\right) = \Delta^{-1}(5) = (s_5^9, 0.0)$$

$$TF_2^3(s_5^9, 0) = \Delta^{-1}\left(\frac{\Delta(s_5^9, 0)\cdot(5-1)}{9-1}\right) = \Delta^{-1}(2.5) = (s_5^5, -0.5)$$

$$TF_1^2(s_3^5, -0.5) = \Delta^{-1}\left(\frac{\Delta(s_3^5, -0.5)\cdot(3-1)}{5-1}\right) = \Delta^{-1}(1.25) = (s_1^3, 0.25).$$

V. MEDM Problem Defined Over a Linguistic Hierarchy

As application of the linguistic hierarchies presented in this paper we shall solve an MEDM problem defined in a multigranular linguistic context.

We have chosen an MEDM problem for this application due to the fact that it is a very common situation in real-world applications, the case in which the experts express their judgments by using linguistic terms drawn form different scales. Therefore, each one can express his preferences by means of linguistic terms assessed in linguistic terms sets with different granularity of uncertainty and/or semantics.

In the following example, we have chosen as multigranular linguistic context the linguistic hierarchy $LH = \bigcup_t l(1, 3)$, since the granularity of its linguistic term sets are very common in decision-making problems.

In this section a decision model for solving the MEDM problem is presented.

A. Description

Let us suppose an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money:

- x_1 is a car industry;
- x_2 is a computer company;
- x_3 is a food company;
- x_4 is a weapon industry.

The investment company has a group of four consultancy departments:

- e_1 is the risk analysis department;
- e_2 is the growth analysis department;
- e_3 is the social-political analysis department;
- e_4 is the environmental impact analysis department.

Each department is directed by an expert, and thus, each expert is an information source. These experts use to provide their preferences, over the set of alternatives, the different term sets of the linguistic hierarchy. Specifically

- e_1 provides his preferences in $l(3, 9)$;
- e_2 provides his preferences in $l(2, 5)$;
- e_3 provides his preferences in $l(1, 3)$;
- e_4 provides his preferences in $l(3, 9)$.

The linguistic terms can have a syntax adequate to the problem, but in this case we shall use the normalized syntax for a better comprehensiveness of the computation processes.

After a deep study, each expert provides the following preference values:

	x_1	x_2	x_3	x_4
e_1	s_4^9	s_6^9	s_3^9	s_5^9
e_2	s_3^5	s_4^5	s_3^5	s_3^5
e_3	s_1^3	s_2^3	s_2^3	s_1^3
e_4	s_4^9	s_5^9	s_3^9	s_5^9

B. Decision Model

Here we present the decision model used to solve the above problem.

1) *Aggregation Phase.* The information is combined to obtain collective preference values for each alternative. The aggregation of the multigranular linguistic information is carried out in two steps:

   a) Normalization process. A linguistic term set is chosen to make uniform the multigranular linguistic information. Then, all the information is

$$TF_t^{t'}\left(\Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right)\cdot(n(t')-1)}{n(t)-1}\right)\right) = \Delta\left(\frac{\Delta^{-1}\left(\Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right)\cdot(n(t')-1)}{n(t)-1}\right)\right)\cdot(n(t)-1)}{n(t')-1}\right)$$

$$= \Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right)\cdot(n(t')-1)\cdot(n(t)-1)}{(n(t)-1)\cdot(n(t')-1)}\right) = \left(s_i^{n(t)}, \alpha^{n(t)}\right).$$

expressed in that linguistic term set by means of linguistic 2-tuples.

b) Aggregation process. Once the information is unified, a 2-tuple linguistic aggregation operator is used to combine it.

2) *Exploitation phase.* The collective preference values are ordered according to a given criterion and the solution set is composed of the best alternative/s.

C. Linguistic Treatment of the Problem

Aggregation Phase:

a) *Normalization process.* First, we must select a linguistic term set to unify the multigranular linguistic information. We can choose "**any**" linguistic term set to do it. In this case we shall choose the linguistic term set $l(3, 9)$, since the most of experts have expressed their preferences in it and thus we reduce the number of computations. Therefore, we obtain the following preference unified values expressed by means of 2-tuples:

	x_1	x_2	x_3	x_4
e_1	$(s_4^9, 0)$	$(s_6^9, 0)$	$(s_3^9, 0)$	$(s_5^9, 0)$
e_2	$(s_5^9, 0)$	$(s_6^9, 0)$	$(s_5^9, 0)$	$(s_5^9, 0)$
e_3	$(s_5^9, 0)$	$(s_9^9, 0)$	$(s_9^9, 0)$	$(s_5^9, 0)$
e_4	$(s_4^9, 0)$	$(s_5^9, 0)$	$(s_3^9, 0)$	$(s_5^9, 0)$

b) Aggregation process. In this problem all the experts have the same importance in the decision process, therefore we shall use the 2-tuple mean operator [3] to aggregate the preferences, whose expression is

$$\overline{x} = \Delta\left(\frac{\sum_{i=1}^{n}\Delta^{-1}(s_i, \alpha_i)}{n}\right). \tag{8}$$

The collective values obtained for each alternative are

x_1	x_2	x_3	x_4
$(s_5^9, -0.5)$	$(s_7^9, 0.25)$	$(s_5^9, 0.25)$	$(s_3^9, 0)$

These collective values can be expressed in any linguistic term of the linguistic hierarchy:

- $\mathbf{l(2, 5)}$:

x_1	x_2	x_3	x_4
$(s_2^5, 0.25)$	$(s_4^5, -0.38)$	$(s_3^5, -0.38)$	$(s_3^5, -0.5)$

- $\mathbf{l(1, 3)}$:

x_1	x_2	x_3	x_4
$(s_1^3, 0.12)$	$(s_2^3, -0.19)$	$(s_1^3, 0.31)$	$(s_1^3, 0.25)$

In this way, all the experts receive the collective values in their expression domain and the exploitation phase is carried out in their correspondent linguistic term set.

Exploitation Phase: In this phase, we shall choose as best alternative that with the biggest collective value, i.e., the solution set of alternatives in this problem will be

$$\{x_2\}$$

i.e., the best option to invest the money is the *computer company.*

VI. Concluding Remarks

The management of information expressed in multigranular linguistic contexts is a complex task, whose main problem is to unify it without loss of information. In this paper, we have presented a set of multigranular linguistic contexts, *linguistic hierarchy term sets*, that allow us to manage these contexts easily and without loss of information. To do so, we have developed a set of functions that transform linguistic terms between different linguistic term sets of the hierarchy. These functions use the 2-tuple linguistic representation model.

We have applied the linguistic hierarchies to an MEDM problem, but they can be applied to different decision-making problems [11], [13], [26], [38], information retrieval [17], [18], management problems [27], [28].

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