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Linguistic decision analysis: steps for solving decision problems under linguistic information

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Abstract

A study on the steps to follow in linguistic decision analysis is presented in a context of multi-criteria/multi-person decision making. Three steps are established for solving a multi-criteria decision making problem under linguistic information: (i) the choice of the linguistic term set with its semantic in order to express the linguistic performance values according to all the criteria, (ii) the choice of the aggregation operator of linguistic information in order to aggregate the linguistic performance values, and (iii) the choice of the best alternatives, which is made up by two phases: (a) the aggregation of linguistic information for obtaining a collective linguistic performance value on the alternatives, and (b) the exploitation of the collective linguistic performance value in order to establish a rank ordering among the alternatives for choosing the best alternatives. Finally, an example is shown. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

There are decision situations in which the information cannot be assessed precisely in a quantitative form but may be in a qualitative one, and thus, the use of a *linguistic approach* is necessary. For example, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language instead of numerical values. As was pointed out in [8], this may arise for different reasons. There are some situations in which the information may be unquantifiable due to its nature, and thus, it may be stated only in linguistic terms (e.g., when evaluating the "comfort" or "design" of a car [36], terms like "good", "medium", "bad" can be used). In other cases, precise quantitative information may not be stated because either it is unavailable or the cost of its computation is too high, so an "approximate value" may be tolerated (e.g., when evaluating the speed of a car, linguistic terms like "fast", "very fast", "slow" may be used instead of numerical values).

The linguistic approach is an approximate technique which represents qualitative aspects as linguistic values by means of *linguistic variables* [56], that is, variables whose values are not numbers but words or sentences in a natural or artificial language. Each linguistic value is characterized by a *syntactic value* or *label* and a *semantic value* or *meaning*. The label is

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a word or sentence belonging to a linguistic term set and the meaning is a fuzzy subset in a universe of discourse. Since words are less precise than numbers, the concept of a linguistic variable serves the purpose of providing a measure of an approximate characterization of the phenomena which are too complex or ill-defined to be amenable to their description by conventional quantitative terms.

Linguistic decision analysis is based on the use of the linguistic approach and it is applied for solving decision making problems under linguistic information. Its application in the development of the theory and methods in *decision analysis* is very beneficial because it introduces a more flexible framework which allows us to represent the information in a more direct and adequate way when we are unable to express it precisely. In this way, the burden of quantifying a qualitative concept is eliminated.

In the literature, we may find many applications of linguistic decision analysis to solve real-world activities, e.g., group decision making [4,22], multi-criteria decision making [5,7,52], consensus [4,18,25,37], marketing [53], software development [35], education [34], subjective assessment of car evaluation [36], vine pruning [42], material selection [9], personel management [30], etc.

The main aim of this paper is to present a study on the steps to follow in linguistic decision analysis, that is, the steps to follow for solving a decision making problem under linguistic information. Usually, in all real-world decision making processes there are various actors (experts or decision makers) who are called to express their performance values on a predefined set of options (alternatives) in order to select the best one(s). We do not distinguish between "experts" and "criteria" and interpret linguistic decision analysis in a contex of *multi-criteria decision making* (MCDM) [10,21]. In a classical fuzzy decision analysis, the solution scheme of an MCDM problem basically consists of two phases [33,41]: (i) an *aggregation phase* of the performance values with respect to all the criteria for obtaining a collective performance value for the alternatives, followed by an (ii) *exploitation phase* of the collective performance value for obtaining a rank ordering, sorting or choice among the alternatives. In the linguistic decision analysis of an MCDM problem, the solution scheme must be formed by the following three steps:

1. *The Choice of the linguistic term set with its semantic.* It consists of establishing the linguistic expression domain used to provide the linguistic performance values about alternatives according to the different criteria. To do so, we have to choose the granularity of the linguistic term set, its labels and its semantic.
2. *The Choice of the aggregation operator of linguistic information.* It consists of establishing an appropriate aggregation operator of linguistic information for aggregating and combining the linguistic performance values provided.
3. *The Choice of the best alternatives.* It consists of choosing the best alternatives according to the linguistic performance values provided. It is carried out in two phases:
   (a) *Aggregation phase of linguistic information*: It consists of obtaining a collective linguistic performance value on the alternatives by aggregating the linguistic performance values provided according to all the criteria by means of the chosen aggregation operator of linguistic information.
   (b) *Exploitation phase*: It consists of establishing a rank ordering among the alternatives according to the collective linguistic performance value for choosing the best alternatives.

With the objective of analyzing these three steps, the paper is structured as follows. Section 2 presents the first step of the linguistic decision analysis; Section 3 studies the second one showing different approaches on aggregation operators of linguistic information; Section 4 analyzes the third one according to its two phases; Section 5 shows an example of the application of the linguistic decision analysis in an MCDM problem, and finally, some conclusions are pointed out.

2. The choice of the linguistic term set with its semantic

The choice of the linguistic term set with its semantic is the first goal to satisfy in any linguistic approach for solving a problem. It consists of establishing the linguistic variable [56] or linguistic expression domain with a view to provide the linguistic performance values.

Definition 1 (Zadeh [56]). A linguistic variable is characterized by a quintuple $(L,H(L),U,G,M)$ in which L is the name of the variable; $H(L)$ (or simply H) denotes the term set of L, i.e., the set of names of linguistic values of L, with each value being a fuzzy variable denoted generically by X and ranging across a universe of discourse U which is associated with the base variable u; G is a syntactic rule (*which usually takes the form of a grammar*) for generating the names of values of L; and M is a semantic rule for associating its meaning with each $L, M(X)$, which is a fuzzy subset of U.

From a practical point of view, we can find two possibilities to choose the appropriate linguistic descriptors of the term set and their semantic:

- The first possibility defines the linguistic term set by means of a context-free grammar, and the semantic of linguistic terms is represented by fuzzy numbers described by membership functions based on parameters and a semantic rule [1,3,56].
- The second one defines the linguistic term set by means of an ordered structure of linguistic terms, and the semantic of linguistic terms is derived from their own ordered structure which may be either symmetrically distributed on the $[0,1]$ interval or not [4,13,25,45,52,55].

In the following subsections, firstly, we study the two possibilities to obtain the linguistic descriptors of the term set, and then we present the possibilities for defining their semantic.

2.1. *The choice of the linguistic term set*

The main aim of establishing the linguistic descriptors of a linguistic variable is to supply the user with a few words by which he can naturally express his information. In order to accomplish this objective, an important aspect to analyze is the *granularity of uncertainty*, i.e., the level of discrimination among different countings of uncertainty, i.e., the cardinality of the linguistic term set used to express the information.

The cardinality of the term set must be small enough so as not to impose useless precision on the users, and it must be rich enough in order to allow a discrimination of the assessments in a limited number of degrees.

Typical values of cardinality used in the linguistic models are odd ones, such as 7 or 9, with an upper limit of granularity of 11 or no more than 13, where the mid term represents an assessment of "approximately 0.5", and with the rest of the terms being placed symmetrically around it [2]. These classical cardinality values seems to fall in line with Miller's observation about the fact that human beings can reasonably manage to bear in mind seven or so items [38].

When the cardinality of the linguistic term set is established, then we have to provide a mechanism for generating the linguistic descriptors. Two approaches are known: one defines them by means of a context-free grammar and another defines them by means of a total order defined on the linguistic term set. Both are analyzed in the following subsections.

2.1.1. *Approach based on a context-free grammar*

One possibility for generating the linguistic term set consists of supplying it using a context-free grammar G, i.e., the term set is a set of sentences belonging to the language generated by G [1,3,56]. A generative grammar G is a 4-tuple (V_N, V_T, I, P), where V_N is the set of non-terminals, V_T is the set of terminals, I is the starting symbol and P the production rules. Thus, the choice of these elements will determine the cardinality and form of the linguistic term set. Obviously, this will be problem dependent, but the generated language should be large enough such that any possible situation of the problem involved can be described.

According to Miller's observations [38], the generated language does not have to be infinite. Moreover, it must be easily understandable. Thus, complex syntactic structures, such as the unlimited recursive use of the same production rule by means of a cyclic non-terminal (which yields an infinite language) should be avoided [1].

For example, among the terminals and non-terminals of G we can find primary terms (e.g. *high*, *medium*, *low*), hedges (e.g. *not*, *much*, *very*, *rather*, *more or less*), relations (e.g. *higher than*, *lower than*), and connectives (e.g. *and*, *but*, *or*). Then, making as I any term, the linguistic term set $H = \{high, very\ high, not\ high, high\ or\ medium, \ldots\}$ is generated by means of P. P may be defined in an extended Backus Naur Form (see [3] as an example).

2.1.2. Approach based on an ordered structure of linguistic terms

An alternative possibility for reducing the complexity of defining a grammar consists of directly supplying the term set by considering all terms as primary ones and distributed on a scale on which a total order is defined [4,13,25,52,55]. For example, a set of seven terms S could be given as follows:

$$S = \{s_0 = none,\ s_1 = very\ low,\ s_2 = low,\ s_3 = medium,\ s_4 = high,\ s_5 = very\ high,\ s_6 = perfect\},$$

in which $s_a < s_b$ iff $a < b$. Usually, in these cases, it is often required that the linguistic term set satisfies the following additional characteristics:

(1) There is a negation operator, e.g., $Neg(s_i) = s_j$, $j = T - i$ ($T + 1$ is the cardinality).
(2) Maximization operator: $\mathrm{Max}(s_i, s_j) = s_i$ if $s_i \geqslant s_j$.
(3) Minimization operator: $\mathrm{Min}(s_i, s_j) = s_i$ if $s_i \leqslant s_j$.

In most cases, as is shown in the following subsection, the semantic of the ordered terms is defined in a special way, by using as the basis some of these requested properties and the total order established in the linguistic term set.

2.2. The semantic of the linguistic term set

In the literature, we can mainly find three possibilities for defining the semantic of the linguistic term set: *semantic based on membership functions and a semantic rule*, *semantic based on the ordered structure of the linguistic term set*, *mixed semantic*. These are analyzed in the following subsections.

2.2.1. Semantic based on fuzzy sets and a semantic rule

This semantic approach assumes that the meaning of each linguistic term is given by means of a fuzzy subset defined in the [0, 1] interval, which are usually described by membership functions [2,3,7,14,35,43]. A computationally efficient way to characterize a fuzzy number is to use a representation based on parameters of its membership function [1]. Because the linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible and unnecessary to obtain more accurate values [2,14,43,44]. This parametric representation is achieved by the 4-tuple $(a_i, b_i, \alpha_i, \beta_i)$. The first two parameters indicate the interval in which the membership value is 1; the third and fourth parameters indicate the left and right width [1]. For example, in [2] the following semantic is proposed for the set of nine terms (see Fig. 1).

$$C = Certain = (1, 1, 0, 0),$$
$$EL = Extremely\_Likely = (0.98, 0.99, 0.05, 0.01),$$
$$ML = Most\_Likely = (0.78, 0.92, 0.06, 0.05),$$
$$MC = Meaningful\_Chance = (0.63, 0.80, 0.05, 0.06),$$
$$IM = It\_May = (0.41, 0.58, 0.09, 0.07),$$
$$SC = Small\_Chance = (0.22, 0.36, 0.05, 0.06),$$
$$VLC = Very\_Low\_Chance = (0.1, 0.18, 0.06, 0.05),$$
$$EU = Extremely\_Unlikely = (0.01, 0.02, 0.01, 0.05).$$
$$I = Impossible = (0, 0, 0, 0).$$

Other authors use a non-trapezoidal representation, e.g., Gaussian functions [3].

Usually, this semantic approach is used when we generate the descriptors of the linguistic term set by means of a generative grammar. Thus, it is established by means of two elements: (i) the primary fuzzy sets associated to the primary linguistic terms and (ii) a semantic rule M for generating the fuzzy sets of the non-primary linguistic terms from primary fuzzy sets. Then, while the primary terms are labels of primary fuzzy sets, the rest may be seen as labels of different kinds of operators which act on the primary fuzzy sets, modifying their original membership distributions [1,3,56]. Therefore, while the semantic for the primary terms is both subjective and context dependent, the semantic for the non-primary terms is deduced by applying the semantic rule M. M deals with hedges, connectives and relations, as operators which modify the meaning of the primary terms. However, this approach implies first establishing the primary fuzzy sets associated with each term and the semantic rule that modifies them, and these tasks present two problems:

1. In the representations of primary fuzzy sets based on parameters, the problem is how to determine the parameters according to all users' attitudes.

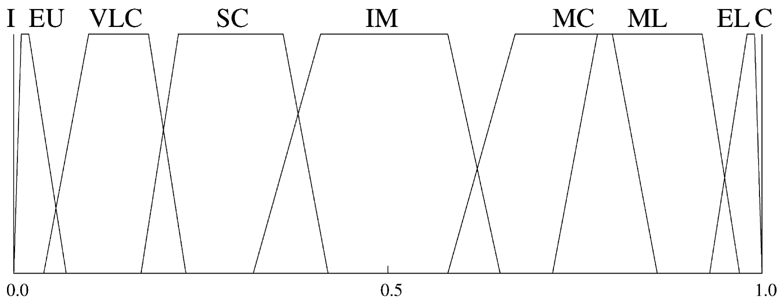

Fig. 1. A set of nine terms with its semantic.

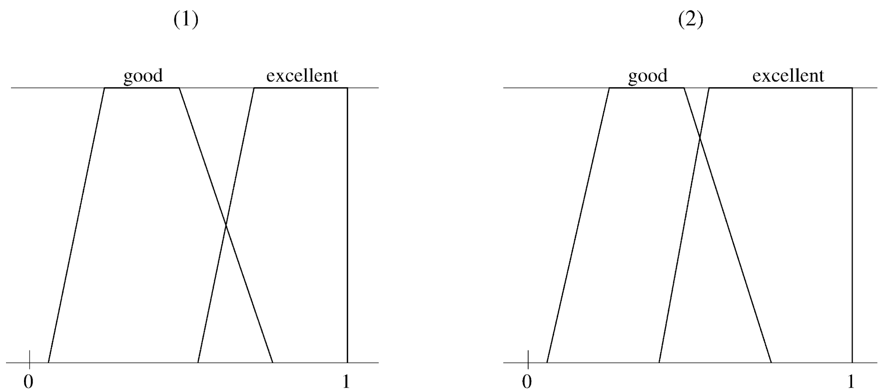

Fig. 2. Different distribution concepts.

Formally speaking, it seems difficult to accept that all users should agree to the same membership functions associated with primary linguistic terms, and therefore, there is no universal distribution of concepts. For example, in Fig. 2 two close perceptions of the same evaluation are shown. Therefore, we can find some situations where primary linguistic term sets with a similar syntax and different semantic are used to evaluate them [21]. Furthermore, it is not always possible for the user to define a fuzzy set for each primary linguistic term because it requires an excess of accuracy that the user cannot always supply. Hence, many times an environment is considered where users can perfectly discriminate the same linguistic term set under a similar conception, taking into account that the concept of a linguistic variable serves the purpose of providing a measure of an approximated characterization of information for an imprecise preference [22].

2. As in our decision framework, we deal with preferences, the semantic rule must be defined in such a way that its application modifies the supports of the primary fuzzy sets.

2.2.2. *Semantic based on the ordered structure of the linguistic term set*

An alternative possibility, which does not use fuzzy sets, introduces the semantic from the structure defined over the linguistic term set. In particular, this happens when the users provide their assessments by using an ordered linguistic term set [4,5,45,52,55]. Under this semantic approach, depending on the distribution of the linguistic terms on a scale ($[0,1]$), there are two possibilites for defining the semantic of the linguistic term set:

1. *Assuming symmetrically distributed terms.* It assumes ordered linguistic term sets which are distributed on a scale, as was mentioned, with an odd cardinal and the mid term representing an assessment of "approximately 0.5" and with the rest of the terms being placed symmetrically around it. Then, the semantic of the linguistic term set is

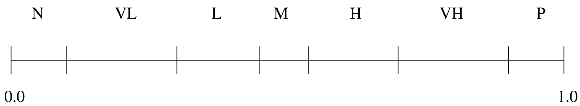

Fig. 3. A symmetrically distributed ordered set of seven linguistic terms.

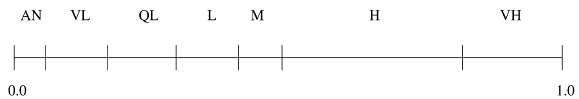

Fig. 4. A non-symmetrically distributed ordered set of seven linguistic terms.

established from the ordered structure of the term set by considering that each linguistic term for the pair (s_i, s_{T-i}) is equally informative [45]. This proposal may be explicitly defined by assigning a subdomain of the reference domain $[0,1]$ to each linguistic term [4,5,52,55] (see Fig. 3).

2. *Assuming non-symmetrically distributed terms.* It assumes that a subdomain of the reference domain may be more informative than the rest of the domain [45]. In this case, the density of linguistic labels in that subdomain would be greater than the density in the rest of the reference domain, i.e., the ordered linguistic term set would not be symmetrically distributed. For instance, suppose that we require a temperature control system with a very precise behavior when the temperature is "Low". Therefore, the linguistic term set would have a distribution over the reference domain similar to that in Fig. 4, (in Fig. 4, AN = Almost-Nil and QL = Quite-Low).

For these situations, in [45] a method was proposed that induces the semantic (the subdomains) by using a negation function defined from the linguistic term set to parts of it. This method is able to establish a semantic for the linguistic term set if the user gives the values of a negation function for each linguistic term. For instance, for the linguistic term set given in Fig. 4, the following negation function may be defined [45]:

$Neg(AN) = Neg(VL) = \{VH\}$,

$Neg(QL) = Neg(L) = \{H\}$,

$Neg(M) = \{M\}$,

$Neg(H) = \{QL, L\}$,

$Neg(VH) = \{AN, VL\}$.

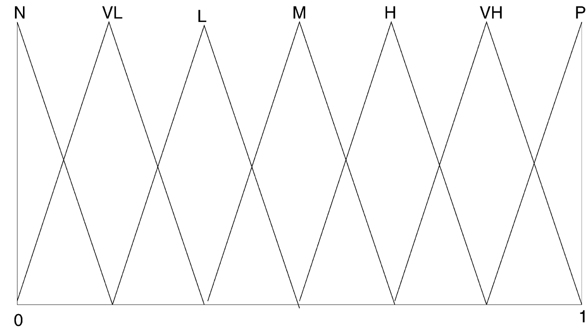

Fig. 5. An uniformly distributed ordered set of seven terms with its semantic.

2.2.3. Mixed semantic

In this semantic approach all linguistic terms are considered primary. It assumes elements from the aforementioned semantic approaches, i.e., an ordered structure of the primary linguistic terms and fuzzy sets for the semantic of the linguistic terms. On the one hand, as in Section 2.2.2, ordered linguistic term sets are assumed which are distributed on a scale, with an odd cardinal and the mid-term representing an assessment of "approximately 0.5", with the rest of the terms being placed symmetrically around it, and assuming that each linguistic term for the pair (s_i, s_{T-i}) is equally informative. On the other hand, as in Section 2.2.1, it defines the semantic of the primary linguistic terms by means of the fuzzy sets represented by trapezoidal or triangular membership functions [13,23,21,25–28]. These membership functions may be uniformly distributed [21] (see Fig. 5) or not [23] (see Fig. 1).

$P = Perfect = (1, 1, 0.16, 0)$,

$VH = Very\_High = (0.84, 0.84, 0.18, 0.16)$,

$H = High = (0.66, 0.66, 0.16, 0.18)$,

$M = Medium = (0.5, 0.5, 0.16, 0.16)$,

$L = Low = (0.34, 0.34, 0.18, 0.16)$,

$VL = Very\_Low = (0.16, 0.16, 0.16, 0.18)$.

$N = None = (0, 0, 0, 0.16)$.

3. The choice of the aggregation operator of linguistic information

In this section, we analyze some linguistic aggregation methods, with their advantages and drawbacks, and present different kinds of linguistic aggregation operators to solve the problem of choosing of a linguistic aggregation operator.

There are two main approaches for carrying out the aggregation of linguistic information. The first one, called *the approximation approach*, uses the membership functions associated with the linguistic terms [1,12,14,16,43,44,56] whereas the second one, called *the symbolic approach*, acts by direct computation on linguistic terms [6,15,19,31,46,50,51]:

1. Approximation approach. The use of the linguistic opertors based on the associated membership functions presents two major problems, namely [1]:
   (a) *How to perform arithmetic operations with fuzzy sets*? *Fuzzy Sets Theory* provides the logical operators (or, and, not, $\rightarrow$) used to build the linguistic model together with the *Extension Principle* that provides the mathematical tool to perform any arithmetic [1,56]. However, the implementation of the Extension Principle generates computational problems in any case, e.g., it enables any non-fuzzy function to accept fuzzy sets as arguments and the resulting function value is also a fuzzy set with a single membership function [1]. The classical solution to this problem consists of using a representation based on parameters for the fuzzy set, and then, defining the arithmetic operations on the basis of these parameters without using the Extension Principle. For example, in this sense, a table of basic arithmetic operations is given in [1].
   (b) *How to associate a linguistic term with an unlabeled fuzzy set on the basis of the semantic similarity* ("*linguistic approximation*")? On the other hand, it is well known that by using extended arithmetic operations to handle fuzzy sets the vagueness of the results increases step by step, and the shape of the membership functions does not hold when the linguistic variables are interactive. Thus, the final results of those methods are fuzzy sets which do not correspond to any label in the original linguistic term set. If we desire to have a label finally, a linguistic approximation is needed [12,43]. This linguistic approximation consists of finding a label whose meaning is the same or the closest (according to some metric) to the meaning of the unlabeled fuzzy set generated by the linguistic computational model. There is no general method for associating a label with a fuzzy set, so specific problems may require specifically developed methods. A review of some methods for the linguistic approximation may be found in [12].
2. Symbolic approach. It acts by direct computation on labels by taking into account the meaning and features of such linguistic assessments. It works assuming that the linguistic term set is an ordered structure uniformly distributed on a scale. These methods seem natural when the linguistic approach is used, because the linguistic assessments are just approximations which are given and handled when it is impossible or unnecessary to obtain of more accurate values. Thus, in this case, the use of membership functions associated to the linguistic terms is unnecessary. Furthermore, they are computationally simple and quick [15].

   Of course, in this approach as in the one above, there is a lack of precision in the results obtained, but it is accepted insofar as these approaches are tools to model non-numerically precise situations.

In the following subsection, we review the kinds of aggregation operators of linguistic information existing in the literature.

3.1. Aggregation operators of linguistic information

In the literature, we can find four kinds of aggregation operators of linguistic information: (i) *aggregation operators of linguistic non-weighted information*, (ii) *aggregation operators of linguistic weighted information*, (iii) *aggregation operators of multi-granularity linguistic information*, (iv) *aggregation operators of numeric and linguistic information.* They are briefly analyzed in the following subsections.

3.1.1. Aggregation operators of linguistic non-weighted information

These operators aggregate linguistic information provided for different criteria with equal importance or relevance, i.e., all criteria are considered equally valuable in the aggregation process.

An example of the aggregation operator of linguistic non-weighted information can be found in [15,24,31,46,50,51]. Among them, we show one of them, the *Linguistic Ordered Weighted Averaging* (*LOWA*) [31] operator. The LOWA operator is a symbolic operator, it presents very good properties (e.g. it is increasingly monotonous, commutative and an "or-and" operator) and has multiple applications [22,25,30].

Definition 2. Let $A=\{a_1,\dots,a_m\}$ be a set of labels to be aggregated, then the LOWA operator, ϕ, is defined as

$$\begin{aligned}\phi(a_1,\dots,a_m) &= W\cdot B^{\mathrm{T}} = \mathscr{C}^m\{w_k,b_k,\ k=1,\dots,m\}\\ &= w_1\odot b_1\oplus(1-w_1)\\ &\quad\odot\mathscr{C}^{m-1}\{\beta_h,b_h,\ h=2,\dots,m\},\end{aligned}$$

where $W=[w_1,\dots,w_m]$, is a weighting vector, such that, (i) $w_i\in[0,1]$ and, (ii) $\sum_i w_i=1$, $\beta_h=w_h/\sum_2^m w_k$, $h=2,\dots,m$, and $B=\{b_1,\dots,b_m\}$ is a vector associated to A, such that,

$$B=\sigma(A)=\{a_{\sigma(1)},\dots,a_{\sigma(n)}\}$$

in which, $a_{\sigma(j)}\leqslant a_{\sigma(i)}\ \forall i\leqslant j$, with σ being a permutation over the set of labels A. $\mathscr{C}^m$ is the convex combination operator of m labels and if $m=2$, then it is defined as

$$\mathscr{C}^2\{w_i,b_i,\ i=1,2\}=w_1\odot s_j\oplus(1-w_1)\odot s_i=s_k,$$

$$s_j,s_i\in S\ \ (j\geqslant i)$$

such that, $k=\min\{T,i+round(w_1\cdot(j-i))\}$, where "round" is the usual round operation, and $b_1=s_j$, $b_2=s_i$.

If $w_j=1$ and $w_i=0$ with $i\neq j\ \forall i$, then the convex combination is defined as

$$\mathscr{C}^m\{w_i,b_i,\ i=1,\dots,m\}=b_j.$$

How to calculate the weighting vector of LOWA operator, W, is a basic question to be solved. A possible solution is that the weights represent the concept of *fuzzy majority* [32] in the aggregation of LOWA operator using *fuzzy linguistic quantifier* [57]. Yager proposed an interesting way to compute the weights by means of a fuzzy linguistic quantifier, which, in the case of a non-decreasing proportional fuzzy linguistic quantifier Q is given by this expression [49]:

$$w_i=Q(i/n)-Q((i-1)/n),\quad i=1,\dots,n,$$

being the membership function of Q, as follows:

$$Q(r)=\begin{cases}0 & \text{if } r<a,\\ \dfrac{r-a}{b-a} & \text{if } a\leqslant r\leqslant b,\\ 1 & \text{if } r>b\end{cases}$$

with $a,b,r\in[0,1]$. Some examples of non-decreasing proportional fuzzy linguistic quantifiers are: "most" $(0.3,0.8)$, "at least half" $(0,0.5)$ and "as many as possible" $(0.5,1)$. When a fuzzy linguistic quantifier, Q, is used to compute the weights of LOWA operator, ϕ, it is symbolized by ϕ_Q.

3.1.2. Aggregation operators of linguistic weighted information

These operators aggregate linguistic information provided for different criteria which are not equally important. Usually, in order to design an aggregation operator of linguistic weighted information, we have to define two aggregations [11]:

- the aggregation of linguistic importance degrees of linguistic weighted information, and
- the aggregation of linguistic weighted information combined with the linguistic importance degrees.

The first aggregation consists of obtaining a collective importance degree from individual importance degrees which characterizes the final result of the aggregation operator [19]. On the other hand, in order to achieve the second aggregation linguistic, we have to combine linguistic weighted information with the linguistic importance degrees, i.e., it involves the transformation of the linguistic weighted information under the linguistic importance degrees by means of a transformation function g.

The transformation function depends upon the type of aggregation of weighted information which is going to be performed [54]. In [47,48] Yager discussed the effect of the importance degrees on the types of aggregation "MAX" and "MIN" and suggested a class of functions for importance transformation in both types of aggregation. For the MIN aggregation, he suggested a family of t-conorms acting on the weighted information and the negation of the importance degree, which presents the non-increasing monotonic property in these importance degrees. For the MAX aggregation, he suggested a family of t-norms acting on weighted information and the importance degree, which presents the non-decreasing monotonic property in these importance degrees.

A general specification of the requirements that any importance transformation function g must satisfy for any type of the aggregation operator is also proposed in [54]. The function g must have the following properties:

1. if $a > b$ then $g(v,a) \geqslant g(v,b)$,
2. $g(v,a)$ is monotone in v,
3. $g(0,a) = \mathrm{ID}$,
4. $g(1,a) = a$

with $a,b \in [0,1]$ expressing the satisfaction with regard to a criterion, $v \in [0,1]$ the importance degree associated with the criterion, and "ID" an identity element, which is such that if we add it to our aggregations it does not change the aggregated value. The first condition means that the function g is monotonically non-decreasing in the second argument, that is, if the satisfaction with regard to the criteria increases the overall satisfaction should not decrease. The second condition may be viewed as a requirement of the effect of the importance of being consistent. It does not specify whether g is monotonically non-increasing or non-decreasing in the first argument. It should be noted that conditions three and four actually determine the type of monotonicity obtained from two. If $a > \mathrm{ID}$, then $g(v,a)$ is monotonically non-decreasing in v, while if $a < \mathrm{ID}$, then it is monotonically non-increasing. The third condition is a manifestation of the imperative that zero importance items do not affect the aggregation process. The final condition is essentially a boundary condition which states that the assumption of all importances equal to one is effectively like not including importances at all [54].

According to the aforementioned ideas, in [19] we presented the *Linguistic Weighted Averaging* (LWA) operator. Other proposals are to be found in [4,6,46,50–52,54,55].

3.1.3. Aggregation operators of multi-granularity linguistic information

These operators aggregate linguistic information assessed in different linguistic expression domains, which present different granularity and/or semantic. The multi-granularity linguistic information is linguistic information assessed in linguistic term sets with a different granularity and/or semantic. We find this situation when the linguistic performance values are not provided using the same linguistic term set. In [21] a proposal was given to deal with these situations.

An aggregation operator of multi-granularity linguistic information has to carry out two tasks [21]:

- making the multi-granularity linguistic information uniform,
- computing the collective or aggregated linguistic performance value from uniform linguistic information.

3.1.4. Aggregation operators of numerical and linguistic information

These operators aggregate linguistic information with numerical ones. They are applied when some performance values are given in a numerical domain, and others in a linguistic one. In [13] a first approach was presented for dealing with these situations. We defined a fusion operator of linguistic (assessed in the same linguistic term set) and numerical information (assessed in the interval $[0,1]$) using the concept of *characteristic values* associated to a fuzzy number.

The characteristic values are crisp ones that summarize the information given by a fuzzy number, i.e., they support its meaning. They allow us to define some transformation functions between different expression domains, i.e., linguistic-numerical transformation functions, which obtain a numerical value from a linguistic term, and numerical-linguistic transformation functions, which obtain a label from a numerical value. Using these transformation functions, in [13] a fusion operator was presented which acts in three steps:

1. it transforms all inputs into a usual linguistic intermediate domain by means of a particular numerical-linguistic transformation function,

2. the transformed information is aggregated by means of a concrete linguistic aggregation operator, and finally,
3. the output information is expressed in each user's expression domain, using an appropriate linguistic-numerical transformation function.

4. The choice of the best alternatives

Assuming a linguistic framework, in an MCDM problem we have linguistic performance values $\{V_1, \dots, V_m\}$ about a set of alternatives $X = \{x_1, \dots, x_n\}$ provided according to a group of criteria $\{P_1, \dots, P_m\}$. Then, the goal consists of finding the best alternatives from the linguistic performance values. This task is achieved by means of a choice process between the alternatives [29]. As is known, basically two approaches may be considered to carry out a choice process [22,32]. A direct approach

$$\{V_1, \dots, V_m\} \to \text{ the best alternatives}$$

according to which, on the basis of the individual preferences, a solution with the best alternatives is derived, and an indirect approach

$$\{V_1, \dots, V_m\} \to V^c \to \text{ the best alternatives}$$

providing the best alternatives on the basis of a collective preference, V^c, which is a preference of the group of criteria as a whole. Here, we assume an indirect approach.

As was aforementioned earlier, the proposed choice process is carried out in two phases: (i) aggregation phase of linguistic information and (ii) the exploitation phase for the aggregated linguistic information. In the following subsections, we analyze both phases.

4.1. Aggregation phase

The goal of the aggregation phase in the linguistic decision analysis of an MCDM problem is to obtain a collective linguistic performance value V^c from the individual ones $\{V_1, \dots, V_m\}$, provided for the criteria, using the aggregation operator chosen in the previous step.

At the beginning of linguistic decision analysis, we should establish what kind of representation to use for providing the linguistic performance values. Traditionally, the linguistic preferences can be provided in either of these two ways [24]:

- *Linguistic preference relation.* In this case, for a criterion a linguistic preference relation is supplied over the set of alternatives $V_k = v_{ij}^k$, reflecting each element of the relation v_{ij}^k, the linguistic degree to which an alternative x_i is prefered to another x_j [22,26].
- *Linguistic utility function.* In this case, for each criterion a utility function $V_k = [v_1^k, \dots, v_n^k]$ is supplied that associates each alternative x_j with a linguistic value v_j^k indicating the performance of that alternative [4,18,52,55].

Thus, if the linguistic performance values $\{V_1, \dots, V_m\}$ are linguistic utility functions then V^c will be a collective linguistic utility function and if $\{V_1, \dots, V_m\}$ are linguistic preference relations then V^c will be a collective linguistic preference relation.

4.2. Exploitation phase

The goal of the exploitation phase is to choose the best alternatives from V^c. Usually, the exploitation is modeled using *choice functions* which allow us to characterize the alternatives and to separate the best alternatives [20,39,40]. Each alternative is characterized by means of a choice degree calculated from a collective performance value and, in such a way, a rank ordering among the alternatives is defined. Later, the alternatives with the maximum choice degree are chosen. Therefore, assuming a linguistic framework, the exploitation step consists of two tasks:

1. Obtain a rank ordering among the alternatives by means of a *linguistic choice function* defined from the collective linguistic performance value V^c. In such a way, a *linguistic choice set of alternatives* is obtained:

   $$X^c = \{(x_j, \mu_{X^c}(x_j)),\ j = 1, \dots, n\}$$

   and

   $$\mu_{X^c} : X \to S.$$

2. Choose the best alternatives according to the established rank ordering. Here, a *solution set of alternatives* is obtained as follows:

   $$X^s = \left\{ x_i \in X \mid \mu_{X^c}(x_i) = \operatorname*{Max}_{x_j \in X} \{\mu_{X^c}(x_j)\} \right\}.$$

The definition of a linguistic choice function depends on the type of representation chosen initially to provide the linguistic performance values. If the linguistic performance values are linguistic utility functions, then V^c will be a collective linguistic utility function and to establish a rank ordering is a direct and easy process since V^c is itself a linguistic choice function [4,18,52,55], i.e., $V^c = X^c$. Then, the solution set of alternatives is obtained as follows:

$$X^s = \left\{ x_i,\ x_i \in X \mid V^c(x_i) = \operatorname*{Max}_{x_j \in X} \{V^c(x_j)\} \right\}.$$

However, if linguistic performance values are linguistic preference relations then V^c will be a collective linguistic preference relation, i.e., $V^c = [v^c_{ij}]$, $\forall x_i, x_j \in X$, and, in this case, to establish a rank ordering is not an easy and direct task. Mainly, we find two problems [20]:

- on the one hand, we have to define a linguistic choice function for a linguistic preference relation and there are different approaches to do so, and
- on the other hand, different solution sets of alternatives can be derived from different linguistic choice functions and we have to achieve a consensus among them.

In the following subsection, we briefly analyze the exploitation phase for a linguistic preference relation assessed in an ordered linguistic term set S.

4.2.1. Exploitation for a linguistic preference relation

Four classical linguistic choice sets of alternatives can be defined for a linguistic preference relation V^c [20]:

1. *A linguistic choice set of greatest alternatives* which assigns a linguistic choice degree of "greatestness" to each alternative $x_j \in X$ with respect to V^c, according to the following expression:

   $$\mu_{X^c} : X \to S, \quad \mu_{X^c}(x_j) = \nabla(v^c_{ji},\ i = 1,\dots,n),$$

   where, traditionally, ∇ is an aggregation operator modeling the linguistic conjunctions.
2. *A linguistic choice set of non-dominated alternatives* which assigns a linguistic choice degree of "non-domination" to each alternative $x_j \in X$ with respect to V^c, according to the following expression:

   $\mu_{X^c} : X \to S,$

   $\mu_{X^c}(x_j) = \nabla(Neg(v^c_{ij}),\ i = 1,\dots,n).$
3. *A linguistic choice set of strictly greatest alternatives* which assigns a linguistic choice degree of "strict greatestness" to each alternative $x_j \in X$ with respect to V^c and depending on some linguistic conjunction operator, according to the following expression:

   $\mu_{X^c} : X \to S,$

   $\mu_{X^c}(x_j) = \nabla(LC^{\to}(Neg(v^c_{ij}), v^c_{ji}),\ i = 1,\dots,n).$

   Some examples of $LC^{\to}$ are [19]:
   (a) *The classical Min linguistic conjunction function*:

   $LC^{\to}_1(w,a) = \mathrm{Min}(w,a).$

   (b) *The nilpotent Min linguistic conjunction function*:

   $$LC^{\to}_2(w,a) = \begin{cases} \mathrm{Min}(w,a) & \text{if } w > Neg(a), \\ s_0 & \text{otherwise.} \end{cases}$$

   (c) *The weakest linguistic conjunction function*:

   $$LC^{\to}_3(w,a) = \begin{cases} \mathrm{Min}(w,a) & \text{if } \mathrm{Max}(w,a) = s_T, \\ s_0 & \text{otherwise.} \end{cases}$$
4. *A linguistic choice set of maximal alternatives* which assigns a linguistic choice degree of "maximality" to each alternative $x_j \in X$ with respect to V^c and depending on some linguistic implication operator $LI^{\to}$, according to the following expression:

   $\mu_{X^c} : X \to S,$

   $\mu_{X^c}(x_j) = \nabla(LI^{\to}(v^c_{ij}, v^c_{ji}),\ i = 1,\dots,n).$

   Some examples of $LI^{\to}$ are [19]:
   (a) *Kleene–Dienes's linguistic implication function*:

   $LI^{\to}_1(w,a) = \mathrm{Max}(Neg(w), a).$

   (b) *Gödel's linguistic implication function*:

   $$LI^{\to}_2(w,a) = \begin{cases} s_T & \text{if } w \leqslant a, \\ a & \text{otherwise.} \end{cases}$$

(c) *Fodor's linguistic implication function*:

$$LI_3^{\rightarrow}(w,a)=\begin{cases} s_T & \text{if } w \leqslant a,\\ \operatorname{Max}(Neg(w),a) & \text{otherwise.}\end{cases}$$

(d) *Lukasiewicz's linguistic implication function*:

$$LI_4^{\rightarrow}(w,a)=\begin{cases} s_T & \text{if } w < a,\\ Neg(w-a) & \text{otherwise,}\end{cases}$$

where $w-a=s_h\in S$ with $w=s_l$, $a=s_t$ and $l=t+h$.

A linguistic choice function for a linguistic preference relation V^{c} in X is a fuzzy set in X defined as

$$C(X,V^{\mathrm{c}})=\{(x_i,\mu_{C(X,V^{\mathrm{c}})}(x_i))\},$$

where $\mu_{C(X,V^{\mathrm{c}})}: S^n \to S$ is a linguistic membership function that assigns a linguistic choice degree to each alternative $x_i\in X$ with respect to V^{c}, according to an expression. Therefore, a linguistic choice function is a generalization of the linguistic choice sets of alternatives, and obviously $\mu_{C(X,V^{\mathrm{c}})}=\mu_{X^{\mathrm{c}}}$. In [20] we may find a complete set of linguistic choice functions.

In many cases, we find that the choice and solution sets of alternatives provided by different linguistic choice functions are very general and different, i.e., we may find a *problem of specificness and consensus* among the solutions provided by different linguistic choice functions. We propose to solve this problem by means of the distinction between two types of linguistic choice mechanisms or ways for applying choice functions [20]:

1. *Simple linguistic choice mechanisms.* They use only one linguistic choice function to obtain the solution set of alternatives. Therefore, this method obtains the solution as

$$X^{\mathrm{s}}=\left\{x_j\in X \mid \mu_{X^{\mathrm{c}}}(x_j)=\operatorname*{Max}_{x_i\in X}\ \mu_{X^{\mathrm{c}}}(x_i)\right\},$$

   i.e., those alternatives with the maximum linguistic choice degree.
2. *Composite linguistic choice mechanisms.* They use various linguistic choice functions to obtain the solution set of alternatives. They are applied when the solution obtained by the application of a simple mechanisms is not precise or specific enough. Futhermore, they also are useful when different simple mechanisms provide very different solutions. Therefore, we may say that a composite mechanism performs a consensus process between different choice functions with a view to achieving more specific solutions. Then, given a set of linguistic choice functions, $\{X_1^{\mathrm{c}},\dots,X_T^{\mathrm{c}}\}$, a composite linguistic choice mechanism obtains the solution set of alternatives, X^{s}, by the combined application of all linguistic choice functions. Usually, the combined application can be done following two different policies:
   (a) *Conjunctive policy*: This policy consists of applying, in a parallel way, all the simple choice mechanisms from each choice function [29], i.e., it obtains the total solution as the intersection of the partial solutions according to the following expression:

$$X^{\mathrm{s}}=\bigcap_{t=1}^{T} X_t^{\mathrm{s}}.$$

   We should point out the existence of a problem, i.e., when it is verified that $\bigcap_{t=1}^{T} X_t^{\mathrm{s}}=\emptyset$. In such a situation, it is necessary to apply another choice policy like the following one.
   (b) *Sequential policy*: This policy consists of applying each one of the simple choice mechanisms of each choice function in sequence according to a previously established order [22]. Therefore, suppose that we have T simple linguistic choice mechanisms, then the total solution is obtained according to the following expression:

$$X^{\mathrm{s}}=\left\{x_j\in X_{T-1}^{\mathrm{s}} \mid \mu_{X_T^{\mathrm{c}}}(x_j) = \operatorname*{Max}_{x_i\in X_{T-1}^{\mathrm{s}}}\{\mu_{X_T^{\mathrm{c}}}(x_i)\}\right\},$$

$$X_{T-1}^{\mathrm{s}}=\left\{x_j\in X_{T-2}^{\mathrm{s}} \mid \mu_{X_{T-1}^{\mathrm{c}}}(x_j) = \operatorname*{Max}_{x_i\in X_{T-2}^{\mathrm{s}}}\{\mu_{X_{T-1}^{\mathrm{c}}}(x_i)\}\right\},$$

$$\vdots$$

$$X_1^{\mathrm{s}}=\left\{x_j\in X \mid \mu_{X_1^{\mathrm{c}}}(x_j)=\operatorname*{Max}_{x_i\in X}\{\mu_{X_1^{\mathrm{c}}}(x_i)\}\right\}.$$

On the other hand, we should also point out that due to the lack of a transitivity property in the collective linguistic preference relation, sometimes a *problem of* *consistency* appears in the solution set of alternatives. Then, a *complete consistent linguistic choice mechanisms*, based on the concept of *linguistic covering relation* to find more precise and coherent solution sets of alternatives, may be defined as:

1. Obtain a linguistic covering relation for V^c, called $CC(V^c)$, which is a transitive linguistic preference relation.

$$\forall (x_i, x_j) \in X^2 \quad CC(V^c)(x_i, x_j) = \mathrm{Min}\{FC(V^c)(x_i, x_j), BC(V^c)(x_i, x_j)\},$$

   where $FC(V^c)$ and $BC(V^c)$ are the linguistic forward and backward covering relations of V^c defined as

$$FC(V^c) = \begin{cases} s_T & \text{if } \forall x_h \in X,\ v^c_{jh} \leqslant v^c_{ih}, \\ \underset{\{x_h \in X \mid v^c_{jh} > v^c_{ih}\}}{\mathrm{Min}} \{v^c_{ih}\} & \text{otherwise} \end{cases}$$

   and

$$BC(V^c) = \begin{cases} s_T & \text{if } \forall x_h \in X,\ v^c_{hi} \leqslant v^c_{hj}, \\ \underset{\{x_h \in X \mid v^c_{hi} > v^c_{hj}\}}{\mathrm{Min}} \{v^c_{hj}\} & \text{otherwise,} \end{cases}$$

   respectively.
2. Choose various simple linguistic choice mechanisms.
3. Apply a *conjunction linguistic choice mechanism* on $CC(V^c)$.
4. If $X^s = \emptyset$ then apply a *sequential linguistic choice mechanism* on $CC(V^c)$. Otherwise, X^s is the solution.

In the following section, we study the application of linguistic decision analysis in a particular MCDM problem.

5. Example

Let us suppose an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible options in which to invest the money:

- x_1 is a car company,
- x_2 is a food company,
- x_3 is a computer company,
- x_4 is an arms company.

The investment company must make a decision according to four criteria:

- P_1 is the risk analysis,
- P_2 is the growth analysis,
- P_3 is the social–political impact analysis, and
- P_4 is the environmental impact analysis.

Below, we show how to model this MCDM problem following an indirect linguistic approach.

5.1. The choice of a linguistic term set with its semantic

We consider the second possibility shown in Section 2, which defines the linguistic expression domain by means of an ordered set of linguistic terms. Then, we characterize the linguistic expression domain as follows:

- The value of granularity chosen is 7.
- We consider a linguistic term set on which a total order is defined and distributed on the scale [0, 1], with the mid term representing an assessment of "approximately 0.5", with the rest of the terms being placed symmetrically around it.
- We define the semantic by considering that each linguistic term for the pair (s_i, s_{6-i}) is equally informative and by assigning triangular membership functions to each linguistic term.
- Furthermore, we assume a negation operator, a maximization one and a minimization one defined in S, as was shown in Section 2.

For example, we can use the set of seven linguistic terms shown in Fig. 1, i.e.,

$$S = \{s_6 = P,\ s_5 = VH,\ s_4 = H,\ s_3 = M,\ s_2 = L,\ s_1 = VL,\ s_0 = N\}.$$

On the other hand, we assume that for each criterion, linguistic performance values about the alternatives are provided by means of reciprocal linguistic preference relations ($v^k_{ij} = Neg(v^k_{ji})$ and $v^k_{ii} = -$) [22], i.e.,

$$V_1 = \begin{bmatrix} - & VL & VH & VL \\ VH & - & H & H \\ VL & L & - & VL \\ VH & L & VH & - \end{bmatrix},$$

$$V_2 = \begin{bmatrix} - & L & H & VL \\ H & - & VH & L \\ L & VL & - & VL \\ VH & H & VH & - \end{bmatrix},$$

$$V_3 = \begin{bmatrix} - & M & VH & N \\ M & - & VH & L \\ VL & VL & - & VL \\ P & H & VH & - \end{bmatrix},$$

$$V_4 = \begin{bmatrix} - & L & VH & VL \\ H & - & L & VL \\ VL & H & - & VL \\ VH & VH & VH & - \end{bmatrix}.$$

5.2. *The choice of aggregation operator of linguistic information*

An aggregation operator of linguistic non-weighted information, in particular, the LOWA operator ϕ_Q presented in Section 3.1.1, is used to aggregate the individual linguistic performance values. It is an operator guided by a fuzzy linguistic quantifier, Q, representing the concept of "*fuzzy majority*" [24]. We propose to use the linguistic quantifier "*At least half*" with the pair $(0, 0.5)$. For the LOWA operator this quantifier establishes the following weighting vector: $W = [0.5, 0.5, 0, 0]$.

5.3. *Choice process*

5.3.1. *Aggregation phase of linguistic information*

Using this aggregation operator the collective linguistic preference relation obtained is the following:

$$V^{\mathrm{c}} = \begin{bmatrix} - & L & VH & VL \\ H & - & VH & M \\ VL & M & - & VL \\ VH & H & VH & - \end{bmatrix}.$$

5.3.2. *Exploitation phase*

Applying the following linguistic choice function [20]:

$$\mu_{X^{\mathrm{c}}} : X \to S, \quad \mu_{X^{\mathrm{c}}}(x_j) = \mathrm{Min}(v^{\mathrm{c}}_{ji},\ i = 1,\ldots,n,\ i \neq j),$$

we obtain the following choice set of alternatives, which is a choice set of greatest alternatives

$$X^{\mathrm{c}} = \{(x_1, VL), (x_2, M), (x_3, VL), (x_4, H)\}.$$

Then, the rank ordering among the alternatives is (x_4, x_2, x_1, x_3), and thus, the alternative, x_4, is the best assessed one, i.e., the solution set of alternatives is

$$X^{\mathrm{s}} = \{x_4\}.$$

6. Conclusions

In this paper, we have analyzed the steps to follow in the linguistic decision analysis of an MCDM problem, showing different approaches to model this problem linguistically.

The use of linguistic models in decision problems is highly beneficial when the performance values cannot be expressed by means of numerical values. The linguistic approach gives a more flexible framework to deal with decision problems using qualitative information.

As we said at the beginning, an important aspect of linguistic decision analysis is its applicability and usefulness in different decision frameworks. Therefore, it is an appropriate tool to model qualitative information in multiple real-world decision situations.

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