Soft Comput (2010) 14:451–463
DOI 10.1007/s00500-009-0453-x

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Analyzing consensus approaches in fuzzy group decision making: advantages and drawbacks

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Published online: 4 June 2009

Abstract Two processes are necessary to solve group decision making problems: a consensus process and a selection process. The consensus process is necessary to obtain a final solution with a certain level of agreement between the experts, while the selection process is necessary to obtain such a final solution. Clearly, it is preferable that the set of experts reach a high degree of consensus before applying the selection process. In order to measure the degree of consensus, different approaches have been proposed. For example, we can use hard consensus measures, which vary between 0 (no consensus or partial consensus) and 1 (full consensus), or soft consensus measures, which assess the consensus degree in a more flexible way. The aim of this paper is to analyze the different consensus approaches in fuzzy group decision making problems and discuss their advantages and drawbacks. Additionally, we study the future trends.

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Keywords Group decision making · Consensus process · Soft consensus measures · Future trends

1 Introduction

In a classical group decision making (GDM) situation there is a problem to solve, a solution set of possible alternatives, $X = \{x_1, \ldots, x_n\}$, and a group of two or more experts, $E = \{e_1, \ldots, e_m\}$, characterized by their own ideas, attitudes, motivations and knowledge, who express their opinions about this set of alternatives to achieve a common solution (Lu et al. 2008; Montero 2008; Nurmi 2008). To do this, experts have to express their preferences by means of a set of evaluations over the set of alternatives.

GDM problems arise from many real-world situations (Chen and Hwang 1992). To solve these problems, experts apply two processes before obtaining a final solution (Herrera-Viedma et al. 2005; Kacprzyk et al. 1992; Kacprzyk et al. 1997): *consensus process* and *selection process* (see Fig. 1). The former consists in how to obtain the maximum degree of consensus or agreement between the set of experts on the solution set of alternatives. Normally, the consensus process is guided by a human figure called moderator (Herrera et al. 1996; Kacprzyk et al. 1992) who does not participate in the discussion but knows the agreement in each moment of the consensus process and is in charge of supervising and addressing the consensus process toward success, i.e., to achieve the maximum possible agreement and to reduce the number of experts outside the consensus in each new consensus round. The latter refers to how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts. It involves two different steps (Herrera et al. 1998; Roubens 1997): aggregation of individual opinions and exploitation of the collective opinion. Clearly, it is preferable that the set of experts achieves a great agreement among their opinions before applying the selection process.

A consensus process is defined as a dynamic and iterative group discussion process, coordinated by a moderator helping experts bring their opinions closer. At the beginning of every GDM problem, the set of experts has diverging opinions, then the consensus process is applied and, in each step, the degree of existing consensus among experts' opinions is measured. If the consensus degree is lower than a specified threshold, the moderator would urge experts to discuss their opinions further in an effort to bring them closer. Otherwise, the moderator would apply the selection process in order to obtain the final consensus solution to the GDM problem.

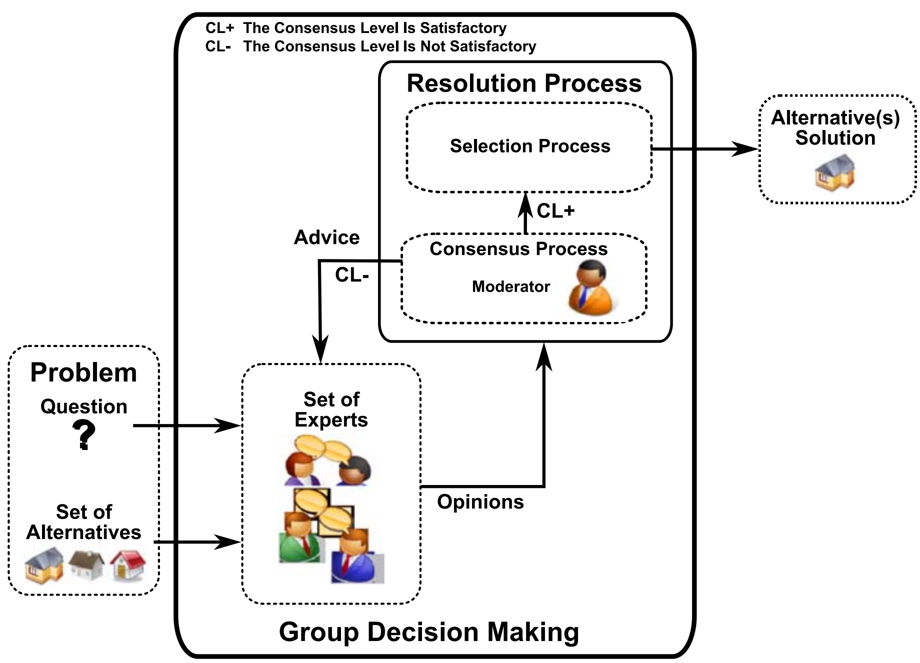
A natural question in the consensus process is how to measure the closeness among experts' opinions in order to obtain the consensus level. To do so, different approaches have been proposed. For instance, several authors have introduced *hard consensus measures* varying between 0 (no consensus or partial consensus) and 1 (full consensus) (Bezdek et al. 1977, 1978; Spillman et al. 1979, 1980). In this way, using *hard consensus measures*, a distance from consensus as a difference between some average preference matrix and one of several possible consensus preference matrices is determined in Bezdek et al. (1977, 1978). In Spillman et al. (1979), some measures of attitudinal similarity between individuals that is an extension of the classical Tanimoto coefficient are derived. Finally, a consensus measure based on *a*-cuts of the respective individual fuzzy preference matrices is derived in Spillman et al. (1980). However, consensus as a full and unanimous agreement is far from being achieved in real situations and, even if it is, in such a situation, the consensus reaching process could be unacceptably expensive. A more realistic approach is to use *soft consensus measures* (Kacprzyk 1987; Kacprzyk and Fedrizzi 1986, 1988), which assess the consensus degree in a more flexible way and, therefore, reflect the large spectrum of possible partial agreements and guide the consensus process until widespread agreement (not always full) is achieved among experts. Soft consensus measures are based on the concept of coincidence (Herrera et al. 1997), measured by means of similarity criteria defined among experts' opinions.

The aim of this paper is to analyze consensus approaches in fuzzy GDM problems to compute soft consensus measures and discuss their advantages and drawbacks. We identify three different coincidence criteria to compute soft consensus measures: (1) *strict coincidence among preferences*, (2) *soft coincidence among preferences* and (3) *coincidence among solutions*. Using these coincidence criteria, two advanced consensus approaches have been proposed:

- Approaches allowing to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible (Herrera-Viedma et al. 2002, 2005, 2007), and
- approaches adapting the consensus process to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round (Mata et al. 2009).

The rest of the paper is organized as follows. In Sect. 2, we analyze the different approaches to obtain soft

Fig. 1 Resolution process of a GDM problem



consensus measures in fuzzy GDM problems and illustrate an example of application. In Sect. 3, we discuss their advantages and drawbacks. The advanced consensus approaches are shown in Sect. 4. Finally, some concluding remarks are pointed out in Sect. 5.

2 Approaches to obtain soft consensus measures in fuzzy GDM problems

As aforementioned, soft consensus measures are based on the coincidence concept (Herrera et al. 1997), and we can identify three different consensus approaches to compute them: (1) *consensus models based on strict coincidence among preferences*, (2) *consensus models based on soft coincidence among preferences*, and (3) *consensus models based on coincidence among solutions*. We describe them in more detail in the following subsections.

2.1 Consensus models based on strict coincidence among preferences

In this case, similarity criteria among preferences are used to compute the coincidence concept. Only two possible results are assumed: the total coincidence (value 1) or null coincidence (value 0). Some examples of this approach are the following:

– In Kacprzyk (1987), assuming fuzzy preference relations to represent experts' preferences, the first consensus model based on strict coincidence was defined. Given a particular alternative pair and two experts, if their preferences are equal, then they are in agreement (value 1), and, otherwise, they are in disagreement (value 0). Then, consensus measures are calculated across the global set of the alternatives in a hierarchical pooling process from the coincidence measured on experts' preferences and using the fuzzy majority concept represented by a linguistic quantifier (Zadeh 1983).
– In Herrera et al. (1996, 1997), different consensus measures based on strict coincidence were presented assuming that experts' preferences are provided by means of linguistic preference relations. Applying the strict coincidence on preferences provided by the experts for each alternative pair, the expert group is divided into subsets, one subset for each possible linguistic label used to qualify the preference on the alternative pair. Then, using the cardinalities of the subsets of experts, three kinds of consensus measures are defined, each one associated with the three different levels of representation of a preference relation: alternative pair, individual alternative and global relation.

Assume a fuzzy GDM problem based on linguistic preference relations as in Herrera et al. (1996, 1997), i.e., a GDM problem where the experts $E = \{e_1, \ldots, e_m\}$ express their preferences relations $P = \{P^1, \ldots, P^m\}$ on the set of alternatives X, using a linguistic term set $S = \{s_0, \ldots, s_g\}$ whose cardinality or granularity $\#S = g + 1$, being $p^h_{ik} \in S$ the preference degree of alternative x_i over alternative x_k for the expert e_h. Additionally, the following properties are assumed (Herrera-Viedma 2001, 2006):

1. The set S is ordered: $s_i \geq s_j$ if $i \geq j$.
2. Negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g - i$.
3. Min operator: $\mathrm{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$.
4. Max operator: $\mathrm{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$.

Then, a consensus model based on strict coincidence could be carried out in the following steps:

1. First, for each pair of experts (e_h, e_l) $(h = 1, \ldots, m - 1,\ l = h + 1, \ldots, m)$, a strict similarity matrix $\mathrm{SM}^{hl} = [sm^{hl}_{ik}], i, k = 1, \ldots, n$, is obtained as follows:

$$sm^{hl}_{ik} = \begin{cases} 1, & \text{if } p^h_{ik} = p^l_{ik} \\ 0, & \text{otherwise} \end{cases}. \tag{1}$$

2. Then, a collective similarity matrix, $\mathrm{SM} = [sm_{ik}]$, is obtained by aggregating all the similarity matrices using the arithmetic mean ϕ as the aggregation function:

$$sm_{ik} = \phi(sm^{hl}_{ik}, h = 1, \ldots, m - 1, l = h + 1, \ldots, m). \tag{2}$$

Note 1: In this case, we have used the arithmetic mean as aggregation function ϕ, although, different aggregation operators could be used according to the particular properties that we want to implement.

3. Computing the consensus degrees and proximity measures as in Herrera et al. (1996):

(a) **Consensus degrees**: once the similarity matrices are computed, the consensus degrees are calculated as follows:

1. **Level 1**. *Consensus degree on pairs of alternatives*. The consensus degree, cop_{ik}, on a pair of alternatives, (x_i, x_k), is defined to measure the consensus degree among all the experts on that pair of alternatives. In this case, this is expressed by the element of the collective similarity matrix SM:

$$\mathrm{cop}_{ik} = sm_{ik} \tag{3}$$

The closer cop_{ik} is to 1, the greater the agreement among all the experts on the pair of alternatives (x_i, x_k). This measure will allow the identification of those pairs of alternatives with a poor level of consensus.

2. **Level 2**. *Consensus degree on alternatives*. The consensus degree on the alternative x_i, called ca_i, is defined

to measure the consensus degree among all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1;k\neq i}^{n}(\text{cop}_{ik} + \text{cop}_{ki})}{2n-2} \tag{4}$$

These values can be used to propose the modification of preferences associated with those alternatives with a consensus degree lower than a minimal consensus threshold γ.

3. **Level 3**. *Consensus degree on the relation*. The consensus degree on the relation, called CR, is defined to measure the global consensus degree among all the experts' opinions. It is computed as the average of all the consensus degrees on the alternatives:

$$\text{CR} = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{5}$$

This is the value used to control the consensus situation.

Note 2: In Herrera et al. (1996) three kinds of consensus are proposed because they allow us to know the current state of consensus from different viewpoints and, therefore, to guide more correctly the consensus reaching process.

(b) **Proximity measures**: to compute the proximity measures for each expert, we need to obtain the collective preference relation, $P^c = [p^c_{ik}]$, which summarizes preferences given by all the experts and is calculated by means of the aggregation of the set of individual preference relations $\{P^1, \ldots, P^m\}$ as follows:

$$p^c_{ik} = \phi(p^1_{ik}, \ldots, p^m_{ik}). \tag{6}$$

To do so, the *linguistic ordered weighted averaging* (LOWA) operator (Herrera et al. 1996) can be used. The LOWA operator is based on the ordered weighted averaging (OWA) operator defined in Yager (1988), and on the convex combination of linguistic labels defined in Delgado et al. (1993). In Herrera et al. (1996), it was shown that it is a rational operator to aggregate linguistic information that satisfies some important properties as commutativity, monotony, unanimity and neutrality.

Definition 1 Let $A = \{a_1, \ldots, a_m\}$ be a set of labels to be aggregated, then the LOWA operator, ϕ, is defined as:

$$\begin{aligned}\phi(a_1, \ldots, a_m) &= W \cdot B^T = \mathcal{C}^m\{w_k, b_k, k = 1, \ldots, m\} \\ &= w_1 \odot b_1 \oplus (1 - w_1) \\ &\quad \odot \mathcal{C}^{m-1}\{\beta_h, b_h, h = 2, \ldots, m\}\end{aligned} \tag{7}$$

where $W = [w_1, \ldots, w_m]$ is a weighting vector, such that $w_i \in [0, 1]$ and $\Sigma_i w_i = 1$. $\beta_h = w_h / \Sigma_2^m w_k$, $h = 2, \ldots, m$, and $B = \{b_1, \ldots, b_m\}$ is a vector associated with A, such that $B = \sigma(A) = \{a_{\sigma(1)}, \ldots, a_{\sigma(m)}\}$, where, $a_{\sigma(j)} \leq a_{\sigma(i)}$ $\forall i \leq j$, with σ being a permutation over the set of labels A. $\mathcal{C}^m$ is the convex combination operator of m labels and if $m = 2$, then it is defined as $\mathcal{C}^2\{w_i, b_i, i = 1, 2\} = w_1 \odot s_j \oplus (1 - w_1) \odot s_i = s_k$, such that $k = \min\{\mathcal{T}, i + \text{round}(w_1 \cdot (j - i))\} s_j$, $s_i \in S$, $(j \geq i)$, where "round" is the usual round operation, and $b_1 = s_j$, $b_2 = s_i$. If $w_j = 1$ and $w_i = 0$ *with* $i \neq j \forall i$, then the convex combination is defined as: $\mathcal{C}^m\{w_i, b_i, i = 1, \ldots, m\} = b_j$.

Using P^c, for each expert, e_h, a proximity matrix, $\text{PM}^h = [pm^h_{ik}]$, is obtained:

$$pm^h_{ik} = \begin{cases} 1, & \text{if } p^h_{ik} = p^c_{ik} \\ 0, & \text{otherwise} \end{cases}. \tag{8}$$

Finally, the computation of the proximity measures is carried out at three different level as follows:

1. **Level 1**. *Proximity measure on pairs of alternatives*. The proximity measure of an expert e_h on a pair of alternatives (x_i, x_k) to the group's one, called pp^h_{ik}, is expressed by the element (i, k) of the proximity matrix PM^h:
$$pp^h_{ik} = pm^h_{ik}. \tag{9}$$
2. **Level 2**. *Proximity measure on alternatives*. The proximity measure of an expert e_h on an alternative x_i to the group's one, called pa^h_i, is calculated as follows:
$$pa^h_i = \frac{\sum_{k=1,k\neq i}^{n}(pp^h_{ik} + pp^h_{ki})}{2n-2}. \tag{10}$$
3. **Level 3**. *Proximity measure on the relation*. The proximity measure of an expert e_h on his/her preference relation to the group's one, called pr^h, is calculated as the average of all proximity measures on the alternatives:
$$pr^h = \frac{\sum_{i=1}^{n} pa^h_i}{n}. \tag{11}$$

Given an expert, if his or her proximity measure is close to 1, then he or she has a positive contribution for the consensus to be high, while if it is close to 0, then he or she has a negative contribution to the consensus.

Example 1 Suppose four experts $E = \{e_1, e_2, e_3, e_4\}$ use the linguistic term set $S = \{Null(N), Very\,Low(VL), Low(L), Medium(M), High(H), Very\,High(VH), Total(T)\}$ to provide their linguistic preference relations on a set of four alternatives:

$$P^1 = \begin{pmatrix} - & H & VH & L \\ L & - & T & VH \\ L & N & - & L \\ H & L & VH & - \end{pmatrix}; \quad P^2 = \begin{pmatrix} - & H & H & M \\ L & - & VH & T \\ VL & L & - & H \\ M & N & L & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & H & M & VH \\ L & - & M & L \\ L & L & - & T \\ VL & H & N & - \end{pmatrix}; \quad P^4 = \begin{pmatrix} - & L & H & M \\ VH & - & M & VH \\ L & M & - & L \\ M & L & T & - \end{pmatrix}.$$

As aforementioned, to obtain the consensus degrees, we compute the different strict similarity matrix for each pair of experts using Eq. (1):

$$SM^{12} = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 1.0 & - & 0.0 & 0.0 \\ 0.0 & 0.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}; \quad SM^{13} = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 1.0 & - & 0.0 & 0.0 \\ 1.0 & 0.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}$$

$$SM^{14} = \begin{pmatrix} - & 0.0 & 0.0 & 0.0 \\ 0.0 & - & 0.0 & 1.0 \\ 1.0 & 0.0 & - & 1.0 \\ 0.0 & 1.0 & 0.0 & - \end{pmatrix}; \quad SM^{23} = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 1.0 & - & 0.0 & 0.0 \\ 0.0 & 1.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}$$

$$SM^{24} = \begin{pmatrix} - & 0.0 & 1.0 & 1.0 \\ 0.0 & - & 0.0 & 0.0 \\ 0.0 & 0.0 & - & 0.0 \\ 1.0 & 0.0 & 0.0 & - \end{pmatrix}; \quad SM^{34} = \begin{pmatrix} - & 0.0 & 0.0 & 0.0 \\ 0.0 & - & 1.0 & 0.0 \\ 1.0 & 0.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}.$$

Then, we compute the collective similarity matrix using the ϕ:

$$SM = \begin{pmatrix} - & 0.50 & 0.17 & 0.17 \\ 0.50 & - & 0.17 & 0.17 \\ 0.50 & 0.17 & - & 0.17 \\ 0.17 & 0.17 & 0.00 & - \end{pmatrix}.$$

From SM, we obtain the following consensus degrees:

1. *Consensus degrees on pairs of alternatives*. The element (i, k) of SM represents the consensus degrees, cop_{ik}, on the pair of alternatives (x_i, x_k).
2. *Consensus degrees on alternatives*:

   $ca_1 = 0.34,\ ca_2 = 0.28,\ ca_3 = 0.20,\ ca_4 = 0.09.$

3. *Consensus degrees on the relation*:
   CR = 0.23.

Clearly, we have a low consensus degree among experts and, therefore, in a decision situation we would have to continue the negotiation process. To do so, as in Herrera-Viedma et al. (2002, 2005, 2007), we could guide the negotiation process by means of the proximities measure. To obtain the proximity measures, we need to compute the collective fuzzy linguistic preference relation by aggregating all individual linguistic preference relations.

Using the LOWA operator (Herrera et al. 1996) with the weighting vector $W = \{0.5, 0.20, 0.16, 0.14\}$, we obtain the following P^c

$$P^c = \begin{pmatrix} - & H & H & M \\ M & - & VH & VH \\ L & L & - & H \\ M & L & H & - \end{pmatrix}.$$

The proximity matrices for each expert are:

$$PM^1 = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 0.0 & - & 0.0 & 1.0 \\ 1.0 & 0.0 & - & 0.0 \\ 0.0 & 1.0 & 0.0 & - \end{pmatrix}; \quad PM^2 = \begin{pmatrix} - & 1.0 & 1.0 & 1.0 \\ 0.0 & - & 1.0 & 0.0 \\ 0.0 & 1.0 & - & 1.0 \\ 1.0 & 0.0 & 0.0 & - \end{pmatrix}$$

$$PM^3 = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 0.0 & - & 0.0 & 0.0 \\ 1.0 & 1.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}; \quad PM^4 = \begin{pmatrix} - & 0.0 & 1.0 & 1.0 \\ 0.0 & - & 0.0 & 1.0 \\ 1.0 & 0.0 & - & 0.0 \\ 1.0 & 1.0 & 0.0 & - \end{pmatrix}.$$

And then, the proximity measures are:

1. *Proximity measure on pairs of alternatives*. The proximity measure of an expert e_h on a pair of alternatives (x_i, x_k) to the group's one, pp^h_{ik}, is expressed by the element (i, k) of the proximity matrix PM^h.
2. *Proximity measure on alternatives*:

$$\{pa^1_1, pa^1_2, pa^1_3, pa^1_4\} = \{0.33, 0.50, 0.17, 0.33\}$$
$$\{pa^2_1, pa^2_2, pa^2_3, pa^2_4\} = \{0.67, 0.50, 0.67, 0.50\}$$
$$\{pa^3_1, pa^3_2, pa^3_3, pa^3_4\} = \{0.33, 0.33, 0.33, 0.00\}$$
$$\{pa^4_1, pa^4_2, pa^4_3, pa^4_4\} = \{0.67, 0.33, 0.33, 0.67\}.$$

3. *Proximity measure on the relation*:

$$pr^1 = 0.33,\ pr^2 = 0.58,\ pr^3 = 0.25,\ pr^4 = 0.50.$$

With these scores, the experts 1 and 3 should change highly their positions to increase the level of consensus in the next consensus rounds.

2.2 Consensus models based on soft coincidence among preferences

As above, similarity criteria among preferences are used to compute the coincidence concept. However, in this case, a major number of possible coincidence degrees is considered. It is assumed that the coincidence concept is a gradual concept, which could be assessed with different degrees defined in the unit interval [0,1]. These are the more popular consensus models. Some examples of this approach are the following:

- In Kacprzyk (1987), a first consensus model based on soft coincidence was also defined. But in this case, given a particular alternative pair and two experts, the coincidence among their preference is measured using a closeness function $s : [0, 1] \rightarrow [0, 1]$.
- In Kacprzyk and Fedrizzi (1986, 1988), some soft consensus measures are introduced and defined as extensions of those presented in Kacprzyk (1987), considering GDM problems with heterogeneous set of alternatives and heterogeneous groups of experts, respectively.
- An extension of these models is presented in Fedrizzi et al. (1993), which consists in the computation of consensus measures using the ordered weighted averaging (OWA) operator (Yager 1988).
- In Bordogna et al. (1997), a soft consensus model for multi-criteria GDM problems defined in a ordinal fuzzy linguistic approach was defined. In this case, coincidence values are obtained by means of a linguistic similarity function defined directly on linguistic assessments given on the alternatives.
- In Herrera et al. (1997), the fuzzification of soft coincidence concept was presented. The soft coincidence is defined in each alternative pair of a linguistic preference relation as a fuzzy set defined on the set of expert pairs and characterized by closeness observed among their preferences. The closeness among preferences is established by means of ad hoc closeness table defined among all the possible labels of linguistic term set used to represent the preferences.
- In Herrera-Viedma et al. (2005), a soft consensus approach is presented to deal with GDM problems in a multi-granular fuzzy linguistic context. Three kinds of soft consensus measures are considered as in Herrera et al. (1996, 1997, 1997). In this case, the soft coincidence among multi-granular linguistic preferences is obtained using a similarity function defined on transformation of such preferences in a basic linguistic term set.
- In Herrera-Viedma et al. (2007), a soft consensus model based on three consensus measures was also proposed. In this case, experts provide their preferences by means of incomplete fuzzy preference relations assessed in [0,1] and the soft coincidence is defined using a similarity function among preferences in [0,1].
- Finally, in Cabrerizo et al. (2009), a soft consensus model is presented for GDM problems in an unbalanced fuzzy linguistic context (Herrera et al. 2008; Herrera-Viedma and López-Herrera 2007). In this case, as in Herrera et al. (1996, 1997, 1997), the soft coincidence is computed using a similarity function defined on transformation of unbalanced fuzzy linguistic preferences in a basic linguistic term set.

In the framework used previously, we could apply a consensus model based on soft coincidence in a fuzzy GDM problem based on linguistic preference relations as follows:

1. Compute the similarity matrices $SM^{hl} = [sm^{hl}_{ik}]$, $i, k = 1, \ldots, n$,:

$$sm^{hl}_{ik} = s(p^h_{ik}, p^l_{ik}) \qquad (12)$$

where $s(p^h_{ik}, p^l_{ik})$ is a similarity function which measures the coincidence between the opinions p^h_{ik} and p^l_{ik}. Depending on the fuzzy context, different similarity functions can be used (Herrera-Viedma et al. 2005, 2007).

2. Then, a collective similarity matrix, $SM = [sm_{ik}]$, is obtained by aggregating all the similarity matrices using the arithmetic mean ϕ:

$$sm_{ik} = \phi(sm^{hl}_{ik}, h = 1, \ldots, m-1, l = h+1, \ldots, m). \qquad (13)$$

3. Compute the consensus degrees and proximity measures:

(a) **Consensus degrees**: once the similarity matrices are computed, the consensus degrees are calculated at three different levels as in the consensus models based on strict coincidence among preferences:

1. **Level 1**. *Consensus degree on pairs of alternatives*:

$$\mathrm{cop}_{ik} = sm_{ik}. \tag{14}$$

2. **Level 2**. *Consensus degree on alternatives*:

$$ca_i = \frac{\sum_{k=1;k\neq i}^{n} (cop_{ik} + cop_{ki})}{2n-2}. \tag{15}$$

3. **Level 3**. *Consensus degree on the relation*:

$$\mathrm{CR} = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{16}$$

(b) **Proximity measures**: to compute the proximity measures for each expert, we need to obtain the collective preference relation, $P^c = [p^c_{ik}]$, which is computed as follows:

$$p^c_{ik} = \phi(p^1_{ik}, \ldots, p^m_{ik}). \tag{17}$$

$$\mathrm{SM}^{12} = \begin{pmatrix} - & 1.00 & 0.83 & 0.83 \\ 1.00 & - & 0.83 & 0.83 \\ 0.83 & 0.67 & - & 0.67 \\ 0.83 & 0.67 & 0.50 & - \end{pmatrix}; \quad \mathrm{SM}^{13} = \begin{pmatrix} - & 1.00 & 0.67 & 0.50 \\ 1.00 & - & 0.50 & 0.50 \\ 1.00 & 0.67 & - & 0.33 \\ 0.50 & 0.67 & 0.17 & - \end{pmatrix}$$

$$\mathrm{SM}^{14} = \begin{pmatrix} - & 0.67 & 0.83 & 0.83 \\ 0.50 & - & 0.50 & 1.00 \\ 1.00 & 0.50 & - & 1.00 \\ 0.83 & 1.00 & 0.83 & - \end{pmatrix}; \quad \mathrm{SM}^{23} = \begin{pmatrix} - & 1.00 & 0.83 & 0.67 \\ 1.00 & - & 0.67 & 0.33 \\ 0.83 & 1.00 & - & 0.67 \\ 0.67 & 0.33 & 0.67 & - \end{pmatrix}$$

$$\mathrm{SM}^{24} = \begin{pmatrix} - & 0.50 & 1.00 & 1.00 \\ 0.33 & - & 0.67 & 0.83 \\ 0.83 & 0.83 & - & 0.67 \\ 1.00 & 0.67 & 0.33 & - \end{pmatrix}; \quad \mathrm{SM}^{34} = \begin{pmatrix} - & 0.67 & 0.83 & 0.67 \\ 0.50 & - & 1.00 & 0.50 \\ 1.00 & 0.83 & - & 0.33 \\ 0.67 & 0.67 & 0.00 & - \end{pmatrix}.$$

To do so, the LOWA operator (Herrera et al. 1996) can be used.

Using P^c, for each expert, e_h, a proximity matrix, $PM^h = [pm^h_{ik}]$, is obtained:

$$pm^h_{ik} = s(p^h_{ik}, p^c_{ik}). \tag{18}$$

Finally, the computation of the proximity measures is carried out at three different levels as follows:

1. **Level 1**. *Proximity measure on pairs of alternatives*:

$$pp^h_{ik} = pm^h_{ik}. \tag{19}$$

2. **Level 2**. *Proximity measure on alternatives*:

$$pa^h_i = \frac{\sum_{k=1,k\neq i}^{n} (pp^h_{ik} + pp^h_{ki})}{2n-2}. \tag{20}$$

3. **Level 3**. *Proximity measure on the relation*:

$$pr^h = \frac{\sum_{i=1}^{n} pa^h_i}{n}. \tag{21}$$

Example 2 Assuming the same linguistic preference relations provided by the experts in the above example, the soft consensus degrees are obtained as follows.

To obtain the consensus degrees, first, we compute the different similarity matrix for each pair of experts. In this case, we need to define a similarity function. As we assume a fuzzy linguistic framework, the following similarity function can be used:

$$s(s_i, s_j) = 1 - \frac{|i-j|}{g}. \tag{22}$$

Using this similarity function, the following similarity matrices are obtained:

Then, we compute the collective similarity matrix:

$$\mathrm{SM} = \begin{pmatrix} - & 0.81 & 0.83 & 0.75 \\ 0.72 & - & 0.62 & 0.66 \\ 0.92 & 0.75 & - & 0.27 \\ 0.75 & 0.67 & 0.42 & - \end{pmatrix}.$$

Finally, we obtain the following consensus degrees:

1. *Consensus degrees on pairs of alternatives*. The element (i, k) of sm represents the consensus degrees on the pair of alternatives (x_i, x_k).
2. *Consensus degrees on alternatives*:

$ca_1 = 0.80,\ ca_2 = 0.71,\ ca_3 = 0.63,\ ca_4 = 0.59.$

3. *Consensus degrees on the relation*:

$CR = 0.68.$

According to this score, we can affirm that the consensus level is acceptable in contrast to Example 1 based on the strict coincidence.

Proximity measures are obtained from the collective fuzzy linguistic preference relation, which using the LOWA operator with the weighting vector $W = \{0.5, 0.20, 0.16, 0.14\}$, is the following:

$$P^c = \begin{pmatrix} - & H & H & M \\ M & - & VH & VH \\ L & L & - & H \\ M & L & H & - \end{pmatrix}$$

From P^c, the proximity matrices for each expert are:

$$PM^1 = \begin{pmatrix} - & 1.00 & 0.83 & 0.83 \\ 0.83 & - & 0.83 & 1.00 \\ 1.00 & 0.67 & - & 0.67 \\ 0.83 & 1.00 & 0.83 & - \end{pmatrix}; \quad PM^2 = \begin{pmatrix} - & 1.00 & 1.00 & 1.00 \\ 0.83 & - & 1.00 & 0.83 \\ 0.83 & 1.00 & - & 1.00 \\ 1.00 & 0.67 & 0.67 & - \end{pmatrix}$$

$$PM^3 = \begin{pmatrix} - & 1.00 & 0.83 & 0.67 \\ 0.83 & - & 0.67 & 0.50 \\ 1.00 & 1.00 & - & 0.67 \\ 0.67 & 0.67 & 0.33 & - \end{pmatrix}; \quad PM^4 = \begin{pmatrix} - & 0.67 & 1.00 & 1.00 \\ 0.67 & - & 0.67 & 1.00 \\ 1.00 & 0.83 & - & 0.67 \\ 1.00 & 1.00 & 0.67 & - \end{pmatrix}.$$

1. *Proximity measure on pairs of alternatives*. The proximity measure of an expert e_h on a pair of alternatives (x_i, x_k) to the group's one, pp_{ik}^h, is expressed by the element (i, k) of the proximity matrix PM^h.
2. *Proximity measure on alternatives*:

$$\{pa_1^1, pa_2^1, pa_3^1, pa_4^1\} = \{0.89, 0.89, 0.80, 0.86\}$$
$$\{pa_1^2, pa_2^2, pa_3^2, pa_4^2\} = \{0.94, 0.89, 0.92, 0.86\}$$
$$\{pa_1^3, pa_2^3, pa_3^3, pa_4^3\} = \{0.83, 0.79, 0.75, 0.58\}$$
$$\{pa_1^4, pa_2^4, pa_3^4, pa_4^4\} = \{0.89, 0.81, 0.81, 0.89\}.$$

3. *Proximity measure on the relation*:

$$pr^1 = 0.86, \quad pr^2 = 0.90, \quad pr^3 = 0.74, \quad pr^4 = 0.85$$

In this case, unlike Example 1 all experts present adequate proximity measures, and the experts with worse scores are e_3 and e_4.

2.3 Consensus models based on coincidence among solutions

In this case, similarity criteria among the solutions obtained from the experts' preferences are used to compute the coincidence concept and different degrees assessed in [0,1] are assumed (Ben-Arieh and Chen 2006; Herrera-Viedma et al. 2002). Basically, we compare the positions of the alternatives between the individual solutions and the collective solution, which allows to know better the real consensus situation in each moment of the consensus process. Some examples of this approach are the following:

– In Herrera-Viedma et al. (2002) was defined the first consensus model based on the measurement of the coincidence degree between individual solutions and collective solution. It is assumed that experts represent their preferences by means of different elements of representation (relation, ordering and utilities) and then it is not possible to compare preferences. To overcome this problem, authors propose to compare solutions to obtain the coincidence degrees. This means that the first step of the consensus process to measure coincidence degrees is to apply a selection process to obtain a temporary collective solution and temporary individual solutions, and measure the closeness among them. An important characteristic of this consensus model was the introduction of a recommendation system to aid experts to change their preferences in the consensus reaching process and, in such a way, to substitute the moderator's actions.
– In Ben-Arieh and Chen (2006), a similar consensus model is presented but assuming heterogeneous GDM problems, i.e., experts with different importance degrees.

Following the consensus model defined in Herrera-Viedma et al. (2002), which is based only on consensus degrees not proximity measures, we can define a consensus model based on coincidence among solutions for fuzzy GDM problems with linguistic preference relations as follows:

1. To obtain the collective ordered vector of alternatives (temporary collective solution) V^c. To do so, we apply a selection process in two steps the selection process (Alonso et al. 2009; Chiclana et al. 1998; Roubens 1997):
   (a) *Aggregation*. In this step, a collective preference relation $P^c = (p_{ik}^c)$ is obtained by means of the aggregation of all individual preference relations $\{P^1, P^2, \ldots, P^m\}$. This collective relation indicates the global preference between every ordered pair of alternatives according to the majority of experts' opinions.
   (b) *Exploitation*. In this step, the set of solution alternatives is obtained from the collective preference relation. In this consensus model, we call it as the collective ordered vector of alternatives. To do so, different choice degrees of alternatives could be used (Herrera and Herrera-Viedma 2000; Herrera-Viedma et al. 2007).
2. Calculating the individual ordered vector of alternatives (individual solution) V^h for every expert e_h. To do so, we apply directly the exploitation step on each individual linguistic preference relation P^h.
3. Calculating the proximity of each expert e_h for each alternative x_i, called $p^h(x_i)$, by comparing the ranking positions of that alternative in the experts' individual solution V^h (symbolized by V_i^h) and in the collective solution V^c (symbolized by V_i^c) as $p^h(x_i) = p(V^h, V^c)(x_i) = f(|V_i^c - V_i^h|)$. As a general dissimilarity function, $f(x) = (a \cdot x)^b, 1 \geq b \geq 0$ may be considered, and, in particular, the function taking $a = 1/(n-1)$ may be used, and then

$$p^h(x_i) = p(V^h, V^c)(x_i) = f(|V_i^c - V_i^h|) = \left(\frac{|V_j^c - V_i^h|}{n-1}\right)^b \in [0,1]. \tag{23}$$

The parameter b controls the rigorousness of the consensus process, in such a way, that values of b close to one decrease the rigorousness and, therefore, the number of rounds to develop in the group discussion process, and values of b close to zero increase the rigorousness and, therefore, the number of rounds. Appropriate values for b are: 0.5, 0.7, 0.9, 1.

4. Calculating the consensus degree of all experts on each alternative x_i using the following expression:

$$C(x_i) = 1 - \sum_{h=1}^{m} \frac{p^h(x_i)}{m} \tag{24}$$

5. The consensus measure over the set of alternatives, called C_X, will be calculated by the aggregation of the above consensus degrees on the alternatives. It is considered that the consensus degrees about the solution set of alternatives has to take a more important weight in this aggregation. To do so, in Herrera-Viedma et al. (2002) the S-OWA OR-LIKE operator defined by Yager and Filev (1994) was used:

$$\begin{aligned} C_X &= S_{\mathrm{OWAOR-LIKE}}(\{C(x_s);\ x_s \in X_{\mathrm{sol}}\}, \\ &\quad \{C(x_t); x_t \in X - X_{\mathrm{sol}}\}) \\ &= (1-\beta) \cdot \sum_{t=1}^{v} \frac{C(x_t)}{v} + \beta \cdot \sum_{s=1}^{\gamma} \frac{C(x_s)}{\gamma} \end{aligned} \tag{25}$$

where γ is the cardinal of the set X_{sol}; v is the cardinal of the set $X - X_{\mathrm{sol}}$; $\beta \in [0,1]$. β is a parameter to control the OR-LIKE behavior of the aggregation operator. The higher the value of β, the higher is the influence of the consensus degrees of the solution alternatives on the global consensus degree.

Example 3 Assuming the same linguistic preference relations provided by the experts in the above examples, the soft consensus degrees based on coincidence among solutions are obtained as follows:

1. Obtaining the collective ordered vector of alternatives V^c:
   (a) *Aggregation*: Using the LOWA operator (Herrera and Herrera-Viedma 2000) and the weighting vector $W = \{0.5, 0.20, 0.16, 0.14\}$, the following collective linguistic preference relation is obtained:

$$P^c = \begin{pmatrix} - & H & H & M \\ M & - & VH & VH \\ L & L & - & H \\ M & L & H & - \end{pmatrix}.$$

   (b) *Exploitation*: We use a choice degree called *dominance degree* (Herrera and Herrera-Viedma 2000) to characterize the alternatives and compute the ordered vector of alternatives:

$$\mathrm{DD}_i = \phi(p_{i1}^c, p_{i2}^c, \ldots, p_{i(i-1)}^c, p_{i(i+1)}^c, \ldots, p_{in}^c) \tag{26}$$

   To do so, we use the LOWA operator with the weighting vector $W = \{0.54, 0.28, 0.18\}$. Then the dominance degrees $\{DD_1, \ldots, DD_4\}$ are the following:

$$\mathrm{DD}_1 = M, \quad \mathrm{DD}_2 = H$$
$$\mathrm{DD}_3 = M, \quad \mathrm{DD}_4 = M.$$

And thus, the collective ordered vector of alternatives is $\{x_2, x_1, x_3, x_4\}$.

2. Calculating $\{V^h;\ h = 1, \ldots, m\}$:

$$e_1 : \{x_2, x_1, x_4, x_3\}, \quad e_2 : \{x_2, x_1, x_3, x_4\}$$
$$e_3 : \{x_1, x_3, x_2, x_4\}, \quad e_4 : \{x_2, x_4, x_1, x_3\}.$$

3. The differences between the ranking of alternatives in the temporary collective solution and the individual are as follows:

$V_i^c - V_i^h$	x_1	x_2	x_3	x_4
e_1	0	0	−1	1
e_2	0	0	0	0
e_3	1	1	−2	0
e_4	0	2	−1	−1

4. Consensus degrees on alternatives calculated for $b = 1$:

$$(C(x_1), C(x_2), C(x_3), C(x_4)) = (0.83, 1.0, 0.67, 0.67).$$

5. Consensus measure calculated for $b = 1$ and $\beta = 0.6$ is:

$$C_X = 0.88.$$

As we observe, assuming the same framework considered in Examples 1 and 2, we obtain a higher consensus level with this consensus model, which reflects better the actual decision situation.

3 Discussion

In this section, we analyze the advantages and drawbacks of the different fuzzy soft consensus approaches.

1. *Strict coincidence among preferences*. This consensus approach assumes only two possible values: 1 if the opinions are equal and, otherwise, a value of 0. Therefore, as we have seen in Example 1, the advantage of this approach is that the computation of the consensus degrees is simple and easy. However, the drawback of this approach is that the consensus degrees obtained do not reflect the real consensus situation because it only assigns values of 1 or 0 when comparing the experts' opinions, and, for example, we obtain a consensus value 0 for two different preference situations as (very high, high) and (very high, low), when clearly in the second case the consensus value should be lower than in the first case. It can be seen in Example 1, where the degree of consensus obtained is very low (0.23) although checking the preference relations provided by the experts, we can observe that the consensus among the experts is higher.
2. *Soft coincidence among preferences*. In this approach, similarity criteria among preferences are used to compute the coincidence concept but, in this case, it is assumed that the coincidence concept is a gradual concept, which could be assessed with different degrees defined in [0,1]. The advantage of this approach is that the consensus degrees obtained reflect better the real consensus situation. Comparing Examples 1 and 2, this is clearly observed. However, the drawback of this approach is that the computation of the consensus degrees is more difficult because we need to define similarity criteria to compute the consensus measures, and, sometimes, as it happens in Cabrerizo et al. (2009) and Herrera-Viedma et al. (2005), it is not possible to define these similarity measures directly.
3. *Coincidence among solutions*. The advantage of this approach is that the consensus degrees are obtained comparing not the opinions, but the position of the alternatives in each solution, which allows us to reflect the real consensus situation in each moment of the consensus reaching process, as it happens in the Example 3. However, the drawback of this approach is that the computation of the consensus degrees is more difficult than in the above approaches because we need to define similarity criteria and it is necessary to apply a selection process before obtaining the consensus degrees. As we show in Example 3, the computation of the consensus degrees is more complex.

4 Advanced consensus approaches

In this section, we describe the soft advanced consensus approaches, which have been developed using the above concepts of coincidence. These consensus approaches are mainly two: ones that generate recommendations to help experts and others that develop adaptive consensus processes. We present them in the following subsections in depth.

4.1 Consensus approaches generating recommendations to help experts

These approaches generate simple and easy rules to help experts change their opinions and find out which direction that change should follow in order to obtain the highest degree of consensus possible (Herrera-Viedma et al. 2002, 2007).

To do so, they are based on two consensus criteria: consensus degrees indicating the agreement between experts' opinions and proximity measures used to find out how far the individual opinions are from the group opinion. Thus, proximity measures are used in conjunction with the consensus degrees to build a guidance advice system, which acts as a feedback mechanism that generates recommendations, so that experts can change their opinions. Furthermore, these consensus criteria are computed at three different levels of representation of information of a preference relation: pair of alternatives, alternative and relation. Therefore, we will be able to identify which experts are close to the consensus solution, or in which alternatives the experts have more trouble to reach consensus.

So, the computation of the consensus degrees in this advanced consensus approaches is carried out using Eqs. (3)–(5), i.e., as in the above consensus models. Once consensus degrees are calculated, the proximity measures are obtained. To compute them for each expert, Eqs. (9)–(11) are used.

As aforementioned, if the proximity measures are close to 1, then they have a positive contribution for the consensus to be high, while if they are close to 0, then they have a negative contribution to the consensus. Therefore, we can use them to provide advice to the experts to change their opinions and to find out which direction that change has to follow in order to obtain the highest degree of consensus possible.

Thus, once proximity measures are calculated, the recommendations to help experts change their opinions are generated. The production of advice to achieve a solution with the highest degree of consensus possible is carried out using two kinds of rules (Herrera-Viedma et al. 2005): *identification rules* and *direction rules*.

1. **Identification rules (IR)**. We must identify the experts, alternatives and pairs of alternatives contributing less to reach a high degree of consensus and, therefore, should participate in the change process.

   (a) *Identification rule of experts (IR.1)*. It identifies the set of experts who should receive advice on how to change some of their preference values. This set of experts, called $EXPCH$, who should change their opinions are those whose satisfaction degree on the relation is lower than the minimum consensus threshold γ. Therefore, the identification rule of experts, IR.1, is the following:

   $$EXPCH = \{e_h \mid pr^h < \gamma\}. \quad (27)$$

   (b) *Identification rule of alternatives (IR.2)*. It identifies the alternatives, the associated assessments of which should be taken into account by the above experts in the change process of their preferences. This set of alternatives is denoted as ALT. The identification rule of alternatives, IR.2, is the following:

   $$ALT = \{x_i \in X \mid pa_i^h < \gamma \wedge e_h \in EXPCH\}. \quad (28)$$

   (c) *Identification rule of pairs of alternatives (IR.3)*. It identifies the pairs of alternatives (x_i, x_k) whose associate assessments p_{ik}^h should be changed by expert e_h. This set of pairs of alternatives is denoted as $PALT^h$. The identification rule of pairs of alternatives, IR.3, is the following:

   $$PALT^h = \{(x_i, x_k) \mid x_i \in ALT \wedge e_h \in EXPCH \wedge pp_{ik}^h < \gamma\}. \quad (29)$$

2. **Direction rules (DR)**. We must find out the direction of the change to be recommended in each case, i.e., the direction of change to be applied to the preference assessment p_{ik}^h, with $(x_i, x_k) \in PALT^h$. To do this, we define the following two direction rules.

   (a) *DR.1*. If $p_{ik}^h > p_{ik}^c$, the expert e_h should decrease the assessment associated with the pair of alternatives (x_i, x_k), i.e., p_{ik}^h.

   (b) *DR.2*. If $p_{ik}^h < p_{ik}^c$, the expert e_h should increase the assessment associated with the pair of alternatives (x_i, x_k), i.e., p_{ik}^h.

4.2 Adaptive consensus approaches

These consensus approaches are based on a refinement process of the consensus process that allows to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round (Mata et al. 2009). The refinement process adapts the search for the furthest experts' preferences to the existent agreement in each round of consensus. So, when the agreement is very low (initial rounds of the consensus process), the number of changes of preferences should be bigger than when the agreement is medium or high (final rounds) (see Fig. 2).

These approaches consider that in the first rounds of the consensus process, the agreement is usually very low and it seems logic that many experts' preferences should be changed. However, after several rounds, the agreement should have improved and then just the furthest experts' preferences from the collective preference should be

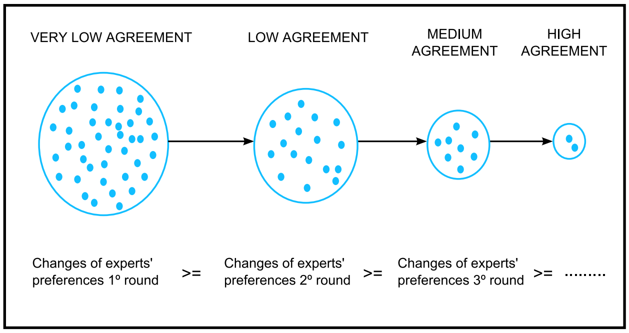


Fig. 2 Reduction in the number of changes of preferences in the consensus process

changed. The procedure to search for the furthest experts' preferences from collective preference should be different according to the achieved agreement in each round. Each Preference Search Procedure (PSp) should have a different behavior and should return a different set of preferences that each expert should change in order to improve the agreement in the next consensus round. In consequence of the adaptation of the consensus process to the existent agreement in each round, the number of changes of preferences suggested to experts after each consensus round will be smaller according to the favorable evolution of the level of agreement.

In the consensus process, if the agreement among experts is low, i.e, there are a lot of experts' preferences with different assessments, the number of experts who should change their preferences in order to make them closer to the collective preference should be great. However, if the agreement is medium or high, it means that the majority of preferences are similar and therefore there exists a low number of experts' preferences far from the collective preference. In this case, only these experts should change them in order to improve the agreement. Keeping in mind this idea, these approaches propose distinguishing among three levels of agreement: very low, low and medium consensus. Each level of consensus involves carryying out the search for the furthest preferences in a different way. So when the consensus degree CR is very low, these approaches will search for the furthest preferences on all experts, while if *CR* is medium, the search will be limited to the furthest experts. To do so, these approaches carry out three different PSps:

– PSp for very low consensus,
– PSp for low consensus, and
– PSp for medium consensus.

The possibility of carrying out different PSps according to the existent consensus degree in each round defines the adaptive character of the model.

5 Concluding remarks

We have analyzed different consensus approaches to compute soft consensus measures in fuzzy GDM problems. Additionally, we have described the new advanced approaches, i.e., those approaches allowing to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible, and, on the other hand, those approaches adapting the consensus process to increase the agreement and reduce the number of experts' preferences that should be changed after each consensus round.

Acknowledgements This paper has been developed with the financing of the SAINFOWEB project (TIC00602), Feder Funds in FUZZYLING project (TIN2007-61079) and PETRI project (PET 2007-0460).

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