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INDIVIDUAL AND SOCIAL STRATEGIES TO DEAL WITH IGNORANCE SITUATIONS IN MULTI-PERSON DECISION MAKING

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Multi-person decision making problems involve the preferences of some experts about a set of alternatives in order to find the best one. However, sometimes experts might not possess a precise or sufficient level of knowledge of part of the problem and as a consequence that expert might not give all the information that is required. Indeed, this may be the case when the number of alternatives is high and experts are using fuzzy preference relations to represent their preferences. In the literature, incomplete information situations have been studied, and as a result, procedures that are able to compute the missing information of a preference relation have been designed. However, these approaches usually need at least a piece of information about every alternative in the problem in order to be successful in estimating all the missing preference values.

In this paper, we address situations in which an expert does not provide any information about a particular alternative, which we call situations of *total ignorance*. We analyze several strategies to deal with these situations. We classify these strategies into: (i) *individual* strategies that can be applied to each individual preference relation without taking into account any information from the rest of experts and (ii) *social* strategies, that is, strategies that make use of the information available from the group of experts. Both individual and social strategies use extra assumptions or knowledge, which could not be directly instantiated in the experts preference relations. We also provide an analysis of the advantages and disadvantages of each one of the strategies presented, and the situations where some of them may be more adequate to be applied than the others.

Keywords: Ignorance; decision making; fuzzy preference relations; consistency; consensus.

1. Introduction

Multi-person decision-making (MPDM) consists of multiple individuals (experts) $E = \{e_1, \ldots, e_m\}$ interacting to reach a decision. Each expert may have unique motivations or goals and may approach the decision process from a different angle, but have a common interest in reaching eventual agreement on selecting the *best* option(s).[1,2] To do this, experts have to express their preferences by means of a set of evaluations over a set of alternatives for the problem $X = \{x_1, \ldots, x_n\}$ $(n \geq 2)$.

To be able to provide their preferences, experts must use a representation format that allows the expression of their opinions. There are several representation formats that have been used for this purpose, being *fuzzy preference relations* (FPRs) widely used and adopted because of their high expressivity, easiness of use (they allow to focus exclusively on two alternatives at a time) and because individual FPRs can be easily aggregated into a collective one.[3,5–10]

In the resolution process for an MPDM problem, it is possible to differentiate several components. Some of these components have a clear structure, and thus, it is possible to develop computer-driven systems and mathematical models to help the experts to reach a "good" final solution for the problem. For example, it is possible to develop several aggregation operators to automatically obtain a global or collective opinion from the individual opinions expressed by the experts[3,11–15]; it is possible to study the consistency of the preferences of the experts in order to evaluate the degree of contradiction introduced in the resolution process[7,16–22]; and it is also possible to study the consensus state of the decision process.[23–35] On the other hand, other components do not have a clear structure and thus cannot be fully modeled mathematically. For example, the phase when experts discuss the alternatives and negotiate about the best solution for the problem or when experts are expressing their preferences cannot easily be held by an automatic system. However, it is possible to provide some tools to help the experts in undertaking unstructured tasks.[31]

As each expert has his/her own experience concerning the problem being studied, we may face situations where a particular expert does not have a perfect knowledge of the problem to be solved.[18,36–43] Indeed, there may be cases in which an expert would not be able to efficiently express any kind of preference degree between two or more of the available options. This may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, because that expert is unable to discriminate the degree to which some options are better than others or even because of imposed time restrictions to solve the problem. Therefore, in these situations we have to deal with *incomplete* preference relations,[18,36,40,41,43] i.e. preference relations with some of their values missing or unknown.

Clearly, the above procedure would not be applicable in a successful way to a missing information situation in which some experts do not provide any information about a particular alternative. Thus, it is necessary to go further and address such situations, which we will call *total ignorance* or simply *ignorance* situations.

In this paper, we present several strategies to deal with ignorance, which can be applied in MPDM problems. These strategies are developed in order to help experts to efficiently express their preferences. We classify these strategies into: (i) *individual* strategies that can be applied to each individual preference relation without taking into account any information from the rest of experts and (ii) *social* strategies, that is, strategies that make use of the information available from the group of experts. Both individual and social strategies use extra assumptions or knowledge which could not by directly instantiated in the experts preference relations. We also provide an analysis of the advantages and disadvantages of each one of the strategies presented, and the situations where some of them may be more adequate to be applied than the others.

The remainder of the paper is set out as follows. Section 2 deals with the preliminaries necessary throughout the paper, i.e. the definition of incomplete FPR, the presentation of an estimation procedure to compute missing values in that relation, and the introduction of the formal concept of an ignorance situation. In Sec. 3, several individual strategies to solve an ignorance situation problem with incomplete FPRs are presented, while social strategies that make use of information about the group of experts are dealt with in Sec. 4. A discussion on the advantages and disadvantages of each of the strategies presented in this paper is given in Sec. 5, together with scenarios where each one of the strategies can be used as the best one. Finally, in Sec. 6 we draw our conclusions.

2. Preliminaries

In this section, we present the definition of incomplete FPRs, a procedure to estimate missing values in an incomplete FPR when we have at least a piece of information for every alternative in the problem, and the concept of ignorance situation.

2.1. *Incomplete FPRs*

When dealing with MPDM problems, a key factor is to model how the experts express their opinions. In the existing decision making models, several representation formats that allow the experts to express their preferences over the alternatives have been used. Pair comparisons of alternatives is usually used in many models because they integrate processes which are linked to some degree of credibility of preference of one alternative over another. In modeling these processes FPRs[3,4,8,44–46] are highly used because of their high expressivity, and their utility and easiness of use when we want to aggregate experts' preferences into group ones.[6–10]

Definition 2.1. An FPR P on a set of alternatives X is a fuzzy set on the product set $X \times X$, i.e. it is characterized by a membership function $\mu_P \colon X \times X \to [0,1]$.

When cardinality of X is finite, the FPR may be conveniently represented by the $n \times n$ matrix $P = (p_{ik})$, being $p_{ik} = \mu_P(x_i, x_k)$ ($\forall i, k \in \{1, \ldots, n\}$) interpreted as the preference degree or intensity of the alternative x_i over x_k: $p_{ik} = 1/2$ indicates indifference between x_i and x_k ($x_i \sim x_k$), $p_{ik} = 1$ indicates that x_i is absolutely preferred to x_k, and $p_{ik} > 1/2$ indicates that x_i is preferred to x_k ($x_i \succ x_k$). Based on this interpretation, we have that $p_{ii} = 1/2 \;\; \forall i \in \{1, \ldots, n\}$ ($x_i \sim x_i$).

Resolution processes of MPDM problems usually assume that experts are always able to provide all the preference values required, that is, to provide all p_{ik} values. This situation is not always possible to achieve. Experts could have some difficulties in giving all their preferences due to lack of knowledge about a part of the problem, or simply because they may not be able to quantify some of their preferences. To model such situations we use the concept of an *incomplete FPR*.[18, 40]

Definition 2.2. A function $f \colon X \to Y$ is *partial* when not every element in the set X necessarily maps onto an element in the set Y. When every element from the set X maps onto one element of the set Y then we have a *total* function.

Definition 2.3 (Herrera-Viedma *et al.*, Ref. 18). An *incomplete FPR P* on a set of alternatives X is a fuzzy set on the product set $X \times X$ that is characterized by a *partial* membership function.

We note that a missing value in an FPR is not equivalent to a lack of preference of one alternative over another. A missing value can be the result of the incapacity of an expert to quantify the degree of preference of one alternative over another, in which case he/she may decide not to "guess" to maintain the consistency of the values already provided. It must be clear then that when an expert is not able to express the particular value p_{ij}, because he/she does not have a clear idea of how better alternative x_i is over alternative x_j, this does not automatically mean that he/she prefers both alternatives with the same intensity, that is, we cannot directly assume that $p_{ik} = 0.5$. Therefore, when a particular preference value p_{ik} is not given by an expert we will call it a *missing value*. Missing values when required will be denoted by $p_{ik} = x$.

Given an incomplete FPR P^h, the following sets are defined[18]:

$$
\begin{aligned}
A &= \{(i,j) \mid i,j \in \{1,\ldots,n\} \wedge i \neq j\},\\
\mathrm{MV}^h &= \{(i,j) \in A \mid p_{ij}^h = x\},\\
\mathrm{EV}^h &= A \setminus \mathrm{MV}^h,\\
\mathrm{EV}_i^h &= \{(a,b) \mid (a,b) \in \mathrm{EV}^h \;\wedge\; (a = i \vee b = i)\},
\end{aligned}
$$

where MV^h is the set of pairs of alternatives for which the preference degree of the first alternative over the second one is not given by expert e_h, that is, the set of *missing values* for expert e_h; EV^h is the set of pairs of alternatives for which the expert e_h provides preference values (we call it the *expert values* for e_h); and EV_i^h is the set of pairs of alternatives involving alternative x_i for which expert e_h provides a preference value.

We must note that given the definition of incomplete FPR it is possible for experts to provide preference relations with many missing values, being the extreme case when an expert provides a completely empty FPR. Although that kind of situation is possible, it is not usually desirable in an MPDM process. In real MPDM situations, experts that provide FPR with many missing values might not being taken into account by the rest of experts to obtain the final solution of the problem. This might be the case when an expert does not provide at least a certain number of preference values. The following completeness measure for an incomplete FPR P^h is defined[18]:

$$CP_{P^h} = \frac{\#\text{EV}^h}{n^2 - n}, \tag{1}$$

where $\#\text{EV}^h$ is the number of values that expert e_h gave and $n^2 - n$ is the maximum number of values that an expert can provide (all p_{ik} with $i \neq k$). Thus, if $CP_{P^h} = 1$ then P^h has no missing values (it is complete) and if $CP_{P^h} = 0$ then all preference values of P^h are unknown.

By using this completeness measure, the experts solving the MPDM problem can decide that only those experts with $CP_{P^h} \geq \psi$ can participate in the decision process, being $\psi \in [0, 1]$ a certain threshold fixed prior to the beginning of the decision process.

2.2. *A procedure to estimate missing values in incomplete FPRs*

In Ref. 18 a procedure was developed to estimate the missing values in an incomplete FPR. This procedure only uses the known preference values in a particular incomplete FPR and is based on the additive transitivity property[9]:

$$p_{ik} = p_{ij} + p_{jk} - 0.5 \quad \forall i, j, k \in \{1, \ldots, n\}. \tag{2}$$

The underlying concept on which the additive transitivity property is based has been applied in both Saaty's AHP[47–51] and Fishburn SSB Utility Theory.[52] In the first case, as shown in Ref. 7, additive transitivity for FPRs can be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations. In the second case, as shown in Ref. 17 if we represent the degree of preference of x_i over x_j by means of a Skew-Symmetric Bilinear function $\phi(x_i, x_j) \in R$ the consistency condition can be stated as

$$\phi(x_i, x_j) + \phi(x_j, x_k) = \phi(x_i, x_k),$$

which corresponds to the additive transitivity property, taking into account that Fishburn represents indifference with the value of 0.

We acknowledge that additive transitivity is a condition difficult to be satisfied by experts' preferences. However, as shown in Refs. 7 and 19, additive transitivity can be used to obtain more consistent FPR from a given one and as shown in Refs. 18 and 53, it is also a valuable concept for incomplete FPRs as it reduces experts' uncertainty when choosing values to estimate their unknown ones, which is not the case if other types of weaker transitivity conditions were to be used.

For a complete FPR, expression 2 can be used to calculate an estimated value cp_{ik} for every p_{ik} as

$$cp_{ik} = \frac{\sum_{j=1; i \neq k \neq j}^{n} (cp_{ik}^{j1} + cp_{ik}^{j2} + cp_{ik}^{j3})}{3(n-2)}, \tag{3}$$

where

$$cp_{ik}^{j1} = p_{ij} + p_{jk} - 0.5, \tag{4}$$

$$cp_{ik}^{j2} = p_{jk} - p_{ji} + 0.5, \tag{5}$$

$$cp_{ik}^{j3} = p_{ij} - p_{kj} + 0.5. \tag{6}$$

Note 1. Clearly, when the values given by an expert fully comply with the additive transitivity property, all the "partial estimations," derived via (4), (5), and (6), coincide with the value to be estimated. When this is not the case, the partial estimation might not be equal, and therefore we take the average value of all possible partial estimations as the final estimation for p_{ik}. This final estimation is the one that will better conform with the additive transitivity property, and thus, the one that maximizes the consistency of the preference relation. For more details, see Ref. 18.

Equation (3) cannot be directly applied to incomplete FPRs because some of the preference values used in (4)–(6) are unknown. As a consequence, not all of the missing values could be estimated in one step. An iterative procedure was developed in which the following two different tasks are carried out in each step of it[18]:

(A) Establish the elements that can be estimated in each step of the procedure.
(B) Produce the expression that will be used to estimate a particular missing value.

(A) Elements to be estimated in step t

The subset of missing values MV^h that can be estimated in step t of this procedure is denoted by EMV_t^h (*estimated missing values*) and defined as follows:

$$\text{EMV}_t^h = \left\{ (i,k) \in \text{MV}^h \Bigg\backslash \bigcup_{l=0}^{t-1} \text{EMV}_l^h \mid \exists j \in (H_{ik}^h)^t \right\},$$

with

$$(H_{ik}^h)^t = (H_{ik}^{h1})^t \cup (H_{ik}^{h2})^t \cup (H_{ik}^{h3})^t,$$

$$(H_{ik}^{h1})^t = \left\{ j \mid (i,j),(j,k) \in \left\{ \mathrm{EV}^h \bigcup_{l=0}^{t-1} \mathrm{EMV}_l^h \right\} \right\},$$

$$(H_{ik}^{h2})^t = \left\{ j \mid (j,i),(j,k) \in \left\{ \mathrm{EV}^h \bigcup_{l=0}^{t-1} \mathrm{EMV}_l^h \right\} \right\},$$

$$(H_{ik}^{h3})^t = \left\{ j \mid (i,j),(k,j) \in \left\{ \mathrm{EV}^h \bigcup_{l=0}^{t-1} \mathrm{EMV}_l^h \right\} \right\},$$

$\mathrm{EMV}_0^h = \emptyset$ (by definition); $(H_{ik}^{h1})^t$, $(H_{ik}^{h2})^t$, $(H_{ik}^{h3})^t$ are the sets of intermediate alternatives x_j $(j \neq i,k)$ that can be used to estimate the preference value p_{ik}^h $(i \neq k)$ using expressions (4)–(6), respectively.

When $\mathrm{EMV}_{\mathrm{maxIter}}^h = \emptyset$ with $\mathrm{maxIter} > 0$ the procedure will stop as there will not be any more missing values to be estimated. Moreover, if $\bigcup_{l=0}^{\mathrm{maxIter}}$, $\mathrm{EMV}_l^h = \mathrm{MV}^h$ then all missing values are estimated, and consequently, the procedure is said to be successful in the completion of the incomplete FPR.

(*B*) *Expression to estimate a particular value* p_{ik} *in step* t

In order to estimate a particular value p_{ik}^h with $(i,k) \in \mathrm{EMV}_t^h$, the following function is applied:

```
function estimate_p(h,i,k,t)
```

(1) $cp_{ik}^{h1} = 0, \;\; cp_{ik}^{h2} = 0, \;\; cp_{ik}^{h3} = 0, \;\; \mathcal{K} = 0$

(2) $if \;\; \#(H_{ik}^{h1})^t \neq 0 \;\; \Rightarrow \;\; cp_{ik}^{h1} = \dfrac{\sum_{j \in (H_{ik}^{h1})^t} (cp^h)_{ik}^{j1}}{\#(H_{ik}^{h1})^t} \;\; ; \;\; \mathcal{K}++.$

(3) $if \;\; \#(H_{ik}^{h2})^t \neq 0 \;\; \Rightarrow \;\; cp_{ik}^{h2} = \dfrac{\sum_{j \in (H_{ik}^{h2})^t} (cp^h)_{ik}^{j2}}{\#(H_{ik}^{h2})^t} \;\; ; \;\; \mathcal{K}++.$

(4) $if \;\; \#(H_{ik}^{h3})^t \neq 0 \;\; \Rightarrow \;\; cp_{ik}^{h3} = \dfrac{\sum_{j \in (H_{ik}^{h3})^t} (cp^h)_{ik}^{j3}}{\#(H_{ik}^{h3})^t} \;\; ; \;\; \mathcal{K}++.$

(5) Calculate $cp_{ik}^h = \frac{1}{\mathcal{K}} \left(cp_{ik}^{h1} + cp_{ik}^{h2} + cp_{ik}^{h3} \right)$

```
end function
```

The function *estimate_p*(h,i,k,t) computes the final estimated value of the missing value, cp_{ik}^h, as the average of all estimated values that can be calculated using all the possible intermediate alternatives x_j and using the three possible expressions (4)–(6).

We should point out that some estimated values of an incomplete FPR could lie outside the unit interval, i.e. for some (i,k) we may have $cp_{ik}^h < 0$ or $cp_{ik}^h > 1$.

In order to normalize the expression domains in the decision model, the following function is used:

$$f(y) = \begin{cases} 0 & \text{if } y < 0, \\ 1 & \text{if } y > 1, \\ y & \text{otherwise.} \end{cases}$$

Then, the *iterative estimation procedure pseudo-code* is as follows:

```
   ITERATIVE ESTIMATION PROCEDURE
0. EMV_0^h = ∅
1. t = 1
2. while (EMV_t^h ≠ ∅) {
3.    for every (i,k) ∈ EMV_t^h {
4.       estimate_p(h,i,k,t)
5.    }
6.    t++
7. }
```

In Ref. 18, the following sufficient conditions that guarantee that the above iterative estimation procedure is successful in estimating all the missing values of an incomplete FPR were established:

Condition 1. If for all $p_{ik}^h \in \mathrm{MV}^h$ $(i \neq k)$ there exists at least a $j \in \{H_{ik}^{h1} \cup H_{ik}^{h2} \cup H_{ik}^{h3}\}$ then all missing preference values can be estimated in the first iteration of the procedure $(\mathrm{EMV}_1^h = \mathrm{MV}^h)$.

Condition 2. Under the assumption of additive consistency property, an incomplete FPR can be completed when it is known the following set of $n-1$ preference values $\{p_{12}, p_{23}, \ldots, p_{(n-1)n}\}$.

Condition 3. An incomplete FPR can be completed if a set of $n-1$ nonleading diagonal preference values, where each one of the alternatives is compared at least once, is known.

This means that for the above procedure to be successful in completing preference relations, at least some preference value that links every alternative of the problem with the others (through a chain of alternatives) is needed.

We must note that the presented estimation procedure is based on the additive transitivity property but it does not require that the experts should comply with this property in order to obtain the estimations, that is, the preferences expressed by the experts do not have to obey this transitivity property. Moreover, in real decision making situations are not uncommon that experts provide preferences, which do not fully comply with any transitivity or even reciprocity properties. In this work, the additive transitivity property is used *as a guide* for the estimation procedure in order to obtain estimations of the missing preference values, which are compatible with information provided by the expert. In addition, if the additive transitivity property is not appropriate for a particular decision problem, the procedure can be adapted to use different transitivity properties in order to estimate the missing values.[54,55]

2.3. *Ignorance situations in MPDM problems*

Clearly, not all MPDM problems with incomplete information will meet the above requirement, i.e. there might be situations in which at least one expert might provide no information about one or more alternatives. We will refer to these as "ignorance situations":

Definition 2.4. Let $E = \{e_1, \ldots, e_m\}$ be a group of experts, which have expressed their preferences on a set of alternatives $X = \{x_1, \ldots, x_n\}$ by means of a set of incomplete FPRs $\{P^1, \ldots, P^m\}$. We define an ignorance situation in MPDM when at least one of the experts, $e_h \in E$, does not provide preference values involving one alternative $x_i \in X$:

$$\exists i, \quad h \mid EV_i^h = \emptyset,$$

x_i is called the unknown or ignored alternative for e_h.

In the following sections, we present several strategies to deal with ignorance, which can be applied in MPDM problems. We classify these strategies into: (i) *individual* strategies that can be applied to each individual preference relation without taking into account any information from the rest of experts and (ii) *social* strategies that make use of the information available from the group of experts.

3. Individual Strategies to Deal with Ignorance Situations

The aim of these strategies is to obtain estimated values of the missing preference values for an individual expert using only the information he/she provided. These estimated values will be obtained by applying the estimation procedure presented in Sec. 2.2, and therefore we will refer to them as consistency guided individual strategies. As aforementioned, for each unknown alternative (x_i) the estimated procedure needs at least one preference value (p'_{ik}) to be known to initiate the estimation of the rest of missing preference values. We refer to the initial preference values as the *seed values*.

The basic structure of all the presented strategies consists of two different phases: in the first one we fix the particular seed values that will be used on the strategy; while in the second one estimated values for the rest of missing values are obtained. The purpose of the first phase is to provide some initial information to the estimation procedure to be able to compute the missing values. With the application of the estimation procedure in the second phase we try to adjust the seed values according to the previously given preference values of the expert by means of the additive consistency property, which is used in the procedure. In this way, the final estimated preference values are more compatible with the initial preferences given by the expert.

Clearly, the strategies to apply here will be influenced by the way the seed values are chosen. We present the following two individual strategies:

$$\text{Consistency Guided Individual Strategies} \begin{cases} \text{Based on Indifference Seed Values} \\ \text{Based on Alternative Proximity Seed Values.} \end{cases}$$

3.1. *Strategy 1: Consistency individual strategy based on indifference seed values*

Let P be an incomplete FPR with an unknown alternative x_i (every $p_{ij} = x$ and $p_{ji} = x$). In this strategy, we start by assuming indifference for the seed values p'_{ij} and p'_{ji}, i.e. we assume that $p'_{ij} = p'_{ji} = 0.5$. Once this assumption is made we apply the consistency-based estimation procedure to obtain a final estimated value for every missing preference value p_{ik} via (4)–(6):

$$\begin{aligned} cp_{ik}^{j1} &= p'_{ij} + p_{jk} - 0.5 \Rightarrow cp_{ik}^{j1} = p_{jk}, \\ cp_{ik}^{j2} &= p_{jk} - p'_{ji} + 0.5 \Rightarrow cp_{ik}^{j2} = p_{jk}, \\ cp_{ik}^{j3} &= p'_{ij} - p_{kj} + 0.5 \Rightarrow cp_{ik}^{j3} = 1 - p_{kj}. \end{aligned}$$

Obviously, the indifference of a preference value can be assumed for any of the possible values of $j \in \{1, \ldots, n\}$ with $j \neq i \neq k$. If this is done, for each unknown preference value p_{ik} we will end with $n-2$ triplets of estimated values $(cp_{ik}^{j1}, cp_{ik}^{j2}, cp_{ik}^{j3})$. Thus, we obtain the following final estimated value of the missing values of the ith row of the incomplete FPR:

$$\begin{aligned} cp_{ik} &= \frac{1}{3}\left(\frac{\sum_{j=1;j\neq i\neq k}^{n} cp_{ik}^{j1}}{n-2} + \frac{\sum_{j=1;j\neq i\neq k}^{n} cp_{ik}^{j2}}{n-2} + \frac{\sum_{j=1;j\neq i\neq k}^{n} cp_{ik}^{j3}}{n-2}\right) \\ &= \frac{1}{3}\left(\frac{\sum_{j=1;j\neq i\neq k}^{n} p_{jk}}{n-2} + \frac{\sum_{j=1;j\neq i\neq k}^{n} p_{jk}}{n-2} + \frac{\sum_{j=1;j\neq i\neq k}^{n} (1-p_{kj})}{n-2}\right) \\ &= \frac{1}{3} + \frac{2 \cdot SC_k}{3} - \frac{SR_k}{3}, \end{aligned}$$

with SC_k and SR_k representing the average of the kth column and kth row of the complete $(n-1) \times (n-1)$ FPR obtained by removing the ith columnn and row. The symmetrical application of the above assumption for the preference value p_{ki} provides the following estimate of the missing values of the ith column of the incomplete FPR:

$$cp_{ki} = \frac{1}{3} + \frac{2 \cdot SR_k}{3} - \frac{SC_k}{3}$$

We note that the truncation step proposed in Sec. 2.2 does not apply in this strategy because the estimated missing values always lie in the unit interval $[0, 1]$. Indeed, because $p_{jk}, p_{kj} \in [0, 1]$ $(\forall j)$ we have that $SC_k, SR_k \in [0, 1]$ $(\forall k)$ and therefore $cp_{ik} \in [0, 1]$ $(\forall k)$.

Example 3.1. We have to solve a decision making problem to find the best of four different alternatives: $X = \{x_1, x_2, x_3, x_4\}$. Expert e_1 gives the following incomplete FPR

$$P^1 = \begin{pmatrix} - & 0.7 & x & 0.4 \\ 0.4 & - & x & 0.3 \\ x & x & - & x \\ 0.6 & 0.75 & x & - \end{pmatrix},$$

with no information about alternative x_3.

The application of this first strategy for the values p^1_{13} and p^1_{34} provides the following estimated values:

$$p^1_{13} = \frac{1}{3} + \frac{2 \cdot (0.7 + 0.4)/2}{3} - \frac{(0.4 + 0.6)/2}{3} = 0.53,$$

$$p^1_{34} = \frac{1}{3} + \frac{2 \cdot (0.4 + 0.3)/2}{3} - \frac{(0.6 + 0.75)/2}{3} = 0.34.$$

The complete preference relation obtained is

$$P^1 = \begin{pmatrix} - & 0.7 & \mathbf{0.53} & 0.4 \\ 0.4 & - & \mathbf{0.33} & 0.3 \\ \mathbf{0.48} & \mathbf{0.7} & - & \mathbf{0.34} \\ 0.6 & 0.75 & \mathbf{0.67} & - \end{pmatrix}.$$

3.2. *Strategy 2: Consistency individual strategy based on alternative proximity seed values*

In this strategy the seed values for an unknown alternative, instead of being taken as the indifference value 0.5, will be obtained from the preference values given to similar alternatives. This is possible if extra information or properties about alternatives are known to strongly suggest that the ignored alternative is similar to another alternative. In this case, we can pick similar preference values of the similar alternative and use them as seed values for the estimation procedure.

In general, if we have an incomplete FPR P^h with an unknown alternative x_i, and we have information that suggests that alternative x_i is similar to alternative x_j, as it could be that of sharing several characteristics, we apply the following scheme:

```
1. ∀k ≠ i ≠ j do {
2.     p_{ik}^{h'} = rand(p_{jk}^h − δ, p_{jk}^h + δ)
3. }
4. ∀k ≠ i ≠ j do {
5.     p_{ki}^{h'} = rand(p_{kj}^h − δ, p_{kj}^h + δ)
6. }
7. Apply the estimation procedure
```

where δ is a small factor (for example, 0.1) that determines a small change from the preference values of the similar alternative. This factor must be fixed prior to the application of the procedure (for example, if there exists a moderator, he/she can fix δ at the beginning of the resolution process). It is used because we do not consider both x_i and x_j identical but similar, and thus, the preference values used as seed values do not have to be completely equal to the similar alternative. This δ factor provides some diversity to the estimation procedure but, if necessary, δ could be set to 0 and thus, the seed values would be identical to the ones of the similar alternative.

We must note that the similarities between the alternatives is additional external information about the problem, and this kind of information might not be available for every decision problem.

Example 3.2. We part from the incomplete FPR of the previous example and assume that alternative x_2 is similar to alternative x_3. We also assume that the moderator fixed $\delta = 0.1$ at the beginning of the resolution process. In the first phase of the strategy, we obtain the seed values as small random variations of the preference values that involve alternative x_2:

$$p_{31} = \text{random}(p_{21} - \delta, p_{21} + \delta) = 0.42,$$

$$p_{34} = \text{random}(p_{24} - \delta, p_{24} + \delta) = 0.32,$$

$$p_{13} = \text{random}(p_{12} - \delta, p_{12} + \delta) = 0.72,$$

$$p_{43} = \text{random}(p_{42} - \delta, p_{42} + \delta) = 0.71.$$

Once the seed values have been fixed we apply the estimation procedure to obtain the final estimations for the missing values:

$$\begin{pmatrix} - & 0.7 & x & 0.4 \\ 0.4 & - & x & 0.3 \\ x & x & - & x \\ 0.6 & 0.75 & x & - \end{pmatrix} \rightarrow \begin{pmatrix} - & 0.7 & [0.72] & 0.4 \\ 0.4 & - & x & 0.3 \\ [0.42] & x & - & [0.32] \\ 0.6 & 0.75 & [0.71] & - \end{pmatrix}$$

$$\underset{\substack{\text{Est.}\\ \text{Proc.}}}{\rightarrow} \begin{pmatrix} - & 0.7 & \mathbf{0.6} & 0.4 \\ 0.4 & - & \mathbf{0.51} & 0.3 \\ \mathbf{0.41} & \mathbf{0.54} & - & \mathbf{0.31} \\ 0.6 & 0.75 & \mathbf{0.77} & - \end{pmatrix}$$

Note that the result of this strategy are not equal, although close, to the results obtained from the application of strategy 1 because they use different approaches to obtain the seed values. In strategy 1 it is assumed that the expert is indifferent between x_3 and the rest of alternatives while in this strategy the expert is considered to have similar opinions about x_3 and x_2 (because both alternatives are assumed to be similar), but not necessarily similar to the rest of alternatives. Moreover, from this example we see that both p_{32} and p_{23} have a value close to 0.5, which agrees with the assumption of this strategy of being x_3 and x_2 similar.

In general, if we have that the unknown alternative x_i is similar to a known one x_j, then the application of this strategy would result in $p^h_{ik} = p^h_{jk} + \alpha;\ \alpha \in [-\delta, +\delta]$. When applying the estimation procedure we can obtain

$$\begin{aligned}
cp^1_{ij} &= p^h_{ik} + p^h_{kj} - 0.5 = p^h_{jk} + \alpha + p^h_{kj} - 0.5,\\
cp^2_{ij} &= p^h_{kj} - p^h_{ki} + 0.5 = p^h_{kj} - p^h_{kj} - \alpha + 0.5 = 0.5 - \alpha,\\
cp^3_{ij} &= p^h_{ik} - p^h_{jk} + 0.5 = p^h_{jk} + \alpha - p^h_{jk} + 0.5 = \alpha + 0.5.
\end{aligned}$$

Under reciprocity, we would have

$$cp^h_{ij} = \frac{1}{3}(cp^1_{ij} + cp^2_{ij} + cp^3_{ij}) = 0.5 + \frac{\alpha}{3}.$$

However, if reciprocity is not guaranteed but the expert's preferences are highly consistent, then it would be $p^h_{kj} + p^h_{jk} = 1 + \beta$, with β a small value. In this case, we would have that

$$cp^h_{ij} = \frac{1}{3}(cp^1_{ij} + cp^2_{ij} + cp^3_{ij}) = 0.5 + \frac{\alpha}{3} + \frac{\beta}{3}.$$

The same reasoning can be applied to show that that cp^h_{ji} is to be close to 0.5.

4. Social Strategies to Deal with Ignorance Situations

In this section, we present three strategies to solve ignorance situations in MPDM that take into account some *social* criteria. The first strategy makes use of the information provided by the set of experts, that is, using consensus preference values of the collective preference relation, which is computed by aggregating all the experts' individual preference relations,[31–33, 43, 56] while the second one uses only the consensus preference values provided by those experts nearest to the expert whose preference relation we try to complete. This second strategy tries to help the decision process to reach a solution of consensus by narrowing the differences between the expert with an ignored alternative and those others who have a similar opinion about the rest of alternatives. As both strategies use consensus preference values we refer to them as consensus-guided social strategies. Finally, a hybrid strategy is defined to combine both social strategies in order to exploit both their advantages.

$$\text{Consensus-Based Social Strategies} \begin{cases} \text{Collective Seed Value} \\ \text{Expert Proximity Seed Value} \\ \text{Hybrid.} \end{cases}$$

4.1. *Strategy 3: Consensus social strategy based on a collective seed value*

This strategy is based on the use of seed values chosen among the consensus preference values of the *collective* FPR. The collective preference relation is computed by aggregating all the individual preference relations given by the experts, and therefore, it represents the consensus opinions about the alternatives of all the experts as a group.[10, 11, 18, 31, 33, 46]

Given an incomplete FPR P^h with an unknown alternative x_i, and the collective preference relation P^c. We apply the following scheme:

```
1. ∀k ≠ i ≠ j do {
2.      p_{ik}^{h'} = p_{ik}^c
3. }
4. ∀k ≠ i ≠ j do {
5.      p_{ki}^{h'} = p_{ki}^c
6. }
7. Apply the estimation procedure
```

Example 4.1. Let us suppose that expert e_1 provides the incomplete FPR given in the first example, P^1, and that the collective preference relation P^c is

$$P^c = \begin{pmatrix} - & 0.43 & 0.57 & 0.42 \\ 0.5 & - & 0.61 & 0.55 \\ 0.38 & 0.5 & - & 0.44 \\ 0.67 & 0.5 & 0.33 & - \end{pmatrix}.$$

To estimate the missing values in P^1 we firstly set as seed values the preference values involving the unknown alternative x_3 from the collective preference relation P^c and then we apply the estimation procedure:

$$\begin{pmatrix} - & 0.7 & x & 0.4 \\ 0.4 & - & x & 0.3 \\ x & x & - & x \\ 0.6 & 0.75 & x & - \end{pmatrix} \to \begin{pmatrix} - & 0.7 & [0.57] & 0.4 \\ 0.4 & - & [0.61] & 0.3 \\ [0.38] & [0.5] & - & [0.44] \\ 0.6 & 0.75 & [0.33] & - \end{pmatrix}$$

$$\underset{\substack{\text{Est.} \\ \text{Proc.}}}{\to} \begin{pmatrix} - & 0.7 & \mathbf{0.52} & 0.4 \\ 0.4 & - & \mathbf{0.32} & 0.3 \\ \mathbf{0.47} & \mathbf{0.66} & - & \mathbf{0.27} \\ 0.6 & 0.75 & \mathbf{0.75} & - \end{pmatrix}.$$

4.2. *Strategy 4: Consensus social strategy based on an expert proximity seed value*

This approach computes the seed values from the preference values provided by those experts nearest to the expert whose preference relation we try to complete. These similar experts' preference values are then used to estimate the unknown values, thus helping the experts to approach faster to a solution of consensus.

In this case, if an incomplete FPR P^h given by expert e_h has an unknown alternative x_i, we apply the following scheme:

```
1. For every expert e_v ∈ E,  e_v ≠ e_h  {
2.     Compute d_v = dist(e_v, e_h)
3. }
4. NE = {e_v | d_v < γ}
5. if (#NE < 2) {
6.     NE = the 2 experts closer to e_h
7. }
8. p^h_ik = φ(p^v_ik),  ∀v|e_v ∈ NE
```

where dist$(\cdot,\cdot)$ is a distance function that is used to find those experts within a γ distance from expert e_h, and ϕ is an aggregation operator. For the sake of simplicity we propose the use of the arithmetic mean as aggregation operator. Note that because the distance function may depend on the particular problem to be solved, we do not provide a particular expression of it in this paper.

Example 4.2. We assume again that expert e_1 provides the incomplete preference relation of the first example (P^1), and that experts e_2, e_3 and e_4 provide the following preference relations:

$$P^2 = \begin{pmatrix} - & 0.6 & 0.4 & 0.7 \\ 0.4 & - & 0.7 & 0.4 \\ 0.6 & 0.35 & - & 0.6 \\ 0.3 & 0.7 & 0.4 & - \end{pmatrix}; \quad P^3 = \begin{pmatrix} - & 0.3 & 0.6 & 0.25 \\ 0.7 & - & 0.55 & 0.5 \\ 0.4 & 0.45 & - & 0.7 \\ 0.8 & 0.5 & 0.3 & - \end{pmatrix};$$

$$P^4 = \begin{pmatrix} - & 0.6 & 0.5 & 0.5 \\ 0.4 & - & 0.65 & 0.4 \\ 0.5 & 0.35 & - & 0.7 \\ 0.5 & 0.7 & 0.3 & - \end{pmatrix}.$$

For this example, we compute the distance between the experts using the arithmetic mean of the difference between the preference values given by each expert:

$$d_2 = \text{dist}(e_2, e_1) = 0.14; \quad d_3 = \text{dist}(e_3, e_1) = 0.25; \quad d_4 = \text{dist}(e_4, e_1) = 0.08.$$

Given a value of $\gamma = 0.15$, then $NE = \{e_2, e_4\}$ and the unknown preference values of P^1 are computed as the arithmetic mean of the corresponding preference values of experts e_2 and e_4:

$$\begin{pmatrix} - & 0.7 & x & 0.4 \\ 0.4 & - & x & 0.3 \\ x & x & - & x \\ 0.6 & 0.75 & x & - \end{pmatrix} \rightarrow \begin{pmatrix} - & 0.7 & \mathbf{0.45} & 0.4 \\ 0.4 & - & \mathbf{0.67} & 0.3 \\ 0.55 & \mathbf{0.35} & - & \mathbf{0.65} \\ 0.6 & 0.75 & \mathbf{0.35} & - \end{pmatrix}.$$

4.3. *Strategy 5: Hybrid strategy*

This strategy integrates the previous two by taking into account both information from the collective preference relation and from the nearest experts. Let x_i be the unknown alternative for expert e_h, and P^c the collective preference relation:

1. Compute $(p_{ik}^h)^1$ and $(p_{ki}^h)^1$ $\forall k \in \{1, \ldots, n\}$ with *strategy 3* (Based on the collective preference relation)
2. Compute $(p_{ik}^h)^2$ and $(p_{ki}^h)^2$ $\forall k \in \{1, \ldots, n\}$ with *strategy 4* (Based on the proximity of experts)
3. Compute $p_{ik}^h = \frac{(p_{ik}^h)^1 + (p_{ik}^h)^2}{2}$, $p_{ki}^h = \frac{(p_{ki}^h)^1 + (p_{ki}^h)^2}{2}$ $\forall k \in \{1, \ldots, n\}$.

We note that a similar scheme could be used to hybridize all the presented strategies, including the individual ones.

Example 4.3. We assume again that we have the same data than in the previous examples. This hybrid strategy leads to

$$P^1 = \begin{pmatrix} - & 0.7 & \mathbf{0.59} & 0.68 \\ 0.4 & - & \mathbf{0.72} & 0.7 \\ \mathbf{0.47} & \mathbf{0.41} & - & \mathbf{0.54} \\ 0.6 & 0.75 & \mathbf{0.54} & - \end{pmatrix}.$$

5. Analysis of the Advantages and Disadvantages of Each Strategy

In this section we will discuss the advantages and disadvantages of each one of the five strategies, and the situations where some of them may be more adequate to be applied than the others.

- *Strategy 1* improves the approach, which considers ignorance equivalent to indifference because it adjusts the estimated preference values to make the preference relation more consistent with the previously existing information. Moreover, the initial indifference, which is assumed for every preference value associated with the unknown alternative, is corrected, by means of the additive consistency property, when there is no indifference between some of the rest of alternatives. This approach is particularly useful when there are no external sources of information about the problem and when a high consistency level is required in the experts' preference relations.
- *Strategy 2* implies having some additional knowledge about the alternatives of the problem. How to obtain this information is not usually an easy task, specially

because it is difficult to quantify similarities among the different options. This information usually requires a study of the problem previous to the application of the decision making process. However, this kind of study might not always be successful in obtaining clear relationships between the alternatives. This strategy could be useful, for example, in decision problems where the alternatives to be evaluated are goods having similar characteristics (similar models). In such a case, this information about their similarities could be exploited to avoid ignorance situations in which an expert is not familiar with one of the alternatives, but has enough knowledge about a similar one.

- *Strategy 3* is appropriate for MPDM problems because most of the decision models to solve them involve the computation of a global opinion from the individual preferences, to finally obtain a solution of consensus. Moreover, this strategy could help to reach a solution of consensus more easily, making the opinions of the experts closer to each other, because the unknown alternatives are completed from global information. Additionally, the use of the estimation procedure assures that the loss of consistency will be minimized. Thus, this kind of approach could be useful in problems where a fast and converging consensus process is needed.
- *Strategy 4* also helps the consensus process to converge because the estimated information comes from the nearest experts in the problem. However, this convergence is achieved in a different way as the ignored information is estimated by using the information of just a part of the experts. This strategy could prove useful in MPDM problems in which the estimated information should be compatible with the information expressed by the expert, which is assured because it is based on just the information of the nearest experts rather than the information from the whole group of experts.
- *Strategy 5* unifies all the advantages of the previous two social strategies. The estimated information will not only help in the consensus process but also will try to maintain a high consistency level for the expert. As it is the strategy that makes use of all the information that is usually present in any group decision making problem we consider this to be the best one for the majority of ignorance situations in MPDM problems.

The following table summarizes the conditions to meet for each one of the strategies to be applied:

	Strategies				
	St. 1	St. 2	St. 3	St. 4	St. 5
Consistency	X	X	X	X	X
Single criteria decision making	X	X			
Additional information about alternatives		X			
Additional collective information			X		X
Fast convergence consensus process			X	X	X

6. Concluding Remarks

In this paper, we have presented several strategies to deal with ignorance situations in MPDM problems. In these situations at least one expert does not provide any information about a particular alternative (the unknown alternative). Usual models do not solve these problems because they need at least one piece of information about each one of the alternatives. The presented strategies help to deal with these situations and have been classified according to their suitability depending on the characteristics and the available information for the problem being solved.

We note that these strategies do not constitute an exhaustive list, and depending on the particular characteristics of a decision problem, different approaches could be given. It is particularly easy to change some of the presented strategies and adapt them to slightly different situations in which they could benefit from different sources of information or particular conditions of the problem.

Finally, we point out some comments on the incorporation of expert satisfaction. The presented strategies work independently from the experts, that is, experts provide their incomplete FPRs and the system that implements the decision process applies one of the strategies in order to complete the unknown information. This means that the presented approaches do not take into account the satisfaction or agreement of the experts with the completed information, and thus, it is possible that some experts might not accept the preference values that have been estimated. Improvements to the strategies to minimize this issue are possible. For example, if the system is to carry out a consensus process, then the usual feedback within this process can be used to give guidance to the experts on how to provide some information about the unknown alternatives. An easy way to implement this would be to present the proximity and/or consensus information to the expert that did have an ignored alternative and allow him/her to update his/her preference relation according to that information. Another possibility would be to present the estimated information (regardless of the strategy applied to solve the ignorance problem) and allow the experts to change the estimated values if they consider that those values do not reflect their opinions in a satisfactory way. This would allow the system to avoid some estimated values from not being accepted by the experts.

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