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Decision Aiding

Managing non-homogeneous information in group decision making

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Abstract

Within the frame of decision aid literature, decision making problems with multiple sources of information have drawn the attention of researchers from a wide spectrum of disciplines. In decision situations with multiple individuals, each one has his own knowledge on the alternatives of the decision problem. The use of information assessed in different domains is not a seldom situation. This non-homogeneous information can be represented as values belonging to domains with different nature as linguistic, numerical and interval valued or can be values assessed in label sets with different granularity, multi-granular linguistic information.

Decision processes for solving these problems are composed by two phases: aggregation and *exploitation*. The main problem to deal with non-homogeneous contexts is *how to aggregate the information assessed in these contexts?* In this paper, taking as base the 2-tuple fuzzy linguistic representation model we shall develop an aggregation process for dealing with non-homogeneous contexts. In first place, we shall develop an aggregation process for combining numerical, interval valued and linguistic information, afterwards we shall propose different extensions of this process to deal with contexts in which can appear other type of information as intuitionistic fuzzy sets or multi-granular linguistic information.

Keywords: Decision making; Interval valued; Linguistic variables; Fusion processes; Granularity of uncertainty; Multi-granular linguistic information

1. Introduction

In decision making problems with multiple experts as group decision making (GDM) problems, each expert expresses his/her preferences depending on the nature of the alternatives and on his/her own knowledge over them:

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- When the alternatives are measurable by their quantitative nature then they are assessed by means of precise numerical values.
- However, when the alternatives are related to qualitative aspects it may be difficult to qualify them using precise values. Usually, this knowledge is not precise and presents uncertainty. Early this uncertainty was expressed in the preference values by means of real values assessed in a predefined range [15,32], soon other approaches based on interval valued [18,30] and on the linguistic approach [8,33] were proposed.

Therefore, the use of non-homogeneous information in decision problems with multiple experts is not an unusual situation (see [4,7,31] with proposals combining numerical preference representations, fuzzy preference relations, multiplicative preference relations, utility preferences, interval numerical preferences, ...). However, most of the proposals for solving decision making problems with multiple experts [8,15] are focused on cases where all the experts express their preferences by means of values from the same type, either real values, or interval values or linguistic labels in the same linguistic term set.

The solution for a GDM problem is derived either from the individual preference relations, without constructing a social preference relation, or by computing first a social fuzzy preference relation and then using it to find a solution [15]. In any of the above approaches, called direct and indirect approaches respectively, the process for reaching a solution of the GDM problems is composed by two steps [27]:

1. *Aggregation phase* that combines the expert preferences.
2. *Exploitation one* that obtains a solution set of alternatives for the decision problem.

The main difficulty for managing GDM problems defined in non-homogeneous contexts is the *aggregation phase*: *how to aggregate this type of information?*, because there don't exist standard operators for combining any type of non-homogeneous information.

In this paper, we propose a method for managing non-homogeneous information in GDM problems. This method unifies the input information in an unique domain. In this case, a linguistic one called *basic linguistic term set* (BLTS), expressing the unified information by means of fuzzy sets over the BLTS.

We must point out that the two classical models for dealing with linguistic information are

- the semantic model [5] that uses the linguistic terms just as labels for fuzzy numbers, while the computation process acts directly over those fuzzy numbers by means of the Principle of Extension; and
- the second one is the symbolic model, an ordinal scale is assumed on which linguistic assessments are to be done [6].

In this paper, we are proposing the use of an extension of the last one, a new approach, the called linguistic 2-tuple representation model [10], where the scale is no longer purely ordinal, but still processing of linguistic information is done directly on labels. It has shown itself as a good choice to manage non-homogeneous information [11,13]. However, the method proposed does not unify the non-homogeneous information into linguistic labels directly, but into fuzzy sets over a BLTS as we have mentioned.

Our proposal for combining non-homogeneous information follows the scheme composed by three phases introduced in [11] *unification, aggregation* and *transformation into 2-tuples*.

To develop the above method, we shall define different transformation functions and operators, that allow us to unify the non-homogeneous information and also to transform fuzzy sets over the BLTS into linguistic 2-tuples. Our proposal is presented over a context with preference relations expressed by means of numerical, interval valued and linguistic values belonging to an unique linguistic term set. Later on, we show how this process can be used in contexts that also present preference relations expressed by means of intuitionistic fuzzy sets or with linguistic preference relations assessed in different linguistic term sets with different granularity or semantics (multi-granular linguistic information).

In order to do so, this paper is structured as follows: In Section 2 we review the scheme of a GDM problem and the different approaches to express the preferences; in Section 3 we propose an aggregation process for combining contexts with information of different nature (numerical, interval valued and linguistic); in Section 4 we solve a GDM problem defined in a non-homogeneous context; in Section 5 we propose different extensions of the aggregation process for dealing with multi-granular linguistic information and intuitionistic fuzzy sets. Finally, some concluding remarks are pointed out.

2. Preliminaries

In GDM problems the experts express their preferences depending on their knowledge over the alternatives by means of preference relations. In this section we shall review different approaches that we can find in the literature to express those preferences. Finally, we shall review the 2-tuple linguistic representation model.

2.1. Group decision making problems

GDM problems, considered in this paper, consist of a decision situation in which two or more individuals express their preferences over some set of alternatives to derive a solution (an alternative or set of alternatives). It is supposed there is a finite set of alternatives:

$$X = \{x_1, \ldots, x_n\}, \quad n \geqslant 2,$$

as well as a finite set of experts:

$$E = \{e_1, \ldots, e_m\}, \quad m \geqslant 2.$$

Depending either on the nature of the alternatives or on the knowledge over the alternatives of the experts, they can express their preferences using different approaches.

In fuzzy contexts, the departure point is a set of fuzzy preference relations where each expert e_k provides his/her preferences on X [15], i.e.,

$$P_{e_k} = \begin{pmatrix} p_{11}^k & \cdots & p_{1n}^k \\ \vdots & \ldots & \vdots \\ p_{n1}^k & \cdots & p_{nn}^k \end{pmatrix},$$

where p_{ij}^k is the degree of preference of alternative x_i over x_j expressed by the expert $k \in \{1, \ldots, m\}$. Let us suppose that $p_{ij}^k \in [0, 1]$, then

1. $p_{ij} = 1$ indicates the maximum degree of preference of x_i over x_j;
2. $0.5 \leqslant p_{ij} \leqslant 1$ indicates a definitive preference of x_i over x_j;
3. $p_{ij} = 0.5$ indicates the indifference between x_i and x_j.

The fuzzy preference relations may satisfy some of the following properties:

- *reciprocity*: $p_{ij} + p_{ji} = 1 \ \forall i, j$;
- *completeness*: $p_{ij} + p_{ji} \geqslant 1 \ \forall i, j$;
- *max–min transitivity*: $p_{ik} \geqslant \min(p_{ij}, p_{jk}) \ \forall i, j, k$;
- *max–max transitivity*: $p_{ik} \geqslant \max(p_{ij}, p_{jk}) \ \forall i, j, k$;
- *restricted max–min transitivity*: $p_{ij} \geqslant 0.5, p_{ik} \geqslant 0.5 \Rightarrow p_{ik} \geqslant \min(p_{ij}, p_{jk}) \ \forall i, j, k$;
- *restricted max–max transitivity*: $p_{ij} \geqslant 0.5, p_{ik} \geqslant 0.5 \Rightarrow p_{ik} \geqslant \max(p_{ij}, p_{jk}) \ \forall i, j, k$;
- *additive transitivity*: $p_{ij} + p_{jk} - 0.5 = p_{ik} \ \forall i, j, k$.

The equivalent properties for linguistic preference relations can be found in [9].

Fuzzy preference relations can represent different types of preference:

1. *Weak preference relation*. Every pairwise comparison value denotes the extent to which an alternative is "at least as good as the other".
2. *Strict preference relation*. Implies "if x is at least as good as y, then y is not as good as x", for all the alternatives.
3. *Indifference relation*. Represent that "x is as good as y and y as good as x".
4. *Incomparability relation*. Means that "neither is x at least as good as y, not is y as good as x".

However, we can find a lot of ways of defining strict preference, indifference and incomparability relations [24].

In this paper, we do not assume by default any property for the preference relations.

2.2. Approaches for modelling preference relations

2.2.1. Fuzzy binary preference relations

A valued (fuzzy) binary relation R on X is defined as a fuzzy subset of the direct product $X \times X$, i.e., $R : X \times X \to [0,1]$. The value, $R(x_i, x_j) = p_{ij}$, of a valued relation R denotes the degree to which $x_i R x_j$, i.e., the degree to which elements x_i and x_j are in relation R for all $x_i, x_j \in X$. Particularly, in preference analysis, p_{ij}, denotes the *degree to which an alternative* x_i *is preferred to alternative* x_j.

$$P_{e_k} = \begin{pmatrix} 0.5 & \cdots & 0.7 \\ \vdots & \cdots & \vdots \\ 0.3 & \cdots & 0.5 \end{pmatrix}.$$

These were the first type of fuzzy preference relations used in decision making [15] to deal with uncertainty, but soon appeared other approaches to express the uncertainty in the preference relations that will be reviewed in the following subsections.

2.2.2. Interval-valued preference relations

A first approach to add some flexibility to the uncertainty representation problem was by means of interval-valued relations:

$$R : X \times X \to \wp([0,1]),$$

where $R(x_i, x_j) = p_{ij}$ denotes the interval-valued preference degree of the alternative x_i over x_j. In these approaches [18,30], the preferences provided by the experts consist of interval values assessed in $\wp([0,1])$, where the preference is expressed as $[\underline{a}, \overline{a}]_{ij}$, with $\underline{a} \leqslant \overline{a}$.

$$P_{e_k} = \begin{pmatrix} [0.5, 0.5] & \cdots & [0.7, 0.9] \\ \vdots & \cdots & \vdots \\ [0.1, 0.3] & \cdots & [0.5, 0.5] \end{pmatrix}.$$

2.2.3. Fuzzy linguistic relations

A linguistic preference relation is defined as

$$R : X \times X \to S,$$

with S being a set of labels.

There are situations in which a better approach to qualify aspects of many activities may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents the information as linguistic values by means of linguistic variables [34]. This approach is adequate when attempting to qualify phenomena related to human perception; we are often led to use words in natural language. This may arise for different reasons. There are some situations where the information may be unquantifiable due to its nature, and thus, it may be stated only in linguistic terms (e.g., when evaluating the "comfort" or "design" of a car, terms like "bad", "poor", "tolerable", "average", "good" can be used [19]). In other cases, according to [35] there is a tolerance for imprecision which can be exploited to achieve tractability, robustness, low solution cost, and better rapport with reality (e.g., when evaluating the speed of a car, linguistic terms like "fast", "very fast", "slow" are used instead of numerical values).

We have to choose the appropriate linguistic descriptors for the term set and their semantics. One possibility of generating the linguistic term set consists in directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [33]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 = \text{none}, s_1 = \text{very low}, s_2 = \text{low}, s_3 = \text{medium}, s_4 = \text{high}, s_5 = \text{very high}, s_6 = \text{perfect}\}.$$

In these cases, it is usually required that there exist

1. A negation operator $\text{Neg}(s_i) = s_j$ such that $j = g - i$ ($g + 1$ is the cardinality of the term set);
2. A maximization operator $\max(s_i, s_j) = s_i$ if $s_i \geqslant s_j$;
3. A minimization operator $\min(s_i, s_j) = s_i$ if $s_i \leqslant s_j$.

The semantics of the terms is given by fuzzy numbers defined in the $[0, 1]$ interval. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [3]. For example, we may assign the following semantics to the set of seven terms via triangular fuzzy numbers:

$$\begin{aligned}
&\text{P} = \text{perfect} = (.83, 1, 1), \quad \text{VH} = \text{very_high} = (.67, .83, 1), \quad \text{H} = \text{high} = (.5, .67, .83),\\
&\text{M} = \text{medium} = (.33, .5, .67), \quad \text{L} = \text{low} = (.17, .33, .5), \quad \text{VL} = \text{very_low} = (0, .17, .33),\\
&\text{N} = \text{none} = (0, 0, .17),
\end{aligned}$$

which is graphically shown in Fig. 1.

Therefore a linguistic preference relation $R(x_i, x_j)$ with $x_i, x_j \in S$ denotes the linguistic preference degree of the alternative x_i over x_j. Using the linguistic term set shown in Fig. 1, a linguistic preference relation could be

$$P_{e_k} = \begin{pmatrix} \text{M} & \cdots & \text{VH} \\ \vdots & \cdots & \vdots \\ \text{VL} & \cdots & \text{M} \end{pmatrix}.$$

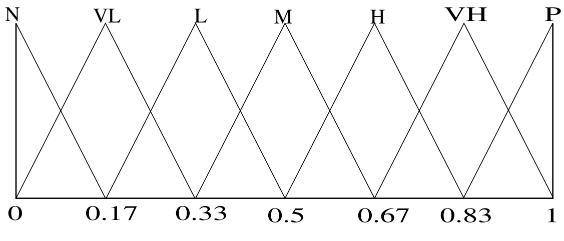

Fig. 1. A set of seven terms with their semantics.

2.3. The 2-tuple fuzzy linguistic representation model

This model has been presented in [10]. Different advantages of this representation to manage linguistic information over semantic and symbolic models were shown in [12]:

1. The linguistic domain can be treated as continuous, while in the symbolic model is treated as discrete.
2. The linguistic computational model based on linguistic 2-tuples carries out processes of computing with words easily and without loss of information.

Due to these advantages, we shall use this linguistic representation model to accomplish our aim, to build an aggregation process for non-homogeneous information.

This linguistic model takes as a basis the symbolic aggregation model [6] and in addition defines the concept of Symbolic Translation and uses it to represent the linguistic information by means of a pair of values called linguistic 2-tuple, (s, α), where s is a linguistic term and α is a numeric value representing the symbolic translation.

Definition 1. Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set $S = \{s_0, \ldots, s_g\}$, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = \text{round}(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a symbolic translation.

Graphically, it is represented in Fig. 2.

From this concept in [10] was developed a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-.5, .5)$.

This model defines a set of transformation functions between linguistic terms and 2-tuples, and between numeric values and 2-tuples.

Definition 2 [10]. Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \to S \times [-0.5, 0.5),$$

$$\Delta(\beta) = \begin{cases} s_i, & i = \text{round}(\beta), \\ \alpha = \beta - i, & \alpha \in [-.5, .5), \end{cases}$$

where "round" is the usual round operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.

Proposition 1 [10]. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

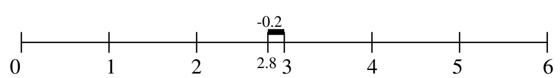

Fig. 2. Example of a symbolic translation.

Proof. It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \rightarrow [0, g],$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \qquad \square$$

Remark 1. From Definitions 1 and 2 and from Proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation:

$$s_i \in S \Rightarrow (s_i, 0).$$

Together with the fuzzy linguistic 2-tuple representation model a wide range of 2-tuple aggregation operators were developed [10], such as, the extended LOWA, the extended weighted average, the extended OWA, ...

The exploitation phase of the decision process ranks the alternatives to obtain the best one(s). The process of comparison between linguistic 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a linguistic assessment:

- If $k < l$ then (s_k, α_1) is smaller than (s_l, α_2).
- If $k = l$ then
  1. if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information;
  2. if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2);
  3. if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2).

3. Aggregation process for non-homogeneous information

Here, we propose a process to carry out the *aggregation step* of a decision process in a GDM problem defined using non-homogeneous information, composed by numerical, interval valued and linguistic values.

Our proposal for combining the above information is developed according to the following process composed by three steps (see Fig. 3):

1. *Making the information uniform.* The non-homogeneous information will be unified into a specific linguistic domain, called BLTS, S_T. Each numerical, interval-valued and linguistic preference value is expressed by means of a fuzzy set on the BLTS, $F(S_T)$. The process is carried out in the following order:
   (a) transforming numerical values in $[0, 1]$ into $F(S_T)$,
   (b) transforming linguistic terms into $F(S_T)$,
   (c) transforming interval valued into $F(S_T)$.

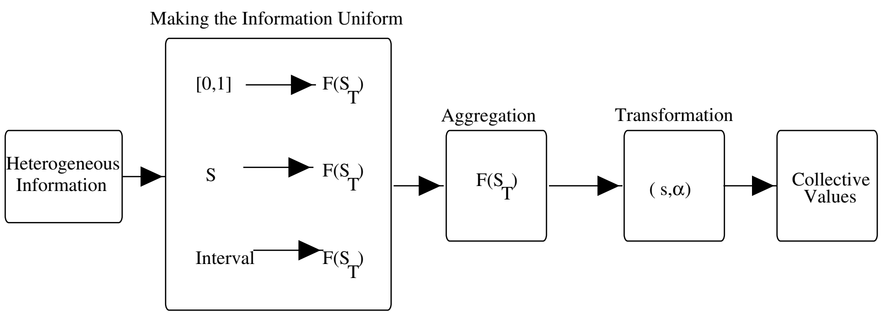

Fig. 3. Aggregation process for heterogeneous information.

2. *Aggregating individual preference values*. For each pair of alternatives, a collective preference value is obtained aggregating the above fuzzy sets on the BLTS that represents the individual preference values assigned by the experts according to his/her preference. Therefore, each collective preference value is a fuzzy set on the specific linguistic domain, the BLTS.

It is clear that the information has been unified into fuzzy sets to be manageable in the aggregation phase. However, in a decision process during the exploitation phase the collective preferences will be rank to obtain the best solution. To facilitate this ranking process we shall transforms these collective fuzzy sets into linguistic 2-tuples [11].

3. *Transforming into 2-tuple*. Then the collective preference values (fuzzy sets on the BLTS) are transformed into linguistic 2-tuples in the BLTS and a collective 2-tuple preference relation is obtained.

In Sections 3.1–3.3 we shall show in depth each step of the different phases of the aggregation process. Afterwards, in Section 4 we shall present an example of a GDM problem defined in a non-homogeneous context.

3.1. Making the information uniform

First of all, the non-homogeneous information is unified in an unique expression domain. In this case, we shall use fuzzy sets over a BLTS, denoted as $F(S_T)$.

Before to transform the input information into fuzzy sets over a BLTS, we have to decide how to choose the BLTS, S_T.

3.1.1. Choosing the BLTS

To choose the BLTS, we shall study the linguistic term set S that belong to the definition context of the GDM problem:

IF

1. S is a fuzzy partition [28], and
2. the membership functions of its terms are triangular, i.e., $s_i = (a_i, b_i, c_i)$

THEN

we select S as the BLTS, due to the fact that, these conditions are necessary and sufficient for the transformation between values in $[0, 1]$ and 2-tuples, being them carried out without loss of information [11].

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We shall choose as BLTS a term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see [21]) and satisfies the above conditions. We choose the BLTS with 15 terms symmetrically distributed whose semantics are (graphically, Fig. 4):

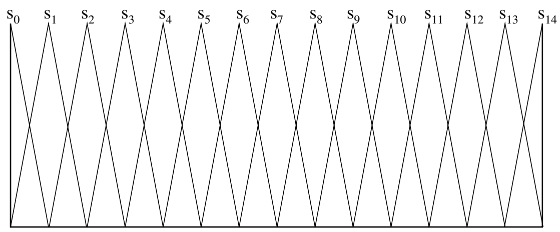

Fig. 4. A BLTS with 15 terms symmetrically distributed.

s_0	(0, 0, 0.07)	s_1	(0, 0.07, 0.14)	s_2	(0.07, 0.14, 0.21)	s_3	(0.14, 0.21, 0.28)
s_4	(0.21, 0.28, 0.35)	s_5	(0.28, 0.35, 0.42)	s_6	(0.35, 0.42, 0.5)	s_7	(0.42, 0.5, 0.58)
s_8	(0.5, 0.58, 0.65)	s_9	(0.58, 0.65, 0.72)	s_{10}	(0.65, 0.72, 0.79)	s_{11}	(0.72, 0.79, 0.86)
s_{12}	(0.79, 0.86, 0.93)	s_{13}	(0.86, 0.93, 1)	s_{14}	(0.93, 1, 1)		

Once we have chosen the BLTS we shall define the transformation functions that we shall need to unify the non-homogeneous information.

The process of unifying the information involves the comparison between fuzzy sets. These comparisons are usually carried out by means of a measure of comparison. We focus in measures of comparison which evaluate the resemblance or likeness of two objects (fuzzy sets in our case) [25]. For simplicity, in this paper we shall choose a measure based on a possibility function $S(A,B) = \max_x \min(\mu_A(x), \mu_B(x))$, where μ_A and μ_B are the membership function of the fuzzy sets A and B, respectively.

3.1.2. Transforming the input information into $F(S_T)$

3.1.2.1. Transforming numerical values in [0,1] into $F(S_T)$. Let $F(S_T)$ be the set of fuzzy sets in $S_T = \{s_0, \ldots, s_g\}$, we shall transform a numerical value $\vartheta \in [0,1]$ into a fuzzy set in $F(S_T)$ computing the membership value of ϑ in the fuzzy number associated with the linguistic terms of S_T.

Definition 3 [11]. The function τ_{NS_T} transforms a numerical value into a fuzzy set in S_T:

$$\tau_{\mathrm{NS}_T} : [0,1] \to F(S_T),$$

$$\tau_{\mathrm{NS}_T}(\vartheta) = \{(s_0, \gamma_0), \ldots, (s_g, \gamma_g)\}, \quad s_i \in S_T \text{ and } \gamma_i \in [0,1],$$

$$\gamma_i = \mu_{s_i}(\vartheta) = \begin{cases} 0 & \text{if } \vartheta \notin \text{support } (\mu_{s_i}(x)), \\ \frac{\vartheta - a_i}{b_i - a_i} & \text{if } a_i \leqslant \vartheta \leqslant b_i, \\ 1 & \text{if } b_i \leqslant \vartheta \leqslant d_i, \\ \frac{c_i - \vartheta}{c_i - d_i} & \text{if } d_i \leqslant \vartheta \leqslant c_i. \end{cases}$$

Remark 2. We consider membership functions, $\mu_{s_i}(\cdot)$, for linguistic labels, $s_i \in S_T$, represented by a parametric function (a_i, b_i, d_i, c_i). A particular case are the linguistic assessments whose membership functions a triangular, i.e., $b_i = d_i$.

Example 1. Let $\vartheta = 0.78$ be a numerical value to be transformed into a fuzzy set in $S = \{s_0, \ldots, s_4\}$. The semantic of this term set is

$$s_0 = (0, 0, 0.25), \quad s_1 = (0, 0.25, 0.5), \quad s_2 = (0.25, 0.5, 0.75), \quad s_3 = (0.5, 0.75, 1), \quad s_4 = (0.75, 1, 1).$$

Therefore, the fuzzy set obtained is (see Fig. 5)

$$\tau_{\mathrm{NS}_T}(0.78) = \{(s_0, 0), (s_1, 0), (s_2, 0), (s_3, 0.88), (s_4, 0.12)\}.$$

3.1.2.2. Transforming linguistic terms in S into $F(S_T)$.

Definition 4. Let $S = \{l_0, \ldots, l_p\}$ and $S_T = \{s_0, \ldots, s_g\}$ be two linguistic term sets, such that, $g \geqslant p$. Then, a linguistic transformation function, τ_{SS_T}, is defined as

$$\tau_{\mathrm{SS}_T} : S \to F(S_T),$$

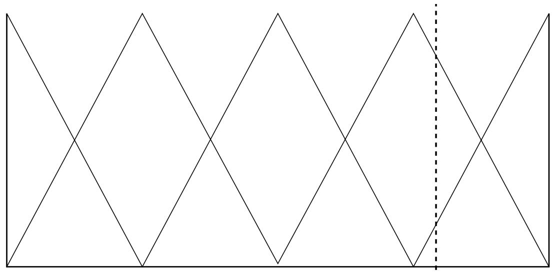

Fig. 5. Transforming a numerical value into a fuzzy set in S.

$$\tau_{SS_T}(l_i) = \{(s_k, \gamma_k^i)/k \in \{0,\dots,g\}\} \quad \forall l_i \in S,$$

$$\gamma_k^i = \max_y \min\{\mu_{l_i}(y), \mu_{s_k}(y)\},$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_i}(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the terms l_i and s_k, respectively.

Therefore, the result of τ_{SS_T} for any linguistic value of S is a fuzzy set defined in the BLTS, S_T.

Remark 3. In the case that the linguistic term set, S, of the non-homogeneous contexts let be chosen as BLTS, then the fuzzy set that represents a linguistic term will be all **0** except the value correspondent to the ordinal of the linguistic label that will be **1**.

Example 2. Let $S = \{l_0, l_1, \dots, l_4\}$ and $S_T = \{s_0, s_1, \dots, s_6\}$ be two term set, with 5 and 7 labels, respectively, and with the following semantics associated:

$l_0 = (0, 0, 0.25)$ $\quad s_0 = (0, 0, 0.16)$
$l_1 = (0, 0.25, 0.5)$ $\quad s_1 = (0, 0.16, 0.34)$
$l_2 = (0.25, 0.5, 0.75)$ $\quad s_2 = (0.16, 0.34, 0.5)$
$l_3 = (0.5, 0.75, 1)$ $\quad s_3 = (0.34, 0.5, 0.66)$
$l_4 = (0.75, 1, 1)$ $\quad s_4 = (0.5, 0.66, 0.84)$
$s_5 = (0.66, 0.84, 1)$
$s_6 = (0.84, 1, 1)$

The fuzzy set obtained after applying τ_{SS_T} for l_1 is (see Fig. 6)

$$\tau_{SS_T}(l_1) = \{(s_0, 0.39), (s_1, 0.85), (s_2, 0.85), (s_3, 0.39), (s_4, 0), (s_5, 0), (s_6, 0)\}.$$

3.1.2.3. Transforming interval valued into $F(S_T)$. Let $I = [\underline{i}, \overline{i}]$ be an interval valued in $[0, 1]$, to carry out this transformation we assume that the interval valued has a representation, inspired in the membership function of fuzzy sets [17], as follows:

$$\mu_I(\vartheta) = \begin{cases} 0 & \text{if } \vartheta < \underline{i}, \\ 1 & \text{if } \underline{i} \leqslant \vartheta \leqslant \overline{i}, \\ 0 & \text{if } \overline{i} < \vartheta, \end{cases}$$

where ϑ is a value in $[0, 1]$. In Fig. 7 can be observed the graphical representation of an interval.

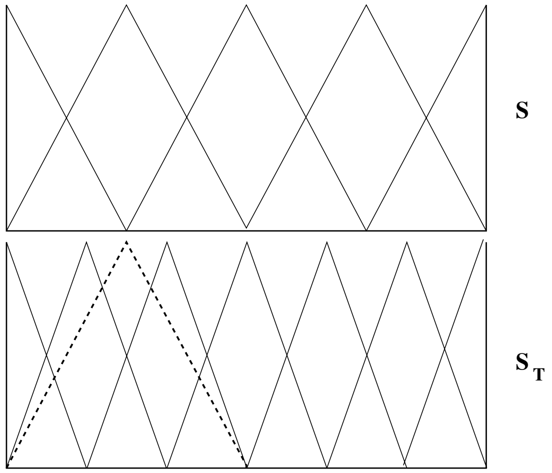

Fig. 6. Transforming $l_1 \in S$ into a fuzzy set in S_T.

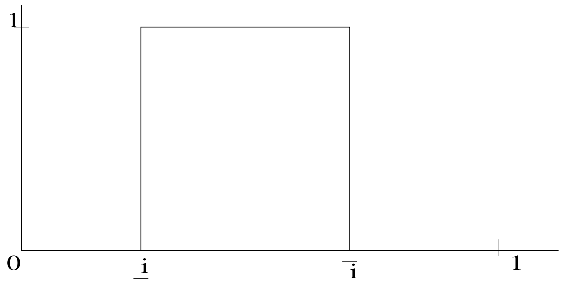

Fig. 7. Membership function of $I = [\underline{i}, \overline{i}]$.

Definition 5. Let $S_T = \{s_0, \ldots, s_g\}$ be a BLTS. Then, the function τ_{IS_T} transforms a interval valued I in $[0, 1]$ into a fuzzy set in S_T.

$$\tau_{\mathrm{IS}_T} : I \rightarrow F(S_T),$$

$$\tau_{\mathrm{IS}_T}(I) = \{(s_k, \gamma_k^i)/k \in \{0, \ldots, g\}\},$$

$$\gamma_k^i = \max_y \min\{\mu_I(y), \mu_{s_k}(y)\},$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_I(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions associated with the interval valued I and terms s_k, respectively.

Example 3. Let $I = [0.6, 0.78]$ be an interval valued to be transformed into a fuzzy set in S_T with five terms symmetrically distributed. The fuzzy set obtained after applying τ_{IS_T} is (see Fig. 8)

$$\tau_{\mathrm{IS}_T}([0.6, 0.78]) = \{(s_0, 0), (s_1, 0), (s_2, 0.6), (s_3, 1), (s_4, 0.2)\}.$$

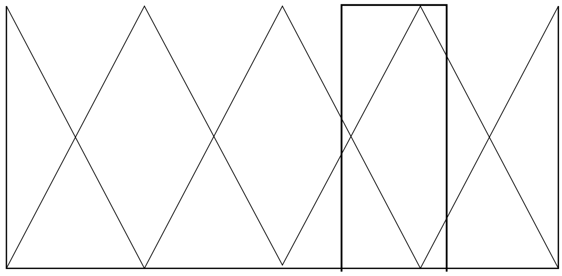

Fig. 8. Transforming [0.6, 0.78] into a fuzzy set in S_T.

3.2. Aggregating individual preference values

Using the above transformation functions the input information is expressed by means of fuzzy sets on the BLTS, $S_T = \{s_0, \ldots, s_g\}$, i.e., the input information is homogeneous (information of the same nature). Now we use an aggregation function for combining the fuzzy sets on the BLTS to obtain a collective preference value for each pair of alternatives that will be a fuzzy set on the BLTS.

At this moment, the preference relations are expressed by means of fuzzy sets on the BLTS as follows:

$$P_{e_k} = \begin{pmatrix} p^k_{11} = \{(s_0, \gamma^{11}_{k_0}), \ldots, (s_g, \gamma^{11}_{k_g})\} & \cdots & p^k_{1n} = \{(s_0, \gamma^{1n}_{k_0}), \ldots, (s_g, \gamma^{1n}_{k_g})\} \\ \vdots & \cdots & \vdots \\ p^k_{n1} = \{(s_0, \gamma^{n1}_{k_0}), \ldots, (s_g, \gamma^{n1}_{k_g})\} & \cdots & p^k_{nn} = \{(s_0, \gamma^{nn}_{k_0}), \ldots, (s_g, \gamma^{nn}_{k_g})\} \end{pmatrix},$$

where p^k_{ij} is the preference degree of the alternative x_i over x_j provided by the expert e_k.

We shall represent each fuzzy set, p^k_{ij}, as $r^k_{ij} = (\gamma^{ij}_{k_0}, \ldots, \gamma^{ij}_{k_g})$ being the values of r^k_{ij}, its respective membership degrees. Then, each preference value of the collective preference relation is obtained aggregating the fuzzy sets provided by each expert $\{r^k_{ij}\ \forall e_k\}$. This collective preference value, denoted as r_{ij}, is a new fuzzy set assessed in S_T, i.e.,

$$r_{ij} = (\gamma^{ij}_0, \ldots, \gamma^{ij}_g)$$

characterized by the following membership function:

$$\gamma^{ij}_v = f(\gamma^{ij}_{1_v}, \ldots, \gamma^{ij}_{k_v}),$$

where f is an "aggregation operator" and k is the number of experts.

3.3. Transforming into 2-tuple linguistic values

In this phase, we transform the fuzzy sets on the BLTS into linguistic 2-tuples over the BLTS, to facilitate the rank process involved in the exploitation phase of the decision process. In [11] was presented a function χ that transforms a fuzzy set into a numerical value in the interval of granularity of S_T, $[0, g]$:

$$\chi : F(S_T) \to [0, g],$$

$$\chi(F(S_T)) = \chi(\{(s_j, \gamma_j), j = 0, \ldots, g\}) = \frac{\sum_{j=0}^{g} j\gamma_j}{\sum_{j=0}^{g} \gamma_j} = \beta,$$

where the fuzzy set, $F(S_T)$ could be obtained from τ_{NS_T}, τ_{SS_T} or τ_{IS_T}.

Therefore, applying the (Definition 2) function Δ to β we shall obtain a collective preference relation whose values are expressed by means of linguistic 2-tuples:

$$\Delta(\chi(\tau(\vartheta))) = \Delta(\beta) = (s, \alpha).$$

4. A GDM problem with non-homogeneous information

Let us suppose that a company wants to renew its cars. There exist four models of car available, {CAR1, CAR2, CAR3, CAR4} and three experts provide his/her preference relations over the four cars (see Table 1). The first expert expresses his/her preference relation using numerical values in [0, 1], P_1^n. The second one expresses the preferences by means of linguistic values in a linguistic term set S (see Fig. 1), P_2^S. And the third expert can express them using interval-valued preference values in [0, 1], P_3^I. The three experts attempt to reach a collective decision.

We shall use a decision process to solve this problem with the two mentioned phases, aggregation and exploitation.

4.1. Aggregation phase

We use the arithmetic mean for aggregating the preference values.

1. *Making the information uniform*

   (a) Choose the BLTS. It will be S, due to the fact, it satisfies the conditions showed in Section 3.1.
   (b) Transforming the input information into $F(S_T)$. Applying the transformation functions from Definitions 3–5, the following fuzzy sets over the BLTS can be obtained:

$$P_1^n = \begin{pmatrix} - & (0,0,0,1,0,0,0) & (0,0,0,0,.19,.81,0) & (0,0,.59,.41,0,0,0) \\ (0,.19,.81,0,0,0,0) & - & (0,0,0,0,0,.59,.41) & (0,.19,.81,0,0,0,0) \\ (0,.19,.81,0,0,0,0) & (0,.81,.19,0,0,0,0) & - & (0,0,.59,.41,0,0,0) \\ (0,0,0,0,0,.59,.41) & (0,0,0,0,.19,.81,0) & (0,0,0,1,0,0,0) & - \end{pmatrix},$$

$$P_2^S = \begin{pmatrix} - & (0,0,0,0,1,0,0) & (0,0,0,0,0,1,0) & (0,0,0,1,0,0,0) \\ (0,0,1,0,0,0,0) & - & (0,0,0,0,1,0,0) & (0,0,0,0,0,1,0) \\ (0,1,0,0,0,0,0) & (1,0,0,0,0,0,0) & - & (0,0,0,0,0,1,0) \\ (0,0,1,0,0,0,0) & (0,1,0,0,0,0,0) & (1,0,0,0,0,0,0) & - \end{pmatrix},$$

$$P_3^I = \begin{pmatrix} - & (0,0,0,0,.81,.81,0) & (0,0,0,.12,1,.19,0) & (0,0,0,0,.19,1,.41) \\ (0,.19,1,.12,0,0,0) & - & (0,0,0,.41,1,.19,0) & (0,0,0,0,.19,1,.12) \\ (0,.19,1,.12,0,0,0) & (0,.19,1,.41,0,0,0) & - & (0,0,0,0,.81,1,.41) \\ (.41,1,.19,0,0,0,0) & (0,.81,1,.41,0,0,0) & (.41,1,.81,0,0,0,0) & - \end{pmatrix}.$$

Table 1
Preference relations

$$P_1^n = \begin{pmatrix} - & 0.5 & 0.8 & 0.4 \\ 0.3 & - & 0.9 & 0.3 \\ 0.3 & 0.2 & - & 0.4 \\ 0.9 & 0.8 & 0.5 & - \end{pmatrix} \quad P_2^S = \begin{pmatrix} - & H & VH & M \\ L & - & H & VH \\ VL & N & - & VH \\ L & VL & N & - \end{pmatrix} \quad P_3^I = \begin{pmatrix} - & [0.7,0.8] & [0.65,0.7] & [0.8,0.9] \\ [0.3,0.35] & - & [0.6,0.7] & [0.8,0.85] \\ [0.3,0.35] & [0.3,0.4] & - & [0.7,0.9] \\ [0.1,0.2] & [0.2,0.4] & [0.1,0.3] & - \end{pmatrix}$$

2. *Aggregating individual preference values*. When all information is expressed by means of fuzzy sets defined in a BLTS we use an aggregation operator for combining it. In this example we shall use as aggregation operator, f, the arithmetic mean obtaining the following collective preference relation:

$$P = \begin{pmatrix} - & (0,0,0,.33,.6,.27,0) & (0,0,0,.04,.4,.19,0) & (0,0,.2,.47,.06,.33,.04) \\ (0,.13,.94,.04,0,0,0) & - & (0,0,0,.14,.67,.26,.14) & (0,.06,.27,0,.06,.67,.04) \\ (0,.46,.6,.04,0,0,0) & (.33,.33,.4,.14,0,0,0) & - & (0,0,.2,.14,.27,.67,.14) \\ (.14,.33,.4,0,0,.20,.14) & (0,.6,.33,.14,.06,.27,0) & (.47,.33,.27,.33,0,0,0) & - \end{pmatrix}.$$

3. *Transforming into* 2-*tuple*. Now we transform the fuzzy sets, expressing the collective preferences, into linguistic 2-tuples using the functions χ and Δ. The result of this transformation is

$$P = \begin{pmatrix} - & (\mathrm{H},-.04) & (\mathrm{H},.24) & (\mathrm{H},-.42) \\ (\mathrm{L},-.08) & - & (\mathrm{H},.33) & (\mathrm{H},.03) \\ (\mathrm{L},-.38) & (\mathrm{VL},.29) & - & (\mathrm{H},.29) \\ (\mathrm{L},.45) & (\mathrm{L},.34) & (\mathrm{VL},.33) & - \end{pmatrix}.$$

4.2. Exploitation phase

The exploitation phase generates a solution set of alternatives (the best ones) for the decision problem. To do so, this phase uses a choice function to obtain the solution set. Different choice functions have been proposed in the choice theory literature [1,23,26].

In this paper, to obtain the solution set of alternatives we shall use a choice function that computes the *dominance degree* for each alternative, x_i, over the rest of alternatives. To do so, we shall use the following function:

$$\Lambda(x_i) = \frac{1}{n-1}\sum_{j=0|j\neq i}^{n} \beta_{ij},$$

where n is the number of alternatives and $\beta_{ij} = \Delta^{-1}(p_{ij})$ being p_{ij} a linguistic 2-tuple representing the collective value that expressed the preference of the alternative x_i over x_j according to the group of experts. Then, we shall choose as solution set of alternatives those with the highest value of dominance degree.

Remark 4. The selection of the dominance degree as choice function is for simplicity in the computations, but we can select any other choice function, based on strict dominance or non-dominance, to obtain a solution set of alternatives.

In this phase we shall calculate the dominance degree for this preference relation (see Table 2).

Then, the dominance degree rank the alternatives and we choose the best alternatives as solution set of GDM problem, in this example the solution set is {CAR1}.

Table 2
Dominance degree of the alternatives

CAR1	CAR2	CAR3	CAR4
(H, −0.08)	(M, 0.43)	(L, 0.4)	(L, 0.04)

5. Combining non-homogeneous information: Extensions

So far, we have presented an aggregation process for combining information of different nature composed by numerical, interval valued and linguistic values. In this section, we explain how to apply the above tools and process to deal with contexts that present linguistic information assessed in linguistic term sets with different granularity of uncertainty or semantics, i.e., multi-granular linguistic information, and with contexts in which the experts express their preferences using intuitionistic fuzzy sets.

5.1. Non-homogeneous contexts with linguistic multi-granular information

Several experts of a GDM problem can express their preference relations in a linguistic way but, using linguistic terms assessed in different linguistic term sets, S_l, with different granularity of uncertainty or semantics: $P^{S_l}_{e_k} : X \times X \rightarrow S_l$, where $P^{S_l}_{e_k}(x_i, x_j) = p^k_{ij}$ denotes the preference degree of the alternative x_i over x_j linguistically expressed, in the term set S_l, provided by the expert e_k.

In these contexts we can use the aggregation process presented in Section 3. But, to deal with multi-granular information, the aggregation process presents one difference: How to choose the BLTS? We show it subsequently.

We consider that S_T must be a linguistic term set which allows us to maintain the uncertainty degree associated to each expert and the ability of discrimination to express the preference values. With this goal in mind, we look for a BLTS with the maximum granularity. We take into consideration two possibilities:

- When there is only one term set with the maximum granularity, then, it is chosen as S_T (see Remark 3).
- If we have two or more linguistic term sets with maximum granularity then, S_T is chosen depending on the semantics of these linguistic term sets, finding two possible situations to establish S_T:
  1. All the linguistic term sets have the same semantics, then S_T is any one of them.
  2. There are some linguistic term sets with different semantics. Then, S_T is a basic linguistic term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see [21]). We define a BLTS with 15 terms and with the semantics (see Fig. 4).

Therefore, to make the information uniform, we shall use the function presented in Definition 4 to transform linguistic labels into fuzzy sets in the BLTS. But in this case the initial term set depends on the linguistic term set, S_l, used by each expert:

$$\tau_{S_l S_T} : S_l \rightarrow F(S_T),$$

$$\tau_{S_l S_T}(l_i) = \{(s_k, \alpha^i_k)/k \in \{0, \ldots, g\}\} \quad \forall l_i \in S_l,$$

$$\alpha^i_k = \max_y \min\{\mu_{l_i}(y), \mu_{s_k}(y)\}.$$

Once the multi-granular linguistic information has been unified into fuzzy sets in S_T the remaining of the aggregation process is similar to the process presented in Section 3.

5.2. Non-homogeneous contexts with intuitionistic fuzzy sets

Also an expert of a GDM problem can express his/her preferences by means of intuitionistic fuzzy sets (IFS) [20,29].

The IFS [2,22] are a tool based on fuzzy sets used to represent uncertainty.

Table 3
Interval-valued preference relation equivalent to intuitionistic fuzzy set preference relation

$$P^{\mathrm{IFS}} = \begin{pmatrix} - & (0.4, 0.3) & (0.3, 0.5) & (0.2, 0.6) \\ (0.6, 0.2) & - & (0.5, 0.4) & (0.25, 0.6) \\ (0.65, 0.3) & (0.6, 0.3) & - & (0.3, 0.6) \\ (0.8, 0.1) & (0.8, 0.15) & (0.7, 0.1) & - \end{pmatrix} \equiv P^{I} = \begin{pmatrix} - & [0.4, 0.7] & [0.3, 0.5] & [0.2, 0.4] \\ [0.6, 0.8] & - & [0.4, 0.6] & [0.25, 0.4] \\ [0.65, 0.7] & [0.6, 0.7] & - & [0.3, 0.4] \\ [0.8, 0.9] & [0.8, 0.85] & [0.7, 0.9] & - \end{pmatrix}$$

Definition 6 [2]. An IFS A in E is defined as an object of the following form:

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle / x \in E\},$$

where the function

$$\mu_A(x) : E \to [0, 1]$$

and

$$v_A(x) : E \to [0, 1]$$

define the degree of membership and the degree of non-membership to A of the element $x \in E$, respectively. And for every $x \in E$,

$$0 \leqslant \mu_A(x) + v_A(x) \leqslant 1.$$

Definition 7 [2]. The value of

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x)$$

is called the degree of non-determinacy (or uncertainty) of the element $x \in E$ to the IFS A.

In [16] is shown that an IFS, instead of describing a truth value as a single number μ, describes it by a pair of values (μ, v), where μ is the degree to which we believe in a given statement, and v is a degree to which we believe in its negation. The pair (μ, v) must satisfy the condition $\mu + v \leqslant 1$. Therefore, instead of using the pair (μ, v), we can describe the same truth values by two numbers μ and $v' = 1 - v$ which satisfy the condition $\mu \leqslant v'$. This condition is exactly the condition under which two real numbers form an interval.

Therefore, to deal with GDM problems that present preferences expressed by means of IFS, we shall transform them into interval-valued preferences (see Table 3), and afterwards, we can apply any of the aggregation processes just presented, depending on the context in which is defined the GDM problem.

6. Concluding remarks

We have presented an aggregation process for managing non-homogeneous information, with contexts composed by numerical, interval valued, and linguistic values, in GDM problems. This aggregation process is based on the unification of the information by means of fuzzy sets on a linguistic term set and afterwards they are transformed into linguistic 2-tuples to facilitate the exploitation phase of the decision model. Finally, we have shown how this aggregation process can be easily extended to deal with non-homogeneous contexts in which appears multi-granular linguistic information and/or intuitionistic fuzzy sets.

In the future we shall study the possibility of extending the process for managing other type of information representation as can be type 2 fuzzy sets [14] or fuzzy selected subsets [36].

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