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Technical Report #DECSAI-94108.
July, 1994
To appear in:
Badania Operacyjne i Decyzje
Operational Research and Decisions

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Abstract

In this paper it is presented the use of preference degrees to solve decision-making problems where linguistic preference relations are used to represent decision-makers' opinions. From a linguistic preference relation are defined a linguistic non-dominance degree, a linguistic dominance degree, and a stric domoninance degree, using linguistic quantifiers and the ordered weight linguistic aggregation operator. These degrees, applied in a selection process, allow us to obtain a solution set in the decision-making problem.

Keywords: Decision making, linguistic preference relation, preference degree.

1. Introduction

Decision making is an usual tarea in humans' activities. Its essence is, basically, to find a best option from among some feasible (relevant, available..) ones. A lot of decision making processes, in the real world, take place in an environment in which the goals, the contraints and the consequences of possible actions are not precisely known. To deal quantitatively with imprecision, concepts and techniques of probability theory are usually employed, and more particularly, the tools provided by decision theory, control theory and information theory. However, if the lack of precision has a qualitative nature, fuzzy set theory serves better than these theories to deal with those human processes. There are many different kinds of imprecision which can not be covered by those theories, that is, inexactness, ill-defined, vagueness, or in short: fuzziness.

Since 1970 fuzzy set theory, created by L. A. Zadeh in 1965 as a mathematical theory of vagueness [18], is being applied in decision-making as aim tool to deal with imprecision. Bellman and Zadeh proposed one of the first models of decision-making in a fuzzy environment [1], which has served as a point of departure for most of the authors in fuzzy decision making theory. Different fuzzy decision-making models have been proposed. A classifications of all of them depending on the number of stages before the decision is reached, is shown in [16]. Some fuzzy models in one-stage decision-making are: fuzzy individual decision-making models, fuzzy decision-making models under constraints, fuzzy multi-person decision-making models applied in group decision theory, and multi-criteria decision-making models. Some models in multi-stage decision-making are: fuzzy dynamic programming models, fuzzy dynamic systems models and linguistic models, [16].

The fuzzy set theory applied on decision-making allows us to work in a most flexible framework, where it is possible to simulate the ability of humans to deal with fuzziness of human judgments quantitatively, and therefore to incorporate more human consistency or "human intelligence" in decision-making models.

Here, our framework is decision-making models where are both linguistic assessments and fuzzy preference relations to represent decision-makers' opinions by means of linguistic preference relations are used. From a linguistic preference relation we define several preference degrees to obtain the solution set in a decision-making problem.

This paper shows the use of the preference degrees to solve decision-making problems under linguistic preference relations, and is structured as follows: section 2 shows the linguistic approach in decision making; section 3 presents the use of linguistic preference relations; section 4 presents the preference degrees over linguistic preference relations; the section 5 shows an example of its application in group decision making, and finally, some conclusions are presented.

2. Linguistic Approach in Decision Making

The linguistic approach considers the variables which participate in the problem assessed by means of linguistic terms instead of numerical values [19]. This approach is appropriated for a lot of problems, since it allows a representation of the decision-makers' information in a more direct and adequate form when they are unable of expressing that information in an exact numerical way.

A linguistic variable differs from a numerical one in that its values are not numbers, they are words or sentences in a natural or artificial language. Since words, in general, are less precise than numbers, the concept of a linguistic variable serves to the purpose of providing a means of approximated characterization of a phenomena being too complex or too ill-defined as to be amenable of being described in conventional quantitative terms.

Therefore, we need a term set defining the uncertainty granularity, that is, the level of distinction among different countings of uncertainty. In [2] was studied the use of term sets with odd cardinal, representing the middle term a probability of "approximately 0.5", being the rest terms placed symmetrically around it and the limit of granularity 11 or no more than 13.

The semantic of the elements of the term set is given by fuzzy numbers defined on the [0,1] interval, which are described by membership functions. As the linguistic assessments are just approximate ones given by the individuals, we can consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since to obtain more accurate values may be impossible or unnecessary. This representation is achieved by the 4-tuple $(a_i, b_i, \alpha_i, \beta_i)$ (the first two paremeters indicate the interval in which the membership value is 1.0; the third and fourth parameters indicate the left and right width of the distribution).

Let $S = \{s_i\}, i \in H = \{0, \ldots, T\}$ be a finite and totally ordered term set on [0,1] in the usual sense [2, 4]. Any label s_i represents a possible value for a linguistic real variable, that is, a vague property or constraint on [0,1]. We shall consider term set like in [**?**], and moreover, we shall require the following properties to this set:

1) The set is ordered: $s_i \geq s_j$ if $i \geq$ j.

2) There is the negation operator: Neg$(s_i) = s_j$ such that $j = T$-i.

3) Maximization operator: Max$(s_i, s_j) = s_i$ if $s_i \geq s_j$.

4) Minimization operator: Min$(s_i, s_j) = s_i$ if $s_i \leq s_j$.

For example, the following nine linguistic term set (with an associated semantic) [2], acomplishes all sense properties:

C	$Certain$	$(1, 1, 0, 0)$
EL	$Extremely\_likely$	$(.98, .99, .05, .01)$
ML	$Most\_likely$	$(.78, .92, .06, .05)$
MC	$Meaningfull\_chance$	$(.63, .80, .05, .06)$
IM	It_may	$(.41, .58, .09, .07)$
SC	$Small\_chance$	$(.22, .36, .05, .06)$
VLC	$Very\_low\_chance$	$(.1, .18, .06, .05)$
EU	$Extremely\_unlikely$	$(.01, .02, .01, .05)$
I	$Impossible$	$(0, 0, 0, 0)$

graphically:

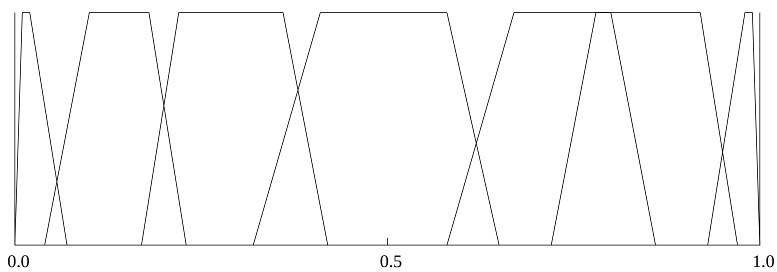

Fig. 1. Distribution of the nine linguistic term set

3. Linguistic Preference Relations

Let X be a set of alternatives over which the fuzzy preference attitude of a decision-maker is defined. Then, according with Tanino [14, 15], the fuzzy preference may be represented as:

1. A fuzzy choice set to represent his total preference attitude. It is described by a fuzzy subset of X, i.e., by a membership function μ on X, whose value $\mu(x)$ denotes the preference degree of x, or degree to which x is chosen as a desirable alternative.

2. A fuzzy utility function. It is described as fuzzy mapping ν, which associates a fuzzy subset of the utility values space (usually the space of real numbers R) with each alternative x, $\nu : X x \mathrm{R} \longrightarrow [0,1]$, where $\nu(x,t)$ denotes the degree to which the utility value of te alternative x is equal to t.

3. A fuzzy preference relation. It is described by a fuzzy binary relation R on X, that is, a fuzzy set on the product set $X x X$, characterized by a membership function $\mu_R : X x X \longrightarrow [0,1]$, where $\mu(x_i, x_j)$ denotes the preference degree of the alternative x_i over x_j.

The use of fuzzy preference relations in decision making situations to voice decision-makers' opinions over an alternative set, with respect to some criteria , seems to be a useful tool in modelling decision processes. Among others, they appear in a very natural way when we want to aggregate decision-makers' preferences into group ones.

Assuming that X is a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$, it can be defined the decision-maker's preference attitude over X, as a nxn matrix whose (i,j) element is given by

$$r_{ij} = \mu_R(x_i, x_j), \quad i,j = 1, \ldots, n.$$

Providing a fuzzy relation R = (r_{ij}) with

$$0 \ \leq \ r_{ij} \ \leq \ 1, \ (i,j = 1, \ldots, n),$$

where:

1. $r_{ij} = 1$ indicates the maximum degree of preference of x_i over x_j.

2. $0.5 \leq r_{ij} \leq 1$ indicates a definite preference of x_i over x_j.

3. $r_{ij} = 0.5$ indicates the indifference between x_i and x_j.

In other to that the fuzzy relation refletcs a preference, it would be desirable to satisfy some of the following properties:

1. Reciprocity: $r_{ij} + r_{ji} = 1, \ \forall i,j$.

2. Max-Min Transitivity: $r_{ik} \geq Min(r_{ij}, r_{jk}),\ \forall i, j, k$.

3. Max-Max Transitivity: $r_{ik} \geq Max(r_{ij}, r_{jk}),\ \forall i, j, k$.

4. Restricted Max-Min Transitivity: $r_{ij} \geq 0.5,\ r_{jk} \geq 0.5, \Rightarrow r_{ik} \geq Min(r_{ij}, r_{jk}),\ \forall i, j, k$.

5. Restricted Max-Max Transitivity: $r_{ij} \geq 0.5,\ r_{jk} \geq 0.5, \Rightarrow r_{ik} \geq Max(r_{ij}, r_{jk}),\ \forall i, j, k$.

6. Additive Transitivity: $r_{ij} + r_{jk} - 0.5 = r_{ik},\ \forall i, j, k$.

7. Multiplicative Transitivity: $\frac{r_{ji}}{r_{ij}} \cdot \frac{r_{kj}}{r_{jk}} = \frac{r_{ki}}{r_{ik}},\ \forall i, j, k$.

More concretely, we are interested in the use of linguistic preference relations, that is, a preference relation defined as: R: Xx$X \longrightarrow S$. Obviously, working in S, the last of the above properties makes not sense, but yes the rest.

Some methods to obtain the solution to a decision process from a fuzzy preference relation have been proposed [13, 12, 14, 15]. In the next section we present some preference degrees to obtain a solution alternative set in the more general case of a decision process under linguistic preference relations.

4. Preference Degrees under Linguistic Preference Relations

In what follows we will describe three different preference degrees under linguistic preference relations. The first is a linguistic non-dominance degree, based in the concept of non-dominanted alternatives by Orlovsky [13]. The other two are dominance degrees based on the concept of fuzzy majority, represented by means of a fuzzy linguistic quantifier. Finally we present a selection process in decision making using the three degrees.

4.1. Linguistic Non-Dominance Degree

Consider the linguistic preference relation $P = (p_{ij}),\ i, j = \{1, \ldots, n\}$. In [6] it is extended the non-dominated alternative concept by Orlovsky [13] to the use of linguistic preference relations.

Let $S = \{s_i\}, i \in H = \{0, \ldots, T\}$ be a finite and totally ordered term set on [0,1], as introduced in section 2. Let P^s be a *linguistic strict preference relation* $\mu_{P^s}(x_i, x_j) = p^s_{ij}$ such that,

$$p^s_{ij} = s_0 \text{ if } p_{ij} < p_{ji},$$

$$\text{or } p^s_{ij} = s_k \in S \text{ if } p_{ij} \geq p_{ji} \text{ with } p_{ij} = s_l, p_{ji} = s_t \text{ and } l = t + k.$$

The *linguistic non-dominance degree* of x_i is defined as

$$\mu_{ND}(x_i) = \text{Min}_{x_j \in X}[\text{Neg}(\mu_{P^s}(x_j, x_i))]$$

where the value $\mu_{ND}(x_i)$ is to be meant as a linguistic degree to which the alternative x_i is dominated by no one of the elements in X.

4.2. Dominance Degrees

As we have mentioned above, we present two dominance degrees based on the concept of fuzzy majority. The first also is based on the use of the linguistic ordered weights aggregation (LOWA) operator [5]. Before to define the dominance degrees, we present the concepts of linguistic quantifier and the LOWA operator.

The fuzzy linguistic quantifiers were introduced by Zadeh in 1983 [20]. Linguistic quantifiers are typified by terms such as *most*, *at least half*, *all*, *as many as possible*, and assumed a quantifier Q to be a fuzzy set in [0,1]. Zadeh distinguished between two types of quantifiers, absolute and proportional or relative. Absolute quantifiers are used to represent amounts that are absolute in nature. These quantifiers are closely related to the concepts of the count of number of elements. Zadeh suggested that these absolute quantifiers values can be represented as fuzzy subsets of the non-negative reals, R^+. In particular, he suggested that an absolute quantifier can be represented by a fuzzy subset Q, where for any $r \in R^+$, $Q(r)$ indicates the degree to which the value r satisfies the concept represented by Q. And, relative quantifiers represent proportion type statements. Thus, if Q is a relative quantifier, then Q can be represented as a fuzzy subset of $[0,1]$ such that for each $r \in [0,1]$, $Q(r)$ indicates the degree to which r portion of objects satisfies the concept devoted by Q.

An absolute quantifier $Q : R^+ \rightarrow [0,1]$, satisfies:

$$Q(0) = 0,$$

$$\exists k \text{ such that } Q(k) = 1$$

A relative quantifier, $Q : [0,1] \rightarrow [0,1]$, satisfies:

$$Q(0) = 0,$$

$$\exists r \in [0,1] \text{ such that } Q(r) = 1.$$

A non-decreasing quantifier satisfies:

$$\forall a, b \text{ if } a > b \text{ then } Q(a) \geq Q(b).$$

The membership function of a non-decreasing relative quantifer can be represented as

$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \frac{r-a}{b-a} & \text{if } a \leq r \leq b \\ 1 & \text{if } r > b \end{cases}$$

with $a, b, r \in [0,1]$.

Let $a_1, \ldots, a_m$ be a set of labels to be aggregated, the LOWA operator is defined using the concepts of ordered weighted averaging operator [17] and the convex combination of linguistic labels [3]. The following expresion shows it [5]:

$$F(a_1, \ldots, a_m) = W \cdot B^T = \mathbf{C}\{w_k, b_k, k = 1, \ldots, m\} =$$

$$= w_1 \odot b_1 \oplus (1 - w_1) \odot \mathbf{C}\{\beta_h, b_h, h = 2, \ldots, m\},$$

where W is a weight vector, $W = [w_1, \ldots, w_n]$, verifying: $i) w_i \in [0, 1]$, and $ii) \Sigma_i w_i = 1$. $\beta_h = w_h / \Sigma_2^m w_k, h = 2, \ldots, m$, and B is the associated ordered label vector, each element $b_i \in B$ is the i-th largest label in the collection of labels $a_1, \ldots, a_m$ and $\mathbf{C}$ is the convex combination of linguistic labels [3]. If $w_j = 1$ y $w_i = 0$ with $i \neq j \; \forall i$, then the convex combination is defined as:

$$\mathbf{C}\{w_i, b_i, i = 1, \ldots, m\} = b_j.$$

The weights vector, W, may be calculated using the relative quantifier Q, which represents a fuzzy majority of dominance over alternatives, by means of the following expresion [17, 7]:

$$w_i = Q(i/m) - Q((i-1)/m), i = 1, \ldots, m.$$

Linguistic Dominance Degree

We define a *linguistic dominance degree*, which acts on the alternative set as [8],

$$LDD(x_i) = F_{Q_{i \neq j}}(p_{ij}),$$

where F_Q is a LOWA operator whose weights are defined using relative quantifer Q, and whose components are the elements of the corresponding row of P, that is, for x_i the set of $n-1$ labels $\{p_{ij} \setminus j = 1, \ldots, n \text{ and } i \neq j\}$.

Strict Dominance Degree

We define *strict dominance degree* as a real degree which acts on the alternative set as [8],

$$SDD(x_i) = Q(\frac{r_i}{n-1}),$$

where

$$r_i = \#\{x_q \in X \setminus p_{iq} > p_{qi}\}.$$

and # stands for the cardinal of term set.

4.3. Preference Degrees in Decision Making: A Selection Process

The above preference degrees can be applied in a decision making problem with a preference relation representing the decision-makers' opinions, by means of a selection process [9], as it is showed in the figure 2.

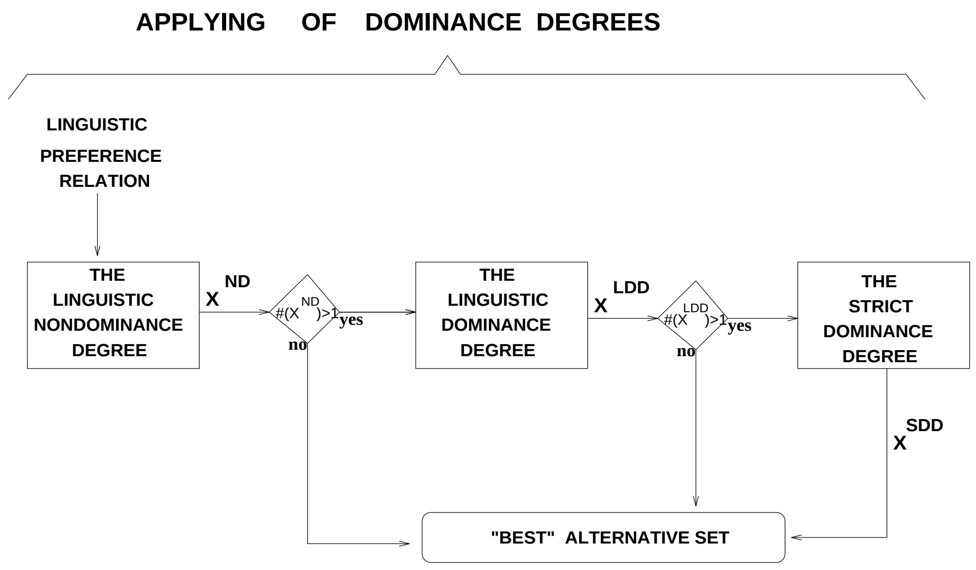

Fig. 2. Selection Process

As it is shown in the above figure, first, applying the linguistic non-dominance degree over the linguistic preference relation, is obtained the set of maximal non-dominated alternatives, X^{ND} as:

$$X^{ND} = \{x \in X \setminus \mu_{ND}(x) = \mathrm{Max}_{y \in X}[\mu_{ND}(y)]\}$$

If $\#X^{ND} > 1$, then we apply the linguistic dominance degree over X^{ND}, and it is obtained the set of nondominated alternatives with maximun linguistic dominance degree, X^{LDD}, as:

$$X^{LDD} = \{x \in X^{ND} \setminus LDD(x) = \mathrm{Max}_{y \in X^{ND}}[LDD(y)]\}.$$

Finally, if $\#X^{LDD} > 1$, then we apply the strict dominance degree over X^{LDD}, and it is obtained the final solution alternative set to the selection process,X^{SDD}, as:

$$X^{SDD} = \{x \in X^{LDD} \setminus SDD(x) = \mathrm{Max}_{y \in X^{LLD}}[SDD(y)]\}.$$

In order to apply the two last degrees does not exist a ordering. It is possible to apply any of them in the first place.

Sometimes, the set of maximal nondominated alternatives is formed by all possible alternatives. This may happen because of the existence of balance among all the alternatives, or the existence of an inconsistency situation among all decision-makers' opinions. In these cases, the use of last two dominance degrees has more interest, because it allows us to jump the inconsistency situation and to identify the best alternative set.

5. The use of Preference Degrees in Group Decision Making

As is well known, a group decision making process can be defined as a decision situation in which: (i) there are two or more individuals, each of them characterized by his own perceptions, attitudes, motivations, and personalities, (ii) who recognize the existence of a common problem, and (iii) attemp to reach a collective decision.

The use of preference relations is usual in group decision making. Several authors have provided interesting results on group decision making with the help of preference relations to represent the decision-makers' opinions[6, 9, 14, 10, 11]. Basically two approaches may be considered. A direct approach

$$\{P_1, \ldots, P_m\} \rightarrow \text{ solution}$$

according to which, on the basis of the individual preference relations, a solution is derived, and an indirect approach

$$\{P_1, \ldots, P_m\} \rightarrow P \rightarrow \text{ solution}$$

according to which, on the basis of the individual preference relations, a collective linguistic preference relation is derived, and then is obtained the solution.

To develop an example of applying the dominance degrees in group decision making, we shall follow the indirect process in [5], based on the use of LOWA operator, for obtaining the collective preference relation.

Suppose we have a set of 4 alternatives $X = \{x_1, \ldots, x_4\}$ and a set of decision-makers $N = \{1, \ldots, 4\}$. Each decision-maker $k \in N$ provides his preference by means of a preference relation P^k, linguistically assessed into the above nine linguistic term set S,

$$\phi_{P^k} : X x X \rightarrow S,$$

where $p_{ij}^k \in S$ represents the linguistically assessed preference degree of the alternative x_i over x_j. Assuming that P^k is reciprocal in the sense, $p_{ij}^k = \text{Neg}(p_{ji}^k)$, and by definition $p_{ii}^k = \textit{Impossible}$ (the minimum label in S), suppose that the linguistic preference relation of each decision-maker is:

$$P_1 = \begin{bmatrix} - & SC & C & I \\ MC & - & EU & EL \\ I & EL & - & VLC \\ C & EU & ML & - \end{bmatrix} \quad P_2 = \begin{bmatrix} - & IM & C & EU \\ IM & - & EU & C \\ I & EL & - & VLC \\ EL & I & ML & - \end{bmatrix}$$

$$P_3 = \begin{bmatrix} - & IM & EL & I \\ IM & - & I & EL \\ EU & C & - & VLC \\ C & EU & ML & - \end{bmatrix} \quad P_4 = \begin{bmatrix} - & IM & C & EU \\ IM & - & EU & C \\ I & EL & - & VLC \\ EL & I & ML & - \end{bmatrix}$$

respectively.

As we consider the indirect derivation, then we obtain a collective linguistic preference relation. Using the linguistic quantifier "As many as possible ", with the pair $(0.5, 1.0)$, and the corresponding LOWA operator with $W = (0, 0, 0.5, 0.5)$, the collective linguistic preference relation is:

$$P = \begin{bmatrix} - & SC & EL & I \\ IM & - & I & EL \\ I & EL & - & VLC \\ EL & I & ML & - \end{bmatrix}$$

Then, the linguistic strict preference relation is

$$P^s = \begin{bmatrix} - & I & EL & I \\ EU & - & I & EL \\ I & EL & - & I \\ EL & I & IM & - \end{bmatrix}$$

being the linguistic nondominance degree of each alternative the following:

$$[\mu_{ND}(x_1), \mu_{ND}(x_2), \mu_{ND}(x_3), \mu_{ND}(x_4)] = [EU, EU, EU, EU].$$

In consequence, the set of maximal non-dominated alternatives is:

$$X^{ND} = \{x_1, x_2, x_3, x_4\}$$

This is an example where the non-dominated alternative set is constituted by all possible alternatives. But, in this case, this is so because of some inconsistencies in the decision-makers' opinions. For example the decision-maker 1 presents this incoherent situation:

$$1 \xrightarrow{C} 3 \xrightarrow{EL} 2 \xrightarrow{C} 4 \xrightarrow{EL} 1$$

and the decision-maker 2 this other one:

$$1 \xrightarrow{C} 3 \xrightarrow{EL} 2 \xrightarrow{EL} 4 \xrightarrow{C} 1.$$

Then, we apply the two dominance degrees, in any ordering, to bridge this inconsistency situation. For example, applying firstly the strict dominance degree on the alternatives of X^{ND}, we obtain:

$$(SDD(x_1), SDD(x_2), SDD(x_3), SDD(x_4)) = [0.0, 0.03, 0.0, 0.03]$$

and therefore,

$$X^{SDD} = \{x_2, x_4\}.$$

As the set X^{SDD} has two alternatives, then we apply the linguistic dominance degree on X^{SDD} using the same linguistic quantifier, obtaning:

$$(LDD(x_2), LDD(x_4)) = [I, SC],$$

and therefore x_4 is the best alternative to this group decision-making process.

6. Conclusions

In this paper we have presented three preference degrees in decision-making problems under linguistic preference relations. We have shown that the use of the preference degrees are useful to solve the decision-making process and to bridge possible inconsistencies of decision-makers' opinions. Finally, we have presented an example of their application in group decision making.

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