Brief Papers

A 2-Tuple Fuzzy Linguistic Representation Model for Computing with Words

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***Abstract*—The fuzzy linguistic approach has been applied successfully to many problems. However, there is a limitation of this approach imposed by its information representation model and the computation methods used when fusion processes are performed on linguistic values. This limitation is the loss of information caused by the need to express the results in the initial expression domain that is discrete via an approximate process. This loss of information implies a lack of precision in the final results from the fusion of linguistic information. In this paper, we present tools for overcoming this limitation. The linguistic information will be expressed by means of 2-tuples, which are composed by a linguistic term and a numeric value assessed in [−0.5, 0.5). This model allows a continuous representation of the linguistic information on its domain, therefore, it can represent any counting of information obtained in a aggregation process. Together with the 2-tuple representation model we shall develop a computational technique for computing with words (CW) without any loss of information. Finally, different classical aggregation operators will be extended to deal with the 2-tuple linguistic model.**

***Index Terms*—Computing with words (CW), information fusion, linguistic modeling, linguistic variables.**

I. Introduction

In day-to-day activities we have to solve different problems and depending on the aspects presented by each problem we can deal with different types of precise numerical values, but in other cases, the problems present qualitative aspects that are complex to assess by means of precise and exact values. In the latter case, the use of the fuzzy linguistic approach [14], [16], [17] has provided very good results. It deals with qualitative aspects that are represented in qualitative terms by means of linguistic variables. When a problem is solved using linguistic information, it implies the need for computing with words (CW). Here, an important limitation for this approach appears because the computational techniques used in the specialized literature present a common drawback, the "loss of information," that implies a lack of precision in the final results. These computational techniques are as follows.

- The first one is based on the *extension principle* [2], [6]. It makes operations on the fuzzy numbers that support the semantics of the linguistic terms.
- The second one is the *symbolic method* [5]. It makes computations on the indexes of the linguistic terms.

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In both approaches, the results usually do not exactly match any of the initial linguistic terms, then an approximation process must be developed to express the result in the initial expression domain. This produces the consequent loss of information and hence the lack of precision [4].

The aim of this paper is to develop a new fuzzy linguistic representation model for overcoming this limitation. This model represents the linguistic information with a pair of values, that we call *2-tuple*, composed by a linguistic term and a number. The main advantage of this representation is to be continuous in its domain, therefore, it can express any counting of information in the universe of the discourse. Together with this representation model, we present a computational technique to deal with the 2-tuples without loss of information. Finally, we shall use this fuzzy linguistic representation model in a decision-making problem, in which, we show that this model is more precise than the previous ones.

In order to do that, this paper is structured as follows. In Section II, we shall present a brief review of the fuzzy linguistic approach and analyze the computational method based on the extension principle and the symbolic one. In Section III, we present the concept of "symbolic translation," subsequently introduce the fuzzy linguistic representation model based on the 2-tuples and develop a computational technique over it. In Section IV, we extend several classical aggregation operators to deal with the 2-tuple linguistic representation model. In Section V, we use the 2-tuple linguistic model in an application over a decision process and, finally, some concluding remarks are included.

II. Analysis of the Linguistic Computational Models

In this section, we shall make a brief review of the fuzzy linguistic approach and of the two computational methods used in processes of CW. Afterwards, we shall present a simple decision-making process to solve a linguistic decision problem by means of the two computational models above.

A. Fuzzy Linguistic Approach

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The variables which participate in these problems are assessed by means of linguistic terms [14], [16], [17]. This approach is adequate in some situations, for example, when attempting to qualify phenomena related to human perception, we

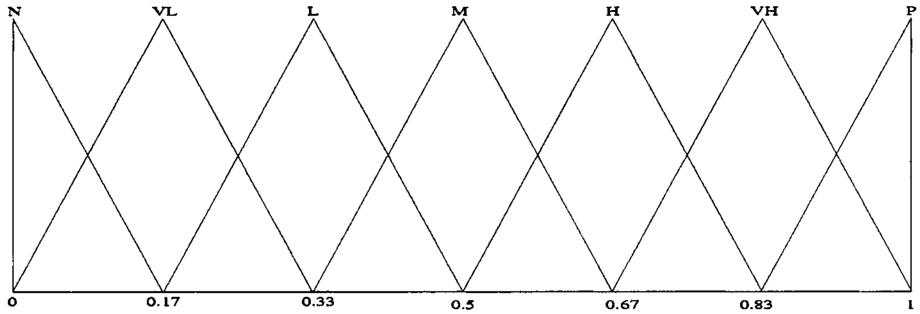

Fig. 1. A set of seven terms with their semantics.

are often led to use words in natural language. The use of linguistic assessements implies to make computations with them. CW has been applied to different areas. Foundations and applications providing the current status of theoretical and empirical developments in CW can be found in [15].

In this paper, we will focus in the use of linguistic information for modeling performance evaluations. In order to do that, we have to choose the appropriate linguistic descriptors for the term set and their semantics. In order to accomplish this objective, an important aspect to analyze is the *granularity of information*, i.e., the cardinality of the term set. One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [13]. For example, a set of seven terms S could be given as follows:

$$S = \{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}.$$

Usually, in these cases, it is required that the linguistic term set satisfies the following additional characteristics.

1) There is a negation operator: $\text{Neg}(s_i) = s_j$ such that $j = g - i$ ($g + 1$ is the cardinality).
2) $s_i \leq s_j \Longleftrightarrow i \leq j$. Therefore, there exists a minimization and a maximization operator.

In this paper, we shall use labels with triangular membership function. For example, we may assign the following semantics to the set of seven terms (graphically, see Fig. 1):

$$H = (.5, .67, .83) \quad VH = (.67, .83, 1) \quad P = (.83, 1, 1)$$
$$VL = (0, .17, .33) \quad L = (.17, .33, .5) \quad M = (.33, .5, .67)$$
$$N = (0, 0, .17).$$

Other authors use a nontrapezoidal representation, e.g., Gaussian functions [3].

B. Linguistic Computational Model Based on the Extension Principle

The extension principle has been introduced to generalize crisp mathematical operations to fuzzy sets. The use of extended arithmetic based on the extension principle [7] increases the vagueness of the results. The results obtained by the fuzzy arithmetic are fuzzy numbers that usually do not match any linguistic term in the initial term set, so a linguistic approximation process is needed to express the result in the original expression domain. In the literature, we can find different linguistic approximation operators [2], [6].

A linguistic aggregation operator based on the extension principle acts according to

$$S^n \xrightarrow{\tilde{F}} F(\mathcal{R}) \xrightarrow{\text{app}_1(\cdot)} S$$

where S^n symbolizes the n Cartesian product of S, $\tilde{F}$ is an aggregation operator based on the extension principle, $F(\mathcal{R})$ the set of fuzzy sets over the set of real numbers $\mathcal{R}$, $app_1 : F(\mathcal{R}) \rightarrow S$ is a linguistic approximation function that returns a label from the linguistic term set S whose meaning is the closest to the obtained unlabeled fuzzy number and S is the initial term set.

C. Linguistic Computational Symbolic Model

A second approach used to operate on linguistic information is the symbolic one [5], that makes computations on the indexes of the linguistic labels. Usually, it uses the ordered structure of the linguistic term sets, $S = \{s_0, \cdots, s_g\}$ where $s_i < s_j$ iff $i < j$, to perform the computations. The intermediate results are numeric values, $\alpha \in [0, g]$, which must be approximated in each step of the process by means of an approximation function $app_2 : [0, g] \rightarrow \{0, \cdots, g\}$ that obtains a numeric value such that it indicates the index of the associated linguistic term $s_{\text{app}_2(\alpha)} \in S$. Formally, it can be expressed as

$$S^n \xrightarrow{C} [0, g] \xrightarrow{\text{app}_2(\cdot)} \{0, \cdots, g\} \longrightarrow S$$

where C is a symbolic linguistic aggregation operator, $app_2(\cdot)$ is an approximation function used to obtain an index $\{0, \cdots, g\}$ associated to a term in $S = \{s_0, \cdots, s_g\}$ from a value in $[0, g]$.

D. Example

Here, we propose a simple decision-making process for solving a linguistic decision problem by means of the two computational models we have just reviewed.

1) Linguistic Decision Problem: A distribution company needs to renew its computing system, so it contracts a consulting company to carry out a survey of the different possibilities existing on the market, to decide which is the best option for its needs. The alternatives are the following:

x_1	x_2	x_3	x_4
UNIX	WINDOWS-NT	AS/400	VMS.

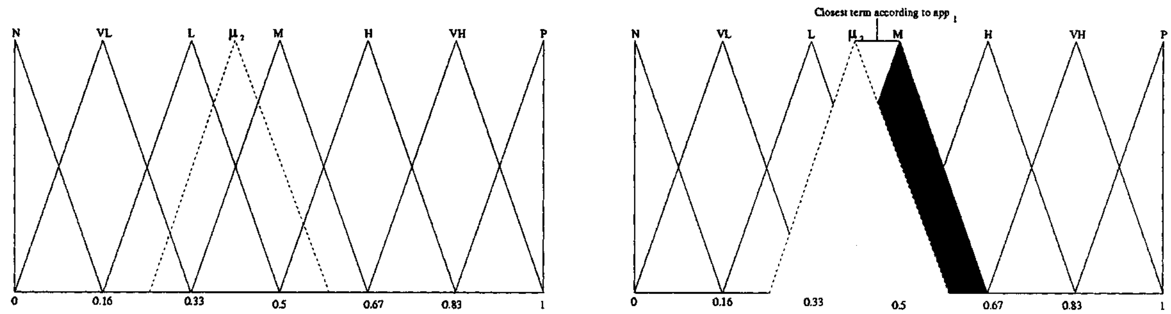

Fig. 2. Linguistic approximation process.

The consulting company has a group of four consultancy departments

p_1	p_2	p_3	p_4
Cost analysis	System analysis	Risk analysis	Techonology analysis.

Each department provides a performance vector expressing its performace evaluations for each alternative. These evaluations are assessed in the term set $S = \{N, VL, L, M, H, VH, P\}$ (see Fig. 1).

		alternatives			
	L_{ij}	x_1	x_2	x_3	x_4
experts	p_1	VL	M	M	L
	p_2	M	L	VL	H
	p_3	H	VL	M	M
	p_4	H	H	L	L

whose membership functions C_{ij} are assumed to be of the triangular type $C_{ij} = (a_{ij}, b_{ij}, c_{ij})$.

2) Selection Model: The selection model we shall use to solve the above problem has the following steps.

1) To obtain a *collective performance value* over each alternative.
2) A selection process over the collective performance vector is applied.

3) Solution Based on the Extension Principle:

a) Collective performance vector: We select the arithmetic mean as an aggregation operator. Obtaining a collective performance value for each alternative "x_j" with

$$C_j = \left(\frac{1}{m}\sum_{i=1}^{m} a_{ij}, \frac{1}{m}\sum_{i=1}^{m} b_{ij}, \frac{1}{m}\sum_{i=1}^{m} c_{ij}\right)$$

being m the number of experts. We obtain the collective preference vector shown at the bottom of the page.

These collective values are fuzzy sets that do not exactly match any linguistic term in S, therefore, we must apply a linguistic approximation process based on the Euclidean distance to each C_j for obtaining the results in the initial term set S

$$d(s_l, C_j) = \sqrt{P_1(a_l - a_j)^2 + P_2(b_l - b_j)^2 + P_3(c_l - c_j)^2}$$

representing (a_l, b_l, c_l) and (a_j, b_j, c_j) the membership functions of "s_l" and "C_j," respectively. Being P_1, P_2, and P_3 weights that measure the representativeness of the parameters a, b, and c of the membership function of the fuzzy set. These weights satisfy

- $P_i \in [0, 1]$;
- $\sum_i P_i = 1$.

Therefore, $\mathrm{app}_1(\cdot)$ chooses s_l^* ($\mathrm{app}_1(C_j) = s_l^*$), such that, $d(s_l^*, C_j) \le d(s_l, C_j)\ \forall s_l \in S$.

This linguistic approximation process is applied to the above fuzzy sets, with $P_1 = 0.2$, $P_2 = 0.6$, $P_3 = 0.2$. We select these values because of the parameter "b_i" is the most representative of the membership function and "a_i," "c_i" are equally representative.

The collective preference vector obtained after the linguistic approximation is

$\mathrm{app}_1(C_1)$	$\mathrm{app}_1(C_2)$	$\mathrm{app}_1(C_3)$	$\mathrm{app}_1(C_4)$
M	M	L	M.

Fig. 2 shows the linguistic approximation for C_2. We can see that C_2 does not match any term in S, then the closest term in S to C_2 according to $\mathrm{app}_1(\cdot)$ is the label M.

b) Selection process: A choice degree is applied to the collective performance vector for obtaining the alternative(s) with the highest collective performance value

$$\{x_1, x_2, x_4\}.$$

C_1	C_2	C_3	C_4
$(0.33, 0.5, 0.66)$	$(0.25, 0.42, 0.58)$	$(0.21, 0.38, 0.54)$	$(0.3, 0.45, 0.625)$

This is not a good solution because due to the lack of precision presented by this method, we are not able to choose only one alternative.

4) Solution Based on the Symbolic Methods: Here, we shall apply the same selection model for solving the problem, but, in this case, we shall deal with the symbolic approach presented in [5]. The symbolic operator we shall use to aggregate linguistic variables is the convex combination [5].

Definition 1: Let $A = \{a_1, \ldots, a_m\}$ be a set of linguistic terms to be aggregated, the convex combination is defined in a recursive way as the following.

For $m = 2$

$$C^2\{\{w_1, 1-w_1\}, \{b_1, b_2\}\} = (w_1 \odot s_j) \oplus ((1-w_1) \odot s_i) = s_k, s_j, s_i \in S, \quad (j \geq i)$$

such that

$$k = \min\{g, i + \text{round}(w_1 \cdot (j-i))\}$$

where $g+1$ is the cardinality of S, round$(\cdot)$ is the usual round operation, and $b_1 = s_j$, $b_2 = s_i$.

For $m > 2$

$$\begin{aligned} C^m\{w_k, b_k, k = 1, \ldots, m\} &= (w_1 \odot b_1) \oplus ((1-w_1) \\ &\quad \odot C^{m-1}\{\eta_h, b_h, h = 2, \ldots, m\}) \\ &= C^2\{\{w_1, 1-w_1\} \\ &\quad \{b_1, C^{m-1}\{\eta_h, b_h, h = 2, \ldots, m\}\}\} \end{aligned}$$

where $W = [w_1, \ldots, w_m]$ is a weighting vector associated with A, such that, (i) $w_i \in [0,1]$, and (ii) $\sum_i w_i = 1$; and $B = \{b_1, \ldots, b_m\}$ is a vector such that $B = \{a_{\sigma(1)}, \ldots, a_{\sigma(m)}\}$, where $a_{\sigma(j)} \leq a_{\sigma(i)}\ \forall i \leq j$, with σ being a permutation over the values a_i. $\eta_h = w_h / \Sigma_2^m w_k$, $h = 2, \ldots, m$. Being $\odot$ and $\oplus$ the product of a number by a label and the addition of two labels, respectively.

a) Collective performance vector: In our example the weighting vector is $\{.25, .25, .25, .25\}$, then the collective performance values obtained are

C_1	C_2	C_3	C_4
M	M	L	M.

b) Selection process: The alternatives with the highest collective performance are

$$\{x_1, x_2, x_4\}.$$

Here, again we find a multiple alternative solution set, that coincides with the above solution. Symbolic methods present a loss of information as well, in this case it is caused by the use of the round operator.

Therefore, both computational models have a common important drawback—the loss of information caused by the need to express the results in the initial expression domain that is discrete. In the following section, we propose a continuous linguistic representation model that can express any counting of information although it does not exactly match any linguistic term.

III. A 2-Tuple Fuzzy Linguistic Representation Model Based on the Symbolic Translation

To develop this linguistic model we shall take as a basis the symbolic model (Section II-C) and, in addition, we define the concept of symbolic translation and use it to represent the linguistic information by means of a pair of values that we call linguistic 2-tuple, (s, α), where s is a linguistic term and α is a numeric value representing the symbolic translation. Together with this representation model we shall present a computational technique to deal with linguistic 2-tuples without loss of information.

A. The Symbolic Translation—2-Tuple Fuzzy Linguistic Representation

Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set, if a symbolic method aggregating linguistic information obtains a value $\beta \in [0, g]$, and $\beta \notin \{0, \ldots, g\}$ then an approximation function ($app_2(\cdot)$) is used to express the index of the result in S.

Definition 2: Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g+1$ the cardinality of S. Let $i = \text{round}(\beta)$ and $\alpha = \beta - i$ be two values such that $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$ then α is called a *symbolic translation*.

Roughly speaking, the symbolic translation of a linguistic term, s_i, is a numerical value assessed in [−0.5, 0.5) that supports the "difference of information" between a counting of information $\beta \in [0, g]$ obtained after a symbolic aggregation operation and the closest value in $\{0, \ldots, g\}$ that indicates the index of the closest linguistic term in S ($i = \text{round}(\beta)$).

From this concept we shall develop a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-.5, .5)$:

- s_i represents the linguistic label center of the information;
- α_i is a numerical value expressing the value of the translation from the original result β to the closest index label, i, in the linguistic term set (s_i), i.e., the symbolic translation.

This model defines a set of transformation functions between linguistic terms and 2-tuples and between numeric values and 2-tuples.

Definition 3: Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta\colon [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases}$$

where round$(\cdot)$ is the usual round operation, s_i has the closest index label to "β," and "α" is the value of the symbolic translation.

1) Example: Let us suppose a symbolic aggregation operation over labels assessed in $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ that

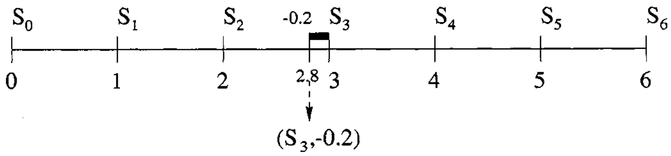

Fig. 3. Example of a symbolic translation computation.

obtains as its result $\beta = 2.8$, then the representation of this counting of information by means of a 2-tuple will be

$$\Delta(2.8) = (s_3, -0.2).$$

Graphically, it is represented in Fig. 3.

Proposition 1: Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.

Proof: It is trivial, we consider the following function:

$$\Delta^{-1}\colon\ S \times [-.5, .5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta.$$

2) Remark: From Definitions 2 and 3 and from Proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value zero as symbolic translation

$$s_i \in S \Longrightarrow (s_i, 0).$$

B. Linguistic Computational Model Based on the Symbolic Translation

In this subsection, we present a computational technique to operate with the 2-tuples without loss of information. We shall present the following computations and operators.

1) Comparison of 2-Tuples: The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a counting of information as follows:

- if $k < l$ then (s_k, α_1) is smaller than (s_l, α_2);
- if $k = l$ then
  1) if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information;
  2) if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2);
  3) if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2).

2) Aggregation of 2-Tuples: The aggregation of information consists of obtaining a value that summarizes a set of values, therefore, the result of the aggregation of a set of 2-tuples must be a 2-tuple. In Section IV, we shall introduce several 2-tuple aggregation operators, that are based on classical aggregation operators.

3) Negation Operator of a 2-Tuple: We define the negation operator over 2-tuples as

$$\mathrm{Neg}((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$$

where $g + 1$ is the cardinality of S, $S = \{s_0, \ldots, s_g\}$.

IV. 2-Tuple Linguistic Aggregation Operators

In this section, we shall present several aggregation operators for linguistic 2-tuples based on classical aggregation operators. In the literature we can find many aggregation operators [1], [12], which allow us to combine the information according to different criteria.

The fuzzy linguistic representation model with 2-tuples has defined the functions Δ and Δ^{-1} that transform numerical values into a 2-tuples and viceversa without loss of information, therefore, any numerical aggregation operator can be easily extended for dealing with linguistic 2-tuples.

A. Arithmetic Mean

The arithmetic mean 1 is a classical aggregation operator, its equivalent operator for linguistic 2-tuples is defined as

Definition 4: Let $x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of 2-tuples, the 2-tuple arithmetic mean $\overline{x}^e$ is computed as

$$\overline{x}^e = \Delta\left(\sum_{i=1}^{n} \frac{1}{n}\, \Delta^{-1}(r_i, \alpha_i)\right) = \Delta\left(\frac{1}{n}\, \sum_{i=1}^{n} \beta_i\right).$$

The arithmetic mean for 2-tuples allows us to compute the mean of a set of linguistic values without any loss of information.

B. Weighted Average Operator

The weighted average [1] allows different values x_i have a different importance in the nature of the variable x. To do so, each value x_i has a weight associated w_i indicating its importance in the nature of the variable. The equivalent operator for linguistic 2-tuples is defined as

Definition 5: Let $x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of 2-tuples and $W = \{w_1, \ldots, w_n\}$ be their associated weights. The 2-tuple weighted average $\overline{x}^e$ is

$$\overline{x}^e = \Delta\left(\frac{\displaystyle\sum_{i=1}^{n} \Delta^{-1}(r_i, \alpha_i) \cdot w_i}{\displaystyle\sum_{i=1}^{n} w_i}\right) = \Delta\left(\frac{\displaystyle\sum_{i=1}^{n} \beta_i \cdot w_i}{\displaystyle\sum_{i=1}^{n} w_i}\right).$$

C. Ordered Weighted Aggregation (OWA) Operator

Yager introduced a weighted aggregation operator [12], in which the weights are not associated with a predetermined value but rather the weights are associated to a determined position.

The OWA operator F^e for dealing with linguistic 2-tuples is defined as

Definition 6: Let $A = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of 2-tuples and $W = (w_1, \ldots, w_n)$ be an associated weighting vector that satisfies: 1) $w_i \in [0, 1]$ and 2) $\sum w_i = 1$. The 2-tuple OWA operator F^e for linguistic 2-tuples is computed as

$$F^e((r_1, \alpha_1), \ldots, (r_n, \alpha_n)) = \Delta\left(\sum_{j=1}^{n} w_j \cdot \beta_j^*\right)$$

where β_j^* is the jth largest of the β_i values.

		alternatives			
		x_1	x_2	x_3	x_4
experts	p_1	$(VL,0)$	$(M,0)$	$(M,0)$	$(L,0)$
	p_2	$(M,0)$	$(L,0)$	$(VL,0)$	$(H,0)$
	p_3	$(H,0)$	$(VL,0)$	$(M,0)$	$(M,0)$
	p_4	$(H,0)$	$(H,0)$	$(L,0)$	$(L,0)$

V. Example

Here, we shall use the linguistic representation model with 2-tuples to solve the decision-making problem presented in Section II-D. To do this, we follow the same selection model.

- The performance vectors of the experts are transformed into 2-tuples, as shown at the top of the page.
- Now we aggregate these 2-tuples using the 2-tuple arithmetic mean, obtaining the collective performance values

$\overline{x}_1^e$	$\overline{x}_2^e$	$\overline{x}_3^e$	$\overline{x}_4^e$
$(\mathrm{M}, 0)$	$(\mathrm{M}, -0.5)$	$(\mathrm{L}, 0.25)$	$(\mathrm{M}, -0.25)$.

- We obtain as solution set of alternatives

$$\{x_1\}.$$

Therefore, the distribution company will receive a survey where the best computing system for their needs is the "UNIX-based system."

A. Comparative Analysis

Throughout this paper, we have solved a decision problem using three different linguistic computational methods, obtaining the following results.

An important difference appears in the "*solution set*" column in Table I. In this column the result obtained by the 2-tuple arithmetic mean is more precise (is a subset) than the sets obtained by the other ones.

- Collective values obtained by both classical methods (extension principle and symbolic) are discrete, then, when different alternatives have the same linguistic term as collective value, we cannot discern which alternative is better than the others.
- In the method based on the 2-tuple representation, the collective values are managed as continuous ones, therefore, if several alternatives have the same linguistic term but a different value for the symbolic translation, we can choose the best one among those alternatives.

TABLE I
RESULTS USING THE THREE METHODS

	Collective Values				Solution Set
	x_1	x_2	x_3	x_4	
E.P.	M	M	L	M	$\{x_1, x_2, x_4\}$
Symbolic	M	M	L	M	$\{x_1, x_2, x_4\}$
2-tuples	(M,0)	(M,-.5)	(L,-.25)	(M,-.25)	$\{x_1\}$

VI. Concluding Remarks

In this paper, we have presented a new fuzzy linguistic representation model based on the symbolic translation. It represents the information by means of 2-tuples, which are composed by a linguistic term and the symbolic translation represented by a numeric value assessed in $[-0.5, 0.5)$. In this way, the linguistic information is managed as a continuous range instead of a discrete one. This approach has no loss of information when we apply it to computation with words processes.

We wish point out two aspects.

- In first place, we must remark that all the 2-tuple operators work on the numeric values associated with the 2-tuples, i.e., β values and then their results are still numeric values in $[0, g]$ that must be translated into linguistic 2-tuples. So the linguistic 2-tuples representation model and its computational model are a mean to maintain the level of granularity of the results produced by the operations performed on the label indexes (values in $[0, g]$) and express them in a linguistic way.
- On the other hand, making a depth analysis of the label indexes is equivalent to consider equidistant labels. If we have in mind to use linguistic terms that are not equidistant and we want to keep this information then it would be neccesary to use another parameter in the representation.

Finally, we should point out that using the 2-tuple fuzzy linguistic representation, we can tackle the problems of multigranularity [9], [10] and combination of linguistic-numerical information [8], [11] in a common context. These aspects have centered the continuation of this work.

References

[1] J. Aczél, "On weighted synthesis of judgements," *Aequationes Math.*, no. 27, pp. 288–307, 1984.

[2] P. P. Bonissone and K. S. Decker, "Selecting uncertainty calculi and granularity: An experiment in trading-off precision and complexity," in *Uncertainty in Artificial Intelligence*, L. H. Kanal and J. F. Lemmer, Eds. Amsterdam, The Netherlands: North-Holland, 1986, pp. 217–247.

[3] G. Bordogna and G. Pasi, "A fuzzy linguistic approach generalizing boolean information retrieval: A model and its evaluation," *J. Amer. Soc. Inform. Sci.*, no. 27, pp. 288–307, 1984.

[4] C. Carlsson and R. Fuller, "Benchmarking and linguistic importance weighted aggregations," *Fuzzy Sets Syst.*, vol. 114, no. 1, pp. 35–42, 2000.

[5] M. Delgado, J. L. Verdegay, and M. A. Vila, "On aggregation operations of linguistic labels," *Int. J. Intell. Syst.*, no. 8, pp. 351–370, 1993.

[6] R. Degani and G. Bortolan, "The problem of linguistic approximation in clinical decision making," *Int. J. Approximate Reasoning*, no. 2, pp. 143–162, 1988.

[7] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*. New York: Academic, 1980.

[8] M. Delgado, F. Herrera, E. Herrera-Viedma, and L. Martínez, "Combining numerical and linguistic information in group decision making," *Inform. Sci.*, no. 107, pp. 177–194, 1998.

[9] F. Herrera, E. Herrera-Viedma, and L. Martínez, "A fusion approach for managing multi-granularity linguistic terms sets in decision making," *Fuzzy Sets Syst.*, vol. 114, no. 1, pp. 43–58, 2000.

[10] F. Herrera and L. Martínez, "A model based on linguistic 2-tuples for dealing with multigranularity hierarchical linguistic contexts in multi-expert decision making," IEEE Trans. Syst., Man, Cybern., pt. B, to be published.

[11] F. Herrera and L. Martínez, "An approach for combining linguistic and numerical information based on 2-tuple fuzzy linguistic representation model in decision-making," *Int. J. Uncertainty, Fuzziness, Knowledge-Based Syst.*, vol. 8, no. 5, pp. 539–562, 2000.

[12] R. R. Yager, "On ordered weighted averaging aggregation operators in multicriteria decision making," *IEEE Trans. Syst., Man, Cybern.*, no. 18, pp. 183–190, 1988.

[13] R. R. Yager, "An approach to ordinal decision making," *Int. J. Approximate Reasoning*, no. 12, pp. 233–261, 1995.

[14] L. A. Zadeh, "The concept of a linguistic variable and its applications to approximate reasoning," *Inform. Sci.*, pt. I, no. 8, pp. 199–249, 1975.

[15] L. Zadeh and J. Kacprzyk, *Computing with Words in Information/Intelligent Systems—Part 1: Foundations; Part 2: Applications*. Heidelberg, Germany: Physica-Verlag, 1999, vol. I.

[16] L. A. Zadeh, "The concept of a linguistic variable and its applications to approximate reasoning," *Inform. Sci.*, pt. II, no. 8, pp. 301–357, 1975.

[17] ——, "The concept of a linguistic variable and its applications to approximate reasoning," *Inform. Sci.*, pt. III, no. 9, pp. 43–80, 1975.