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A Linguistic Decision Process in Group Decision Making

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Abstract

Assuming a set of linguistic preferences representing the preferences of the individuals, a linguistic choice process is presented. This is developed using the concept of fuzzy majority for deriving a collective linguistic preference; and the concept of nondominated alternatives for deriving the selected alternatives in the linguistic choice process. The fuzzy majorities are equated with fuzzy linguistic quantifiers. The collective linguistic preference is derived by means of a linguistic ordered weighted averaging operator whose weights are defined using a fuzzy linguistic quantifier. In order to obtain the nondominated alternatives we present a novel reformulation of Orlovski's nondominance degree under linguistic information.

Keywords: Group decision making, fuzzy logic, linguistic preferences, fuzzy majority, fuzzy linguistic quantifiers, nondominance degree.

1. Introduction

A group decision making process can be defined as a decision situation: (i) there are two or more individuals, each of them characterized by his or her own perceptions, attitudes, motivations, and personalities; (ii) all of them recognize the existence of a common problem; and (iii) they attemp to reach a collective decision (Bui 1987).

The use of preference relations is usual in group decision making. Moreover, since human judgments including preferences are often vague, fuzzy logic plays an important role in decision making. Several authors have provided interesting results on group decision making or social choice theory with the help of fuzzy sets. They proved that fuzzy sets provided a more flexible framework for discussing group decision making (Spillman and Spillman 1987; Kacprzyk and Roubens 1988; Kacprzyk 1990; Nurmi and Kacprzyk 1991; Kacprzyk, Fedrizzi and Nurmi 1992).

In a fuzzy environment it is supposed that there exists a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ as well as a finite set of individuals $N = \{1, \ldots, m\}$, and each individual $k \in N$ provides his preference relation on X, i.e., $P_k \subset$ XxX, and $\mu_{P_k}(x_i, x_j)$ denotes the degree of preference of alternative x_i over x_j, $\mu_{P_k}(x_i, x_j) \in [0, 1]$.

Sometimes, however, an individual may have vague information about the preference degree of the alternative x_i over x_j and cannot estimate his preference with an exact numerical value. Then a more realistic approach is to use linguistic assessments instead of numerical values, that is, to suppose that the variables (preference relations) in the problem are assessed by means of linguistic terms (Fedrizzi and Mich 1992; Yager 1992a; Delgado, Verdegay and Vila 1993a; Herrera and Verdegay 1993; Mich, Gaio and Fedrizzi 1993). A scale of certainty expressions (linguistically assessed) is presented to the individual who could then use it to describe his degree of certainty in a preference.

The aim of this paper is to present a linguistic decision process in group decision making. Assuming a set of linguistic preferences, representing the preferences of the individuals, we develop a linguistic decision process.

We define a linguistic ordered weighted averaging (LOWA) operator based on two concepts: the ordered weighted averaging operator (Yager 1988), and the convex combination of linguistic labels (Delgado, Verdegay and Vila 1993b). The fuzzy majority concept, represented by a fuzzy linguistic quantifier and the LOWA operator, is used to obtain a collective linguistic preference. Finally, the nondominated alternative concept is used for defining a linguistic nondominance degree, which allows us to obtain the solution alternative(s) in the choice process. Figure 1 summarizes the linguistic decision process.

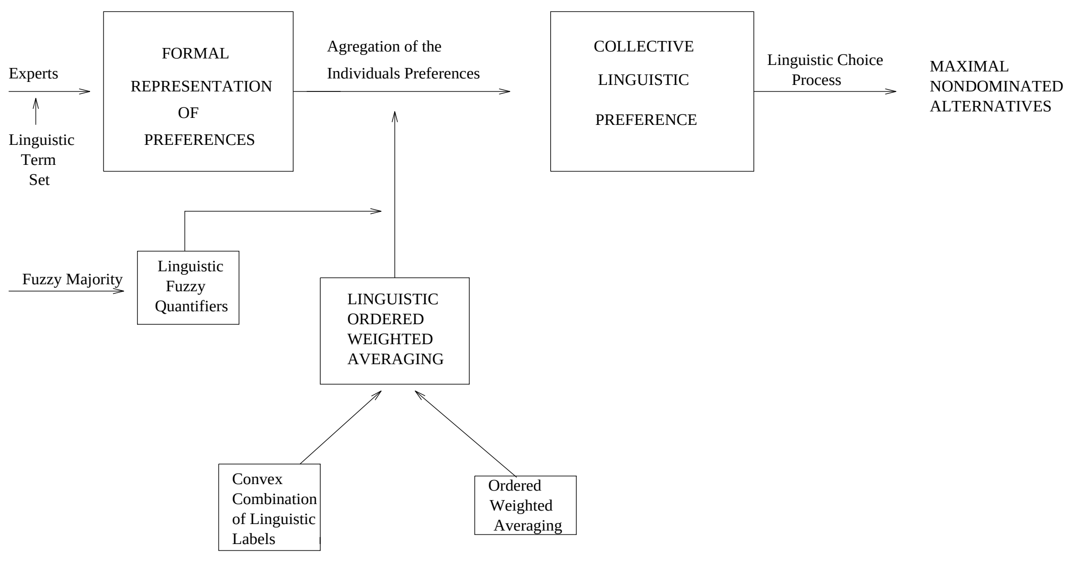

Fig. 1. Linguistic decision process

The paper is organized as follows: section 2 shows the linguistic approach; section 3 is devoted to the use of linguistic preference relations in group decision making and a discussion of the linguistic choice process; in section 4, an example is presented; and finally, concluding remarks in section 5.

2. The linguistic approach

2.1. Linguistic assessments

The linguistic approach assesses the variables in the problem by means of linguistic terms instead of numerical values (Zadeh 1975). This approach is appropriate for several problems, since it allows a representation of the experts' information in a more direct and adequate form, whether or not they are unable to express the preferences with precision.

We need a term set defining the uncertainty granularity, i.e. the finest level of distinction among different quantifications of uncertainty. The elements of the term set will determine the granularity of the uncertainty. Bonissone and Decker (1986) studied the use of term sets with odd cardinality, representing the middle term by a probability of "approximately 0.5", with the remaining terms placed symmetrically around it and the limit of granularity 11 or at most 13.

The semantic of the elements of the term set is given by fuzzy numbers defined in the [0,1] interval, described by membership functions. Provided that the linguistic assessments are estimates given by the experts or decision-makers, we can consider the linear trapezoidal membership functions good enough to capture the vagueness of those linguistic assessments. To obtain more accurate values may be impossible or unnecessary.

For instance, as an illustration, Figure 2 shows the hierarchical structure of linguistic values or labels.

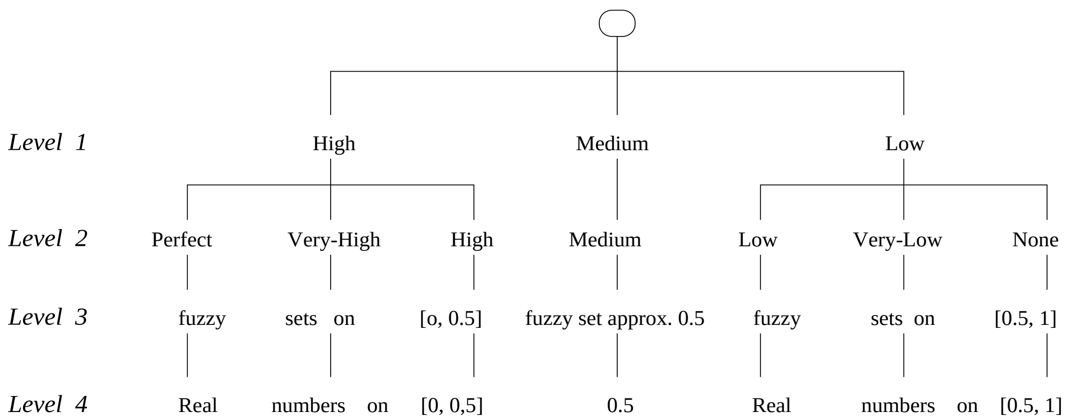

Fig. 2. Hierarchy of labels

Clearly, level 1 provides a granularity containing three elements, level 2 a granularity with nine elements. Different granularity levels can be presented. In Figure 2 the level 4 presents the finest granularity in a decision process, the numerical values.

Obviously, the kind of label set to be used ought to be established first. Let $S = \{l_i\}, i \in H = \{0, \ldots, T\}$ be a finite and totally ordered term set on [0,1] in the usual sense (Bonissone and Decker 1986; Delgado, Verdegay and Vila 1993a; Zadeh 1979). A label l_i represents a possible value for a linguistic real variable, that is, a vague property or constraint on [0,1]. The representation is achieved by the 4-tuple $(a_i, b_i, \alpha_i, \beta_i)$. The first two paremeters indicate the interval in which the membership value is 1.0 and the third and fourth parameters indicate the left and right width of the distribution. We consider a term set with odd cardinality, where the middle label represents an uncertainty of "approximately 0.5", and the remaining terms are placed symmetrically around it. Moreover, we require the following properties for the term set:

1) The set is ordered: $l_i \geq l_j$ if $i \geq j$.

2) The negation operator is defined as: $\text{Neg}(l_i) = l_j$ such that $j = T - i$.

3) $\text{Max}(l_i, l_j) = l_i$ if $l_i \geq l_j$.

4) $\text{Min}(l_i, l_j) = l_i$ if $l_i \leq l_j$.

For example, consider the term set of level 2:

$$S = \{l_6 = P, l_5 = VH, l_4 = H, l_3 = M, l_2 = L, l_1 = VL, l_0 = N\}$$

where

$$\begin{array}{lll} P = Perfect & VH = Very\_High & H = High \\ M = Medium & L = Low & VL = Very\_Low \\ N = None & & \end{array}$$

It is obvious that this term set verifies each of the above properties. Figure 3 shows a possible domain of the term set.

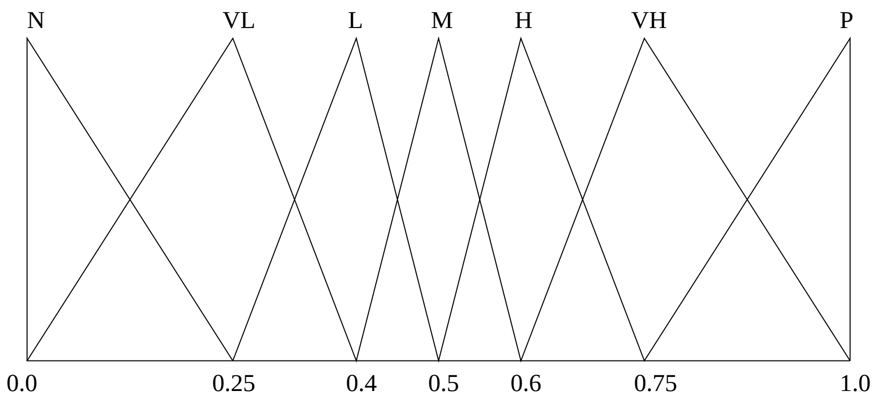

Fig. 3. Domain at level 2

The representation values are:

$P = (1,1,0,0)$ $VH = (.75,.75,.15,.25)$ $H = (.6,.6,.1,.15)$
$M = (.5,.5,.1,.1)$ $L = (.4,.4,.15,.1)$ $VL = (.25,.25,.25,.15)$
$N = (0,0,0,.25)$

2.2. Combination of linguistic values

Since aggregation of uncertainty information is a recurrent need in the decision process, combinations of linguistic values are needed. Two main different approaches are used to aggregate and compare linguistic values: the first acts by direct computation on labels (Delgado, Verdegay and Vila 1993b); and the second uses the associated membership functions. Most of the available techniques belong to the latter. However, the final results of these are fuzzy sets which do not correspond to any label in the original term set. To obtain a label a "linguistic approximation" is needed (Zadeh 1975, 1979; Bonissone and Decker 1986; Fedrizzi and Mich 1992). There are neither general criteria to evaluate the suitability of an approximation, nor general methods to associate a label with a fuzzy set, and therefore specific problems may require the development of tailored methods.

In the following, we present an aggregation operator of linguistic labels by direct computation on labels, based on the ordered weighted averaging (OWA) operator, (Yager 1988), and the convex combination of linguistic labels (Delgado, Verdegay and Vila 1993b).

A mapping F

$$I^n \rightarrow I (\text{where } I = [0,1])$$

is called an OWA operator of dimension n if it is associated with a weighting vector $W = [w_1, \ldots, w_n]$, such that: $i) w_i \in [0,1]$, ii)$\Sigma_i w_i = 1$, and $F(a_1, \ldots, a_n) = w_1 \cdot b_1 + w_2 \cdot b_2 + \ldots + w_n \cdot b_n$, where b_i is the i-th largest element in the collection $a_1, \ldots, a_n$. If we denote B as the vector consisting of the arguments of F in descending order,

$$F(a_1, \ldots, a_n) = W \cdot B^T$$

provides an aggregation type operator that always lies between the "and" and the "or" aggregation. Its properties were presented by Yager (1988).

Yager (1992b) extended the OWA operator to linguistic elements. Here, we will extend it to linguistic arguments using the convex combination of linguistic labels defined in Delgado et al. (1993b). In fact, let M be a collection of linguistic labels, $l_k \in M, k = 1, \ldots, m$, and assume without loss of generality $l_m \leq l_{m-1} \leq \ldots \leq l_1$. For any set of coefficients $\{\lambda_k \in [0,1], k = 1,2,..,m, \Sigma\lambda_k = 1\}$, the convex combination of these m generalized labels is the label given by

$$\mathbf{C}\{\lambda_k, l_k, k = 1, \ldots, m\} = \lambda_1 \odot l_1 \oplus (1 - \lambda_1) \odot \mathbf{C}\{\beta_h, l_h, h = 2, \ldots, m\}$$

with

$$\beta_h = \lambda_h / \sum_{2}^{n} \lambda_k; h = 2, \ldots, m.$$

Delgado et al. (1993b) define the aggregation of labels by addition, the difference of generalized labels, and the product by a positive real number over a *generalized label space* $\mathcal{S}$, based on S, that is, the cartesian product $\mathcal{S} = S\text{x}\mathbf{Z}^+$, with the basic label set $S = \{(l_i, 1), i \in H\}$. In our context all the operations are made over the basic set S. Briefly, the result of the expression $\lambda \odot l_j \oplus (1-\lambda) \odot l_i, j \geq i$, is l_k where $k = \min\{T, i + \text{round}(\lambda \cdot (j-i))\}$.

An example based on the term set of level 2 is the following:

		$1-\lambda = 0.6$			
		VL	**VH**	**P**	**VL**
$\lambda = 0.4$	**P**	M	VH	P	M
	N	VL	M	H	VL
	L	VL	M	H	VL
	M	L	H	VH	L

where, for example:

$$k_{11} = min\{6, 1 + round(0.4 * (6-1))\} = 3 \Rightarrow l_{k_{11}} = \text{M}$$
$$k_{21} = min\{6, 0 + round(0.6 * (1-0))\} = 1 \Rightarrow l_{k_{21}} = \text{VL}$$

Therefore the linguistic ordered weighted aggregation (LOWA) operator can be defined as:

$$F(a_1, \ldots, a_m) = W \cdot B^T = \mathbf{C}\{w_k, b_k, k = 1, \ldots, m\} =$$
$$= w_1 \odot b_1 \oplus (1 - w_1) \odot \mathbf{C}\{\beta_h, b_h, h = 2, \ldots, m\}$$

where $\beta_h = w_h / \Sigma_2^m w_k, h = 2, \ldots, m$, and B is the associated ordered label vector. Each element $b_i \in B$ is the i-th largest label in the collection $a_1, \ldots, a_m$.

As Yager (1988) suggested, there exist at least two methods to obtain the values of w_i. The first approach is to use a learning mechanism. In this approach, sample data are used along with arguments and associated aggregated values, and then wheights are fitted to the sample data. The second approach is to give some semantics or meaning to the weights. Then, based upon these semantics we can directly provide the values for the weights.

In the following section, we study a semantic for the weights based on fuzzy linguistic quantifiers (Zadeh 1983; Yager 1988), in order to obtain a collective linguistic preference relation.

3. Linguistic preference relations and linguistic choice process

Suppose we have a set of n alternatives $X = \{x_1, \ldots, x_n\}$ and a set of individuals $N = \{1, \ldots, m\}$. Each individual $k \in N$ provides a preference relation linguistically assessed into the term set S,

$$\phi_{P_k} : X \mathrm{x} X \rightarrow S,$$

where $\phi_{P_k}(x_i, x_j) = l_{ij}^k \in S$ represents the linguistically assessed preference degree of the alternative x_i over x_j. We assume that P_k is reciprocal in the sense, $l_{ij}^k = \mathrm{Neg}(l_{ji}^k)$, and by definition, $l_{ii}^k = None$ (the minimum label in S).

As is known, two approaches may be considered. First a direct approach

$$\{P_1, \ldots, P_m\} \rightarrow \text{ solution } \subseteq X,$$

according to which, on the basis of the individual preference relations, a solution is derived. Secondly an indirect approach

$$\{P_1, \ldots, P_m\} \rightarrow P \rightarrow \text{ solution } \subseteq X$$

providing the solution on the basis of a collective preference relation, P, which is a preference relation of the group of individuals as a whole.

Here we consider the indirect derivation, and hence we have two issues to study: (i) to derive a collective linguistic preference P from $\{P_1, \ldots, P_m\}$, and (ii) how to develop the linguistic choice process, i.e., to obtain the solution from P.

3.1. The collective linguistic preference relation

For the first question it is necessary to aggregate the linguistic preference relations to obtain $l_{ij} \in S$ from $\{l_{ij}^1, \ldots, l_{ij}^m\}$ for all i, j. Using a *fuzzy majority* specified by a fuzzy linguistic quantifier, we deal with this question. Fuzzy linguistic quantifiers provide tools to formally deal with fuzzy majority, and can be used to define a weight vector necessary for using the LOWA operator. We then use the LOWA operator to obtain the collective preference relation P as

$$P = F(P_1, \ldots, P_m)$$

with $l_{ij} = F(l_{ij}^1, \ldots, l_{ij}^m)$ and the weight vector, W, representing the fuzzy majority over the individuals.

The fuzzy linguistic quantifiers were introduced by Zadeh (1983). Linguistic quantifiers are typified by terms such as *most*, *at least half*, *all*, *as many as possible*. A quantifier Q assumed to be a fuzzy set in [0,1]. Zadeh distinguished between two types of quantifiers; absolute, and proportional or relative. Absolute quantifiers are used to represent amounts that are absolute in nature. These quantifiers are closely related to the number of elements. Zadeh suggested that these absolute quantifier values can be represented as fuzzy subsets of the non-negative reals R^+. In particular, he suggested that an absolute quantifier can be represented by a fuzzy subset Q, where for any $r \in R^+$, $Q(r)$ indicates the degree to which the value r satisfies the concept represented by Q. Relative quantifiers represent proportion type statements. Thus, if Q is a relative quantifier, then Q can be represented as a fuzzy subset of $[0,1]$ such that for each $r \in [0,1]$, $Q(r)$ indicates the degree to which r portion of objects satisfies the concept devoted by Q.

Here, we will focus on relative quantifiers. A relative quantifier, $Q : [0,1] \to [0,1]$, satisfies:

$$Q(0) = 0,$$

$$\exists r \in [0,1] \text{ such that } Q(r) = 1.$$

In addition, it is nondecreasing if it has the following property:

$$\forall a, b \in [0,1], \text{ if } a > b \text{ then } Q(a) \geq Q(b).$$

The membership function of a relative quantifer can be represented as:

$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \frac{r-a}{b-a} & \text{if } b \leq r \leq a \\ 1 & \text{if } r > b \end{cases}$$

with $a, b, r \in [0,1]$.

Some examples of relative quantifiers are shown in Figure 4, where the parameters, (a,b) are $(0.3, 0.8)$, $(0, 0.5)$ and $(0.5, 1)$, respectively.

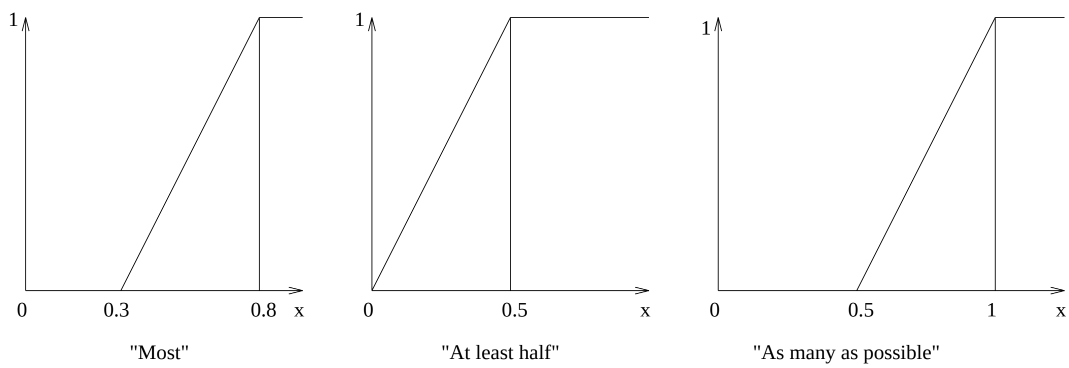

Fig. 4. Relative quantifiers

More generally, Yager (1988) computes the weights w_i of the aggregation from the function Q describing the quantifier. In the case of a relative quantifier

$$w_i = Q(i/m) - Q((i-1)/m), i = 1, \ldots, m, \text{ with } Q(0) = 0.$$

3.2. The linguistic choice process

When fuzzy preferences are used, numerous answers (called the solution concepts) derived from a collective fuzzy preference have been presented in the literature, including: a consensus winner (Kacprzyk 1986); a competitive-like pooling, (Kacprzyk, Fedrizzi and Nurmi 1990b); and a fuzzy α-majority uncovered fuzzy set based on the concept of fuzzy tournaments, (Nurmi and Kacprzyk 1991).

Next, we propose a process to derive a solution on the basis of the collective preference relation, a process based on the concept of non-dominated alternatives (Orlovski 1978).

Let P^s be a *linguistic strict preference relation* $\mu_{P^s}(x_i, x_j) = l_{ij}^s$ such that,

$$l_{ij}^s = None \text{ if } l_{ij} < l_{ji},$$

$$\text{or } l_{ij}^s = l_k \in S \text{ if } l_{ij} \geq l_{ji} \text{ with } l_{ij} = l_l, l_{ji} = l_t \text{ and } l = t + k.$$

The *linguistic nondominance degree* of x_i is defined as

$$\mu_{ND}(x_i) = \text{Min}_{x_j \in X}[\text{Neg}(\mu_{P^s}(x_j, x_i))]$$

where the value $\mu_{ND}(x_i)$ is to be interpreted as the linguistic degree to which the alternative x_i is not dominated by any of the elements in X.

Finally, a set of *maximal nondominated alternatives*, $X^{ND} \subset X$, is obtained as

$$X^{ND} = \{x \in X / \mu_{ND}(x) = \text{Max}_{y \in X}[\mu_{ND}(y)]\}.$$

Therefore, aggregating the knowledge of the experts, X^{ND} is selected as the set of preferred alternatives in the linguistic choice process.

4. Example

We consider the above seven term set (*Fig. 3*), and suppose a situation with four individuals whose linguistic preferences are:

$$P_1 = \begin{bmatrix} - & VL & VH & VL \\ VH & - & H & H \\ VL & L & - & VL \\ VH & L & VH & - \end{bmatrix} P_2 = \begin{bmatrix} - & L & H & VL \\ H & - & VH & L \\ L & VL & - & VL \\ VH & H & VH & - \end{bmatrix}$$

$$P_3 = \begin{bmatrix} - & M & VH & N \\ M & - & VH & L \\ VL & VL & - & VL \\ P & H & VH & - \end{bmatrix} P_4 = \begin{bmatrix} - & L & VH & VL \\ H & - & L & VL \\ VL & H & - & VL \\ VH & VH & VH & - \end{bmatrix}$$

We apply the choice process with different fuzzy quantifiers:

a) Using the linguistic quantifier "As many as possible" with the pair $(0.5, 1.0)$, and the corresponding LOWA operator, with $W = [0, 0, 0.5, 0.5]$, then the collective linguistic preference is:

$$P = \begin{bmatrix} - & VL & H & N \\ M & - & M & VL \\ VL & VL & - & VL \\ VH & M & VH & - \end{bmatrix}$$

The linguistic strict preference relation is:

$$P^s = \begin{bmatrix} - & N & M & N \\ L & - & L & N \\ N & N & - & N \\ VH & L & H & - \end{bmatrix}$$

and the linguistic nondominance degree is:

$$\mu_{ND}(x_1, x_2, x_3, x_4) = [VL, H, L, P].$$

Finally, the set of maximal nondominated alternatives, X^{ND}, is:

$$X^{ND} = \{x_4\}.$$

Therefore, x_4 is the maximal nondominated alternative and the solution to the linguistic decision process.

b) Using the linguistic quantifier "At least half" with the pair $(0.0, 0.5)$, and the corresponding LOWA operator, with $W = [0.5, 0.5, 0.0, 0.0]$, then the collective linguistic preference is:

$$P = \begin{bmatrix} - & L & VH & VL \\ H & - & VH & M \\ VL & M & - & VL \\ VH & H & VH & - \end{bmatrix}$$

The linguistic strict preference relation is:

$$P^s = \begin{bmatrix} - & N & H & N \\ L & - & L & N \\ N & N & - & N \\ H & VL & H & - \end{bmatrix}$$

and the linguistic nondominance degree is:

$$\mu_{ND}(x_1, x_2, x_3, x_4) = [L, VH, L, P].$$

Finally, the set of maximal nondominated alternatives, X^{ND}, is:

$$X^{ND} = \{x_4\}.$$

Similarly, x_4 is the maximal nondominated alternative and the solution to the linguistic decision process.

5. Conclusions

We have presented a representation of commonsense knowledge by means of linguistic labels, and developed a linguistic decision process in group decision making for this representation.

The linguistic decision process has been defined as an indirect approach based on the concepts of fuzzy majority and nondominated alternatives, where fuzzy linguistic quantifiers have been used as tools to deal with fuzzy majority. This model seems to be very consistent to the social choice in an imprecise environment.

Finally, note that in group decision making, the linguistic approach is a tool which provides a framework with more human-consistency than usual ones, and therefore helps the development of decision processes.

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