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A web based consensus support system for group decision making problems and incomplete preferences

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ABSTRACT

Reaching a high level of consensus among experts is critical in group decision making problems. Usually, it is the moderator task to assure that the consensus process is carried out properly and, if possible, to offer recommendations to the expert in order to change their opinions and narrow their differences.

In this paper we present an implemented web based consensus support system that is able to help, or even replace, the moderator in a consensus process where experts are allowed to provide their preferences using one of many types (fuzzy, linguistic and multi-granular linguistic) of incomplete preference relations.

This system is based on both consistency and consensus measures and it has been designed to provide advice to the experts to increase group consensus level while maintaining the individual consistency of each expert. The consistency measures are characterized by and computed using uninorm operators. When appropriate, the system also helps experts to reduce the incompleteness of their preference relations. The web interface allows to carry out distributed consensus processes and thus, experts do not necessarily need to physically meet together.

1. Introduction

In group decision making (GDM) problems there are two processes to carry out before obtaining a final solution [21,24,31,35]: *the selection process* and *the consensus process*. The former [26,48] refers to how to obtain a solution set of alternatives from the opinions on the alternatives given by the experts, while the latter deals with the achievement of the maximum degree of consensus or agreement between the set of experts on the solution set of alternatives. Usually, this process is guided by the figure of a moderator [17,24,34,35] and it is carried out before the selection process. Clearly, the consensus process is an important step in solving GDM problems because it aids to obtain solutions with high level of consensus among experts, which is usually a desirable property.

Good GDM processes require, before their application, the specification of several aspects about the problem to solve as well as the methodology to follow. This includes, but is not restricted to, the definition of the representation format(s) [45] available for the experts to express their preferences about the possible alternatives in the problem: linguistic [2,19,53] or fuzzy [32,33,51,62,63] formats. Many GDM approaches assume an homogeneous representation of the preference information provided by experts and therefore are based on the availability of one single representation format. However, it may

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well be the case that experts might feel more comfortable if different representation formats are available to express their preferences [8,31]. Thus, the use of multiple representation formats has become a major area of research in GDM.

In [8] an approach with three different preference representation formats was proposed. In that approach, preference orderings and utility values are transformed into fuzzy preference relations in order to be able to operate with them. In [9], another preference representation format (multiplicative preference relations) was incorporated to the previous model to enhance it. Additionally, in [31] a consensus model for GDM problems with these four representation formats was presented. Fan et al. [18] proposed a goal programming approach where the preference information on alternatives provided by decision makers is represented in two different formats, multiplicative preference relations and fuzzy preference relations. In [42] an approach that deals with preference information represented in four different formats was also presented. These two approaches differ from the previous ones in that the ranking of alternatives or selection of the most desirable alternative(s) is obtained in a direct way, i.e. no unify process or aggregation of individual preferences are required. In [28] a model that tackles GDM situations with information expressed using 2-tuple linguistic values and interval valued preferences is presented. In [58] an interactive method for multiple attribute GDM dealing with exact numerical values and triangular fuzzy numbers was developed. Finally, in [64] some experimental results that validate the necessity of using multiple preference formats in decision making were presented.

A different issue that needs attention when dealing with real GDM problems is the lack of complete information [44,55]. There might be situations where some of the experts might not not be able to efficiently express any kind of preference degree between two or more of the available options. Indeed, this may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem to be solved, or because that expert is unable to discriminate the degree to which some options are better than others. In such situations experts may prefer not to guess some values and thus, not to give part of the required information, that is, they would provide incomplete information [3,16,30,37,38,56,57,59,54].

Current decision support models and systems incorporate mechanisms to maximize the consensus in the decision process [5,6,4,17,25,24,34–36,65]. For example, in [14] a consensus driven model is used in the selection of advanced technology field; in [22] some consensus reaching ideas are included as part of a decision support system for water resource management; while in [29] the authors developed a consensus model for GDM and incomplete information.

The aim of this paper is to present a new web based consensus support system (WBCSS) to deal with GDM problems under incomplete information situations and with experts' preferences represented with different representation formats: fuzzy preference relations, linguistic preference relations and multi-granular linguistic preference relations. This consensus support system is based on the use of several consensus and consistency measures which are interactively computed when the experts provide their preferences. One of the main novelties in this contribution is the use of uninorms [40] to define the consistency measures as well as to tackle missing information. The consensus support system uses both kinds of measures to offer advice to the experts by means of easy to follow rules, thus providing a feedback mechanism to help experts to change their preferences in order to obtain solutions with a high level of consensus. The system also aims to help experts to maintain a high consistency level in their preferences to avoid self contradiction and, when that is the case, to reduce as much as possible incomplete information situations. This system is designed to help the moderator to carry out his duties during the different steps of the consensus process, but it could takeover the role of the moderator once the initialization steps have been completed. The system has been fully implemented and the experts can use it via a web interface which allows to carry out consensus processes in distributed environments. This means that the usual imperative condition of experts to physically meet together is eliminated and therefore the decision making process for group of experts, living for example in different countries, is facilitated.

The rest of this paper is organized as follows. Section 2 presents the theoretical model on which the WBCSS is based. In Section 3 the WBCSS for GDM problems with different kinds of preference relations is presented, and some of the technical details regarding its implementation and use are discussed. In Section 4 a toy application example of the system is used to illustrate the solving a simple GDM problem. Finally, in Section 5 we draw our conclusions and future improvements to the system are highlighted.

2. Theoretical model for the consensus support system

In [29] a consistency and consensus measures based theoretical model to guide the GDM consensus process with incomplete fuzzy preference relations was developed. This section briefly describes the extension of that model and the necessary improvements needed to allow the use of multi-granular linguistic preference relations, the use of uninorms as a new characterization of consistency and their use to deal with incomplete information. In order for this paper to be as self-contained as possible, in the following subsections we present the representation formats available to the experts to use within the system (fuzzy, linguistic and multi-granular linguistic preference relations), the consistency and consensus measures based theoretical model, and the consensus/consistency control and feedback mechanisms used in the model.

2.1. Preference relations

In a GDM problem a set of experts $E = \{e_1, \ldots, e_m\}$ have to choose the best alternative or alternatives from a set of alternatives $X = \{x_1, \ldots, x_n\}$. Preference relations are usually assumed to model experts' preferences [51,33]. However, in the literature different types of preference relations have been proposed, with fuzzy and linguistic preference relations being predominantly used in the area of decision making under uncertainty because they allow for the implementation of intensity of preferences [2,8,9].

2.1.1. Fuzzy preference relations

Definition 1. A *fuzzy preference relation* P on a set of alternatives X is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P: X \times X \to [0,1]$.

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ik})$, being $p_{ik} = \mu_P(x_i,x_k)$ $(\forall i,k \in \{1,\ldots,n\})$ interpreted as the degree or intensity of preference of alternative x_i over x_k:

- $p_{ik} = 1/2$ indicates indifference between x_i and x_k $(x_i \sim x_k)$,
- $p_{ik} = 1$ indicates that x_i is absolutely preferred to x_k,
- $p_{ik} > 1/2$ indicates that x_i is preferred to x_k $(x_i \succ x_k)$

Obviously, we have that $p_{ii} = 1/2 \forall i \in \{1,\ldots,n\}$ $(x_i \sim x_i)$.

Usual models to solve GDM problems assume that experts are always able to provide all possible preference values. However, this might not be always possible to achieve, specially when experts are unable to quantify the degree of preference of one alternative over another because time pressure, lack of knowledge or data, and their limited expertise related to the problem domain. In order to model these situations, the following definitions express the concept of an *incomplete fuzzy preference relation* [30].

Definition 2. A function f: $X \to Y$ is *partial* when not every element in the set X necessarily maps to an element in the set Y. When every element from the set X maps to one element of the set Y then we have a total function.

Definition 3. A preference relation P on a set of alternatives X with a *partial* membership function is an *incomplete preference relation.*

The degree of completeness of a preference relation is measured as follows:

Definition 4. The *completeness level* C^h for the preference relation P^h given by expert e_h is computed as

$$C^h = \frac{\#EV^h}{n \cdot (n-1)}, \tag{1}$$

where $\#EV^h$ is the number of preference values provided by the expert. When $C^h = 1$ then the preference relation is complete (all values are known).

2.1.2. Linguistic preference relations

In [2] the 2-tuple linguistic model [27] is used to represent a linguistic preference relation. This model is based on the concept of symbolic translation and the linguistic information is represented by means of a pair of values called linguistic 2-tuple:

Definition 5. Let $\beta \in [0,g]$ be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set $S = \{s_0,s_1,\ldots,s_{g-1},s_g\}$ characterized by its cardinality or *granularity* $\#(S) = g + 1$, i.e., the result of a symbolic aggregation operation. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0,g]$ and $\alpha \in [-0.5,0.5)$, then α is called a symbolic translation.

This 2-tuple linguistic model also provides transformation functions between numerical values and 2-tuples (Δ) and vice-versa (Δ^{-1}):

Definition 6. Let $S = \{s_0,s_1,\ldots,s_{g-1},s_g\}$ be a linguistic term set and $\beta \in [0,g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0,g] \to S \times [-0.5, 0.5),$$
$$\Delta(\beta) = \left\{ \begin{array}{cc} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{array} \right\},$$

where "round" is the usual round operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.

A linguistic term can be seen as a linguistic 2-tuple with symbolic translation value 0, $s_i \in S \equiv (s_i, 0)$, and thus, this linguistic model can be used to represent linguistic preference relations [2]:

Definition 7. A *linguistic preference relation P* on a set of alternatives *X* is a set of 2-tuples on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P: X \times X \rightarrow S \times [-0.5, 0.5)$. If it is not possible to provide the 2-tuple values for all pairs of alternatives then we have an *incomplete* linguistic preference relation.

2.1.3. Multi-granular linguistic preference relations

A convenient situation to approach GDM problems in a linguistic context is that where all the experts use the same linguistic term set *S* to express their preferences about the alternatives. However, in some cases, experts might come from different research areas, and thus have different background and levels of knowledge,and therefore it is natural to assume that linguistic term sets of different cardinality and/or semantics could be used to express their opinions on the set of alternatives. Thus, to be able to manage and model multi-granular linguistic information becomes an essential requirement [23,43,46] to correctly model GDM situations.

When we deal with multi-granular linguistic preference relations, an expert e_h may express his preferences over the set of alternatives *X*, using a particular linguistic term set $S_h = \left\{s_0^h, \ldots, s_{g_h}^h\right\}$. Therefore, $p_{ik}^h \in S$ represents the preference of alternative x_i over alternative x_k for the expert e_h assessed on the label set S_h.

2.2. Consensus model based on consensus and consistency measures

A typical consensus process is depicted in Fig. 1 and comprises the following finite set of steps or tasks:

0. We assume that there is a GDM problem to be solved.
1. The problem and the set of feasible alternatives (the possible solutions to the problem) are presented to the experts.
2. The experts provide the moderator with their opinions (preferences) about the alternatives using a particular preference representation format.
3. Once every expert has provided his preferences about the alternatives in the problem, the moderator checks if the level of consensus (agreement) among all experts is high enough.

4.a. If the level of agreement is high enough, the consensus process stops and the selection process is carried out *(go to step 6)*.

4.b. If the consensus level is not high enough, the moderator gives recommendations to the experts to help them change their opinions to obtain a solution of consensus.

5. Taking into account these recommendations the experts change their preferences about the alternatives and a new round of the consensus process starts *(go to step 2)*.
6. The selection process is carried out and a final solution to the problem is computed.

Note that this kind of consensus process could be seen as a Delphi method based one [41]. However, there are several important differences. Among others, we can cite:

- The Delphi method final goal is forecasting whilst our approach tries to find the best alternative from a set of feasible alternatives.
- Our approach does not rely on the use of questionnaires as the Delphi method does. In our case, the experts express their preferences using some particular preference representation formats.
- Our consensus model provides several automatic tools and measures that ease the moderator tasks (even it can be replaced at a certain point of the process) to help and guide experts to provide more consistent and complete preferences towards a solution of consensus.
- The Delphi method assumes anonymity of participants, whilst our method does not impose any constraint of that kind.

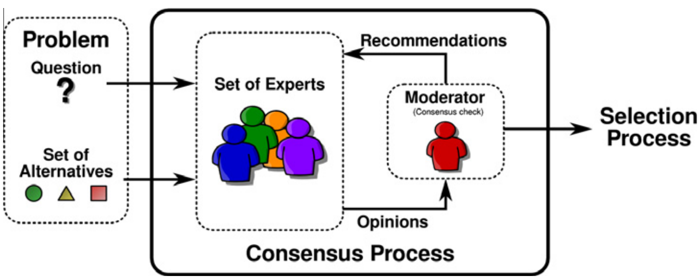

Fig. 1. Consensus process scheme.

The consensus model to be presented in this paper requires the definition of consistency and consensus measures, consistency and consensus control processes and a feedback mechanism, all of which have to address the issue of working with incomplete preference relations.

2.2.1. Consistency measures

The consistency of a preference relation is related to the concept of transitivity [10,15,20,32,50]. Indeed, a preference relation is considered a *consistent* when the pairwise comparisons among every three alternatives satisfy a particular transitivity property. In previous works [2,29,30] we developed some consistency measures (that can be interpreted as a measure of the self-contradiction expressed in the preference relation) for incomplete fuzzy preference relations based on the *additive transitivity property* [51]. However, in [11] it is proved that uninorm operators present better rational properties (monotonicity, associativity, reciprocity, identity element, continuity and cancellative) to model the consistency of fuzzy preference relations:

Definition 8. Let U be a representable uninorm operator with strong negator $N(x) = 1 - x$. A fuzzy preference relation P on a finite set of alternatives is consistent with respect to U (U-consistent) if

$$\forall i,\ j,k:\ (p_{ik},p_{kj}) \notin \{(0,1),(1,0)\} \rightarrow p_{ij} = U(p_{ik},p_{kj}).$$

In particular, Tanino's multiplicative transitivity property [52] under reciprocity is the restriction to the region $[0,1] \times [0,1] \backslash \{(0,1),(1,0)\}$ of the following well known and-like representable uninorm:

$$U(x,y) = \begin{cases} 0, & (x,y) \in \{(0,1),(1,0)\}, \\ \frac{xy}{xy+(1-x)(1-y)}, & \text{Otherwise}. \end{cases} \tag{2}$$

We can use expression (2) to calculate an estimated value of a preference degree using other preference degrees in a fuzzy preference relation. Indeed, the preference value $p_{ik}(i \neq k)$ can be estimated using an intermediate alternative x_j in the following way:

$$cp^j_{ik} = \begin{cases} 0, & (p_{ij},p_{jk}) \in \{(0,1),(1,0)\}, \\ \frac{p_{ij}\cdot p_{jk}}{p_{ij}\cdot p_{jk}+(1-p_{ij})\cdot(1-p_{jk})}, & \text{Otherwise}. \end{cases} \tag{3}$$

The overall estimated value cp_{ik} of p_{ik} is obtained as the average of all possible cp^j_{ik}:

$$cp_{ik} = \frac{\sum_{j=1;i\neq k\neq j}^{n} cp^j_{ik}}{n-2}. \tag{4}$$

When the information provided is completely consistent then $cp^j_{ik} = p_{ik}\ \forall j$. However, because experts are not always fully consistent, the information given by an expert may not verify (2).

Definition 9. The error between a preference value of a fuzzy preference relation P and its estimated one is computed as:

$$\varepsilon p_{ik} = |cp_{ik} - p_{ik}|. \tag{5}$$

We can easily extend the multiplicative transitivity property and previous definitions for a 2-tuple linguistic preference relation P:

$$\Delta^{-1}\left(cp^j_{ik}\right) = \begin{cases} 0, (p_{ij},p_{jk}) \in \{(s_0,s_g),(s_g,s_0)\} \\ \frac{\frac{\Delta^{-1}(p_{ij})\cdot\Delta^{-1}(p_{jk})}{g}}{\frac{\Delta^{-1}(p_{ij})\cdot\Delta^{-1}(p_{jk})}{g^2}+\left(1-\frac{\Delta^{-1}(p_{ij})}{g}\right)\cdot\left(1-\frac{\Delta^{-1}(p_{jk})}{g}\right)}, & \text{Otherwise}, \end{cases} \tag{6}$$

$$cp_{ik} = \Delta\left(\frac{\sum_{j=1;i\neq k\neq j}^{n} \Delta^{-1}\left(cp^j_{ik}\right)}{n-2}\right). \tag{7}$$

Definition 10. The normalised error between a preference value of a linguistic preference relation P and its estimated one is defined as:

$$\varepsilon p_{ik} = \frac{|\Delta^{-1}(cp_{ik}) - \Delta^{-1}(p_{ik})|}{g}. \tag{8}$$

We extend the previous definitions to include consistency measures for preference relations (either fuzzy or linguistic ones) at three different levels: pair of alternatives, alternatives and relation levels (see Fig. 2) [30]:

Definition 11. Given preference relation P^h, the *consistency level* associated to the preference value p^h_{ik} is defined as

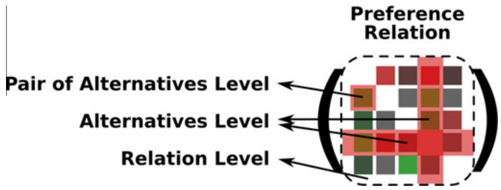

Fig. 2. Three levels at which consistency and consensus measures are computed.

$$cl_{ik}^h = 1 - \varepsilon p_{ik}^h. \tag{9}$$

When $cl_{ik}^h = 1$ then $\varepsilon p_{ik}^h = 0$ and there is no inconsistency at all. The lower the value of cl_{ik}^h, the higher the value of εp_{ik}^h and the more inconsistent is p_{ik}^h with respect to the rest of information.

Definition 12. Given preference relation P^h, the *consistency level* associated to a particular alternative x_i is defined as

$$cl_i^h = \frac{\sum_{k=1, i \neq k}^{n} \left(cl_{ik}^h + cl_{ki}^h \right)}{2(n-1)} \tag{10}$$

with $cl_i^h \in [0, 1]$.

When $cl_i^h = 1$ all the preference values involving alternative x_i are fully consistent, otherwise, the lower cl_i^h the more inconsistent these preference values are.

Definition 13. The *consistency level* of preference relation P^h is defined as follows:

$$cl^h = \frac{\sum_{i=1}^{n} cl_i^h}{n} \tag{11}$$

with $cl^h \in [0,1]$.

When $cl^h = 1$ the preference relation P^h is fully consistent, otherwise, the lower cl^h the more inconsistent is P^h.

Definition 14. In a GDM problem, the *global consistency measure* is computed as follows:

$$CL = \frac{\sum_{h=1}^{m} cl^h}{m}. \tag{12}$$

When $CL = 1$ all the experts are completely consistent. The lower CL is, the more inconsistent the group of experts is.

Finally, from Definitions 4 and 13 we can derive a completeness/consistency measure for each expert [1,30] that synthesizes both measures into a single value:

Definition 15. The completeness/consistency level of preference relation P^h is computed as

$$CC^h = cl^h \cdot C^h. \tag{13}$$

Obviously, when $CC^h = 1$ the preference relation is complete and consistent.

2.2.2. Consensus measures

In order to develop a theoretical consensus model for GDM problems, in [29] two different kinds of measures were defined to determine the current consensus state at a particular stage of the decision making process, and to find out particular values to provide the experts with some advice about how to change their preferences in order to increase the level of consensus. *Consensus degrees* are used to measure the actual levels of consensus amongst all the experts. *Proximity measures* quantify how far is each expert from the consensus solution. Both kinds of measures are computed at the three aforementioned levels (pair of alternatives, alternatives and relation levels).

Consensus degrees are based on the computation of similarity matrices, $SM^{hl} = (sm_{ik}^{hl})$, for each pair of experts $(e_h, e_l)(h < l)$,

$$sm_{ik}^{hl} = 1 - |\overline{p}_{ik}^h - \overline{p}_{ik}^l|, \tag{14}$$

where $\overline{p}_{ik}^h = p_{ik}^h$ if P^h is a fuzzy preference relation and $\overline{p}_{ik}^h = \frac{\Delta^{-1}(p_{ik}^h)}{g_h}$ if P^h is a linguistic preference relation.

A collective similarity matrix, $SM = (sm_{ik})$, is obtained by aggregating the above $(m-1) \times (m-2)$ similarity matrices using the arithmetic mean as the aggregation function ϕ:

$$sm_{ik} = \phi(sm_{ik}^{hl}) \quad \forall h, l = 1, \ldots, m | h < l. \tag{15}$$

Definition 16. The *consensus degree on a pair of alternatives* (x_i, x_k), denoted cop_{ik}, measures the agreement amongst all the experts on that pair of alternatives:

$$cop_{ik} = sm_{ik}. \tag{16}$$

Definition 17. The *consensus degree on alternative* x_i, denoted ca_i, measures the agreement amongst all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1;k\neq i}^{n}(cop_{ik} + cop_{ki})}{2(n-1)} \tag{17}$$

and

Definition 18. The *consensus degree on the relation*, denoted CR, measures the global agreement amongst all the experts' opinions:

$$CR = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{18}$$

Proximity measures for each expert are based on the computation of a collective preference relation, P^c. To do so, a modified version of the Additive Consistency IOWA (AC-IOWA) operator Φ [12,29,60] able to operate with fuzzy and linguistic preference relations is used:

$$p_{ik}^c = \Phi_W\left(\langle z_{ik}^1, \overline{p}_{ik}^1\rangle, \ldots, \langle z_{ik}^m, \overline{p}_{ik}^m\rangle\right) = \sum_{h=1}^{m} w_h \cdot \overline{p}_{ik}^{\sigma(h)}, \tag{19}$$

where

- σ is a permutation of $\{1,\ldots,m\}$ such that $z_{ik}^{\sigma(h)} \geqslant z_{ik}^{\sigma(h+1)}\ \forall h = 1,\ldots,m-1$;
- the weighting vector is computed according to the following expression:

$$w_h = Q\left(\frac{\sum_{j=1}^{h} z_{ik}^{\sigma(j)}}{T}\right) - Q\left(\frac{\sum_{j=1}^{h-1} z_{ik}^{\sigma(j)}}{T}\right) \tag{20}$$

  with $T = \sum_{j=1}^{m} z_{ik}^j$ and Q a linguistic quantifier [61];
- and the set of values of the inducing variable $\{z_{ik}^1, \ldots, z_{ik}^m\}$ are computed as

$$z_{ik}^h = (1-\delta)\cdot cl_{ik}^h + \delta \cdot co_{ik}^h, \tag{21}$$

  being co_{ik}^h a consensus measure for the preference value p_{ik} expressed by expert e_h and $\delta \in [0,1]$ a parameter to control the weight of both consensus and consistency criteria in the inducing variable. Usually $\delta > 0.5$ will be used to give more importance to the consensus criterion. The value co_{ik}^h is defined as

$$co_{ik}^h = \frac{\sum_{l=h+1}^{n} sm_{ik}^{hl} + \sum_{l=1}^{h-1} sm_{ik}^{lh}}{n-1}. \tag{22}$$

Definition 19. The *proximity measure of an expert* e_h *on the pair of alternatives* (x_i, x_k) to the group one, denoted pp_{ik}^h, is calculated as

$$pp_{ik}^h = 1 - |\overline{p}_{ik}^h - p_{ik}^c|. \tag{23}$$

Definition 20. The *proximity measure of an expert* e_h *on alternative* x_i to the group one, denoted pa_i^h, is calculated as:

$$pa_i^h = \frac{\sum_{k=1;k\neq i}^{n}(pp_{ik}^h + pp_{ki}^h)}{2(n-1)}. \tag{24}$$

Definition 21. The *proximity measure of an expert* e_h *on his preference relation* to the group one, denoted pr^h, is calculated as:

$$pr^h = \frac{\sum_{i=1}^{n} pa_i^h}{n}. \tag{25}$$

Note that all the above measures, as well as the consistency ones, have been defined for complete preference relations. They can be extended to the case of incomplete preference relations, and we refer the reader to [29] for the details of such extension.

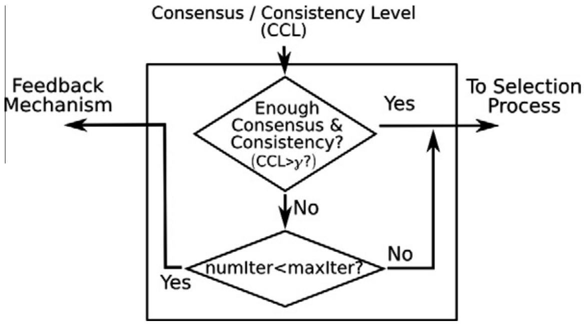

Fig. 3. Controlling the consensus and consistency state.

2.2.3. Consensus/consistency control mechanism

In any rational GDM process, both consensus and consistency should be sought after, i.e. a solution with a high level of consensus is desirable, but also that solution should be derived from consistently enough information [13]. We propose the use of a combined measure called *consensus/consistency level (CCL)* which is used as a control parameter:

$$CCL = (1 - \delta) \cdot CL + \delta \cdot CR. \tag{26}$$

When CCL exceeds a minimum satisfaction threshold value $\gamma \in [0,1]$ the consensus reaching process stops and the selection process is executed. Otherwise, the feedback mechanism is activated. Additionally, a parameter to control the maximum number of consensus rounds, *maxIter*, is used (independently of the current CCL value) to avoid stagnation (see Fig. 3).

2.2.4. The feedback mechanism

The aim of the feedback mechanism is to provide advice to the experts using consensus/consistency criteria. An important aspect of this phase is that the recommendations for a particular expert are easy to follow rules, which are expressed in the same format used by that to represent preference. Therefore, if an expert provided preferences using fuzzy preference relations then the feedback rules for that expert will be expressed using numerical values, while linguistic terms will be used if a linguistic preference relation was the representation format used. Thus, the feedback mechanism consists of two sub-modules: *Identification of the preference values* that need to be modified; and *Generation of advice*.

For the first one, we use a three step process to identify the experts (e_h), the alternatives $\left(X_i^h\right)$ and, finally, the particular preference values $\left(p_{ik}^h\right)$ that contribute less to the consensus/consistency level:

Step 1. The experts that should change their opinions are those whose preference relation consensus/consistency level is lower than the threshold value γ, i.e.,

$$EXPCH = \{h \mid (1 - \delta) \cdot cl^h + \delta \cdot pr^h < \gamma\}. \tag{27}$$

Step 2. The alternatives that the above experts should consider to change are those with a consensus/consistency level lower than the threshold value γ, i.e.,

$$ALT = \left\{(h,i) | h \in EXPCH \wedge (1 - \delta) \cdot cl_i^h + \delta \cdot pa_i^h < \gamma\right\}. \tag{28}$$

Step 3. The preference values to be modified by the above experts are those with an associated consensus/consistency level lower than the threshold value γ, i.e.,

$$APS = \left\{(h,i,k) | (h,i) \in ALT \wedge (1 - \delta) \cdot cl_{ik}^h + \delta \cdot pp_{ik}^h < \gamma\right\}. \tag{29}$$

The feedback process will also provide rules for those preference values that are unknown. Therefore, the final set of preference values for which the generation of advice sub-module will provide rules is

$$APS' = APS \cup \{(h,i,k) | p_{ik}^h \notin EV^h\}. \tag{30}$$

The rules will be expressed as follows:

"*Expert e_h should change his p_{ik}^h preference to a value near rp_{ik}^h*" $\forall (h,i,k) \in APS'$

where rp_{ik}^h is expressed in the same domain used by expert e_h:

- If e_h expressed his preferences using a fuzzy preference relation:

$$rp_{ik}^h = (1 - \delta) \cdot cp_{ik}^h + \delta \cdot p_{ik}^c, \tag{31}$$

- if e_h expressed his preferences using a linguistic preference relation:

$$rp_{ik}^{h} = \Delta\left((1-\delta)\cdot\Delta^{-1}\left(cp_{ik}^{h}\right) + \delta\cdot g_h\cdot p_{ik}^{c}\right). \quad (32)$$

3. Web based consensus support system

Nowadays, many decision and consensus support systems are being implemented in order to aid experts to solve decision problems efficiently [22,49]. Web-based applications are increasingly being used for GDM and Decision Support environments [65] because they offer many advantages. An example of these advantages is the possibility of accessing them from all over the world and thus, the possibility of carrying out distributed decision making processes where experts cannot meet physically together.

In this section we present a web consensus support system for the theoretical consensus model presented in Section 2.2. It will help both the moderator and experts in their tasks (defining the problem, expressing preferences avoiding missing information and so on). The system is programmed and fully implemented using a LAMP stack [39,47] (GNU/Linux operating system, Apache web server, MySQL database server and PHP programming language). It also makes use of Java technologies for the most complex interface tasks. It has been designed in form of different modules that interact with each other. Those different modules have been defined to separate the main logic of the system from the data storage requirements and the interface representation elements. In this way, it is possible to upgrade the system just by making changes in a particular module. These modules are:

- main module (access control)
- moderator's module
- expert's module
- computing module
- storage module

Fig. 4 illustrates a scheme of the WBCSS with the main interactions between the different modules. In the following we describe all the modules of the system, their interactions and how the users of the system (moderator and experts) are supposed to interact with them.

3.1. Main module

The main module of the system is responsible for giving access to the other modules and acts as a starting point to begin using the system. It controls the access of the users of the system assuring that non-moderator users cannot access to the moderator module and that only the registered experts can access to the expert's module for each problem on the system. To do so, this module presents a login screen (see Fig. 5) where the user is prompted for his e-mail address, password and the decision problem in which he is going to participate. Once the user introduces his login information the system checks if the information is correct and redirects the user to his corresponding module (moderator's or expert's module). Usual security measures are taken into account (for example, the moderator is notified via e-mail of several incorrect login attempts from a particular computer) to avoid unauthorized login into the system.

3.2. Moderator's module

Once a moderator has logged into the system he is presented with a screen with several possible actions (see Fig. 6):

- Change his password,
- Create a GDM problem,
- Show the current status of a GDM process and
- Log out.

To create a new GDM problem involves a three step input procedure:

1. *Basic definition of the problem:* problem name, description, maximum number of consensus rounds, several control parameters (stop conditions, consensus/consistency balance), number of experts involved and number of alternatives (see Fig. 7).
2. *Definition of alternatives:* The information about the different alternatives in the problem has to be entered: alternative names, description and an optional image.
3. *Registration of the experts:* name, e-mail and initial password for each expert (see Fig. 8).

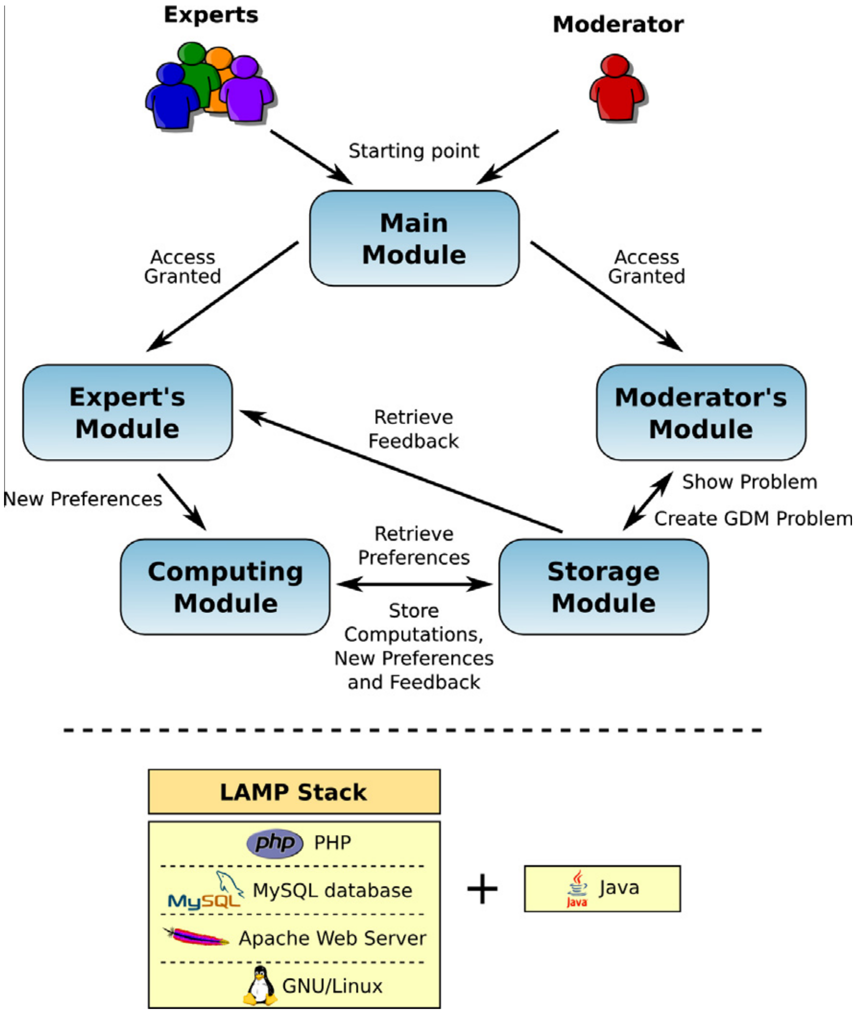

Fig. 4. Scheme of the WBCSS and the main interactions between modules.

Fig. 5. Login screen for WBCSS.

Once a decision problem is created by the moderator, the system automatically sends e-mails to the registered experts providing them with information about the problem, their login details for the system and basic instructions about the use of the web-system.

The moderator can also check the current status of a problem (*Show Problem* button), that is, all the preferences that were given by the experts in every consensus round, the measures that have been computed, the recommendations given to the experts and the final solution of the problem when it is finished.

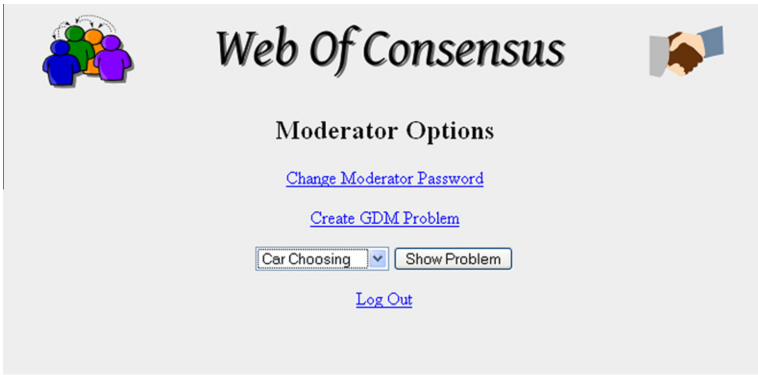

Fig. 6. Moderator's module main screen.

Fig. 7. Create a new GDM problem: basic definition of the problem.

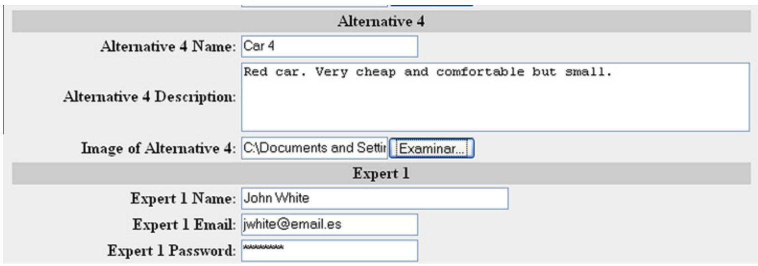

Fig. 8. Create a new GDM problem: definition of alternatives and experts in the problem.

3.3. Expert's module

The main goal of this module is to help experts to provide their preferences to the system assuring that those preferences are as consistent as possible, and also to show them the feedback recommendations that will be generated by the computation module after each round of the consensus process.

The first time an expert logs onto the system a screen is presented in which the type of preference relation to use is to be selected: fuzzy or multi-granular linguistic preference relations (see Fig. 9). Next, the information about the problem is presented, and if recommendations are already available then these will be shown along with the consensus and consistency measures computed for the current consensus round.

A complete preference relation requires $n \cdot (n-1)$ preference values. Obviously, as n increases it might be difficult for the experts to express complete and consistent information. To help experts to express their preferences in a consistent way, an

Fig. 9. The expert chooses the kind of preference relation to use.

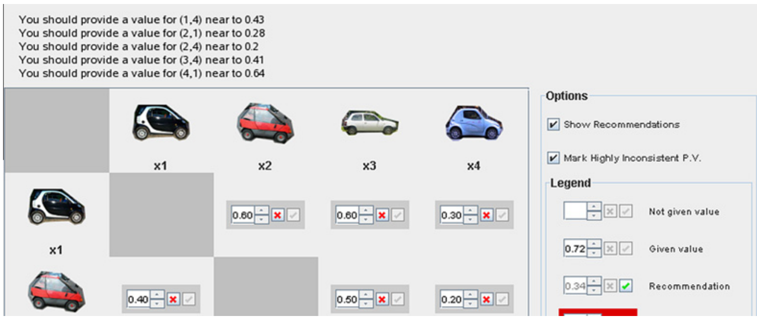

Fig. 10. Expert's module snapshot.

improved version of the Java applet that was developed in [1] has been developed to support not just fuzzy preference relations but also linguistic preference relations. This Java applet inform experts in real time about the possible inconsistencies that they may have introduced. It also provides some hints about the particular preference values that they should provide in order to improve consistency. Fig. 10 shows a partial snapshot of the Java applet within the expert's module containing some recommendations.

In the following we enumerate the *Interface Requirements* and *Logical Goals* that were used in designing the Java applet, and the actual implementation solutions developed will be illustrated with a snapshot of the system (Fig. 11).

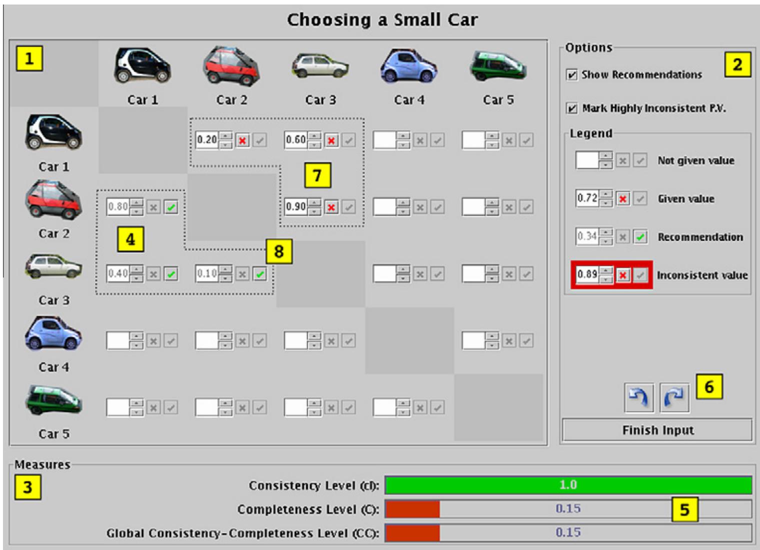

Fig. 11. The Java applet to ease the expression of preference relations.

3.3.1. Interface requirements

These requirements deal with the visual representation of the information and the different controls in the system. The developed applet comply with the so called *"Eight Golden Rules"* [7] for interface design:

GR 1. *Strive for consistency:* To comply with this requirement the interface has been homogenized in order to present an easy to understand view of the process which is being carried out. We have introduced three main areas: In area number **(1)** we present the *preference relation* that the expert is introducing, as well as a brief description of every alternative. Area number **(2)** contains several global controls to activate/ deactivate certain functions, as well as to finish the input process. Area number **(3)** contains different measures that show the overall progress.
GR 2. *Enable frequent users to use shortcuts:* Shortcuts have been added to the most frequent options and the input text-boxes for the preference values have been ordered to access them easily using the keyboard.
GR 3. *Offer informative feedback:* Our systems provides recommendations **(4)** and consistency and completeness measures **(5)**. All controls have tooltips.
GR 4. *Design dialogues to yield closure:* With every change that the user makes to his preferences the system provides new recommendations and measures.
GR 5. *Offer simple error handling:* The system avoids the introduction of incorrect preference values (for example values outside the [0,1] interval for fuzzy preference relations). Additionally, the system provides a way to point out in red colour when a preference is highly inconsistent.
GR 6. *Permit easy reversal of actions (undo action):* We have introduced *undo* and *redo* buttons **(6)** in the system.
GR 7. *Support internal focus of control (user is at charge):* The user can choose at every moment which preference value wants to give or update, as well as to enable/disable options.
GR 8. *Reduce short-term memory load of the user:* All information is presented in a single screen, so the user does not have to remember any data.

3.3.2. Logical goals

The interface comply with the following logical goals:

LG 1. *Offer recommendations to the expert to guide him toward a highly consistent and complete preference relation:* To offer recommendations, the system uses Eqs. (3) and (6)to compute all the missing values that could be estimated (for more details check [1,11,30]) and it presents them in area **(1)**. As the values are computed taking into account the multiplicative transitivity property, the recommendations if accepted will contribute to increment the overall consistency level. They are presented in a different colour (gray) **(4)** to differentiate them from the values entered by the expert **(7)**.
LG 2. *Recommendations must be given interactively:* When the expert introduces or updates a preference value all possible recommendations are recomputed and presented.
LG 3. *Recommendations must be simple to understand and to apply:* Recommendations are given in the same manner as the user inputs his preferences (fuzzy or linguistic). There is also a button that enables the user to easily accept or validate a given recommendation **(8)**.
LG 4. *The user must be able to refuse recommendations:* A user can choose any value for a particular preference degree ignoring all the recommendations. However, if the introduced preference degree is highly inconsistent it will be highlighted (in red color).
LG 5. *The system must provide indicators of the consistency and completeness level achieved at every step:* The applet presents some of the consistency and completeness measures presented in Section 2.1.1 and 2.2.1 **(5)**: C^h, cl^h, and CC^h.
LG 6. *The applet should be easy to adapt to other types of preference relations:* As the system is programmed following the principles of Object Oriented Programming, its extension to support new kinds of preference relations is an easy task. In fact, in this paper we have successfully adapted a previous version to support linguistic preference relations.
LG 7. *The applet should be easy to incorporate to web-based GDM models and decision support systems [65]:* As the system is Java based, it is easy to incorporate it into both new and existing web-based environments, and in fact, in this paper we have successfully integrated it into the expert's module of the WBCSS.

3.4. Computing module

This module computes all the consistency and consensus measures presented in Section 2 and also generates the recommendations which will be presented to the experts at the end of each consensus round. This module also checks if the current consensus/consistency level has reached the minimum satisfaction threshold value, and if so it will activate the selection process to obtain the final solution of the problem. The computations and recommendations are stored, along with the preferences expressed by the experts, in a database and therefore this module communicates with the storage module.

When all the computations have been carried out, e-mails are sent to the experts and the moderator to update them on the progress of the consensus process, and if necessary to request them to initiate a new consensus round. This module automatizes most of the tasks that are usually assumed by the moderator, and therefore the moderator can actually be replaced by it.

3.5. Storage module

A storage module is been developed to store, retrieve and manage all the information produced by the system and that other modules will need. Data is stored in a relational database (in a MySQL database server) as illustrated in Fig. 12.

4. Example of application

In this section we present an example of application of the consensus support system to solve a simple GDM problem. The problem is that of selecting the best car from a set of four different alternatives:

- Black, economic and slow car: *Black Car*.
- Red, very small, fast and comfortable car: *Red Car*.
- White and very fast car. It consumes little but it is very expensive: *White Car*.
- Blue, very small and very cheap car: *Blue Car*.

Four experts (e_1, e_2, e_3, e_4) will give their preferences using different kinds of preference relations (fuzzy and linguistic) and with different granularities. Experts e_1 and e_2 provide their preferences by means of a fuzzy preference relation; experts e_3 and e_4 provides their preferences using linguistic preference relation with a term set of granularity 5 and 7, respectively:

$$S_3 = \{Null(N), Low(L), Med(M), High(H), Total(T)\},$$
$$S_4 = \{Null(N), VeryLow(VL), Low(L), Med(M), High(H), VeryHigh(VH), Total(T)\}.$$

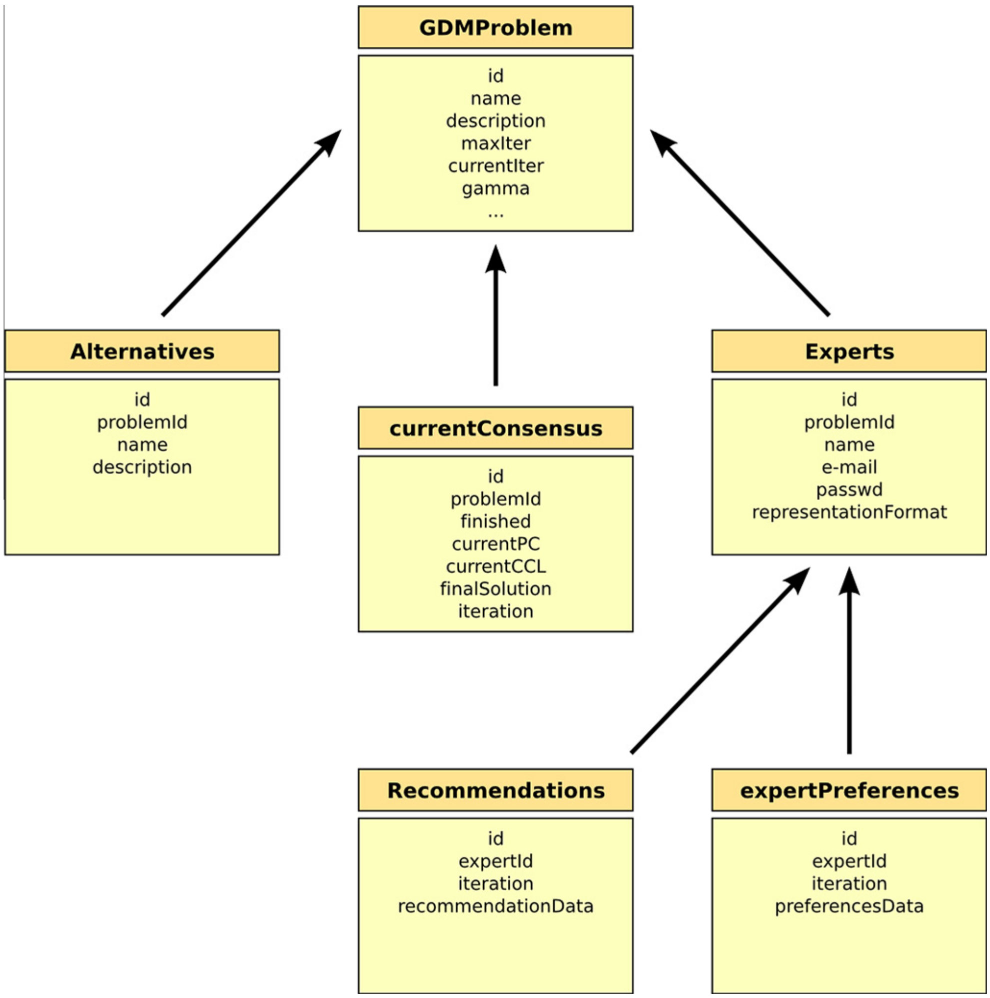

Fig. 12. Main tables in the storage module database.

In addition, we suppose that the moderator has introduced the following parameters to control the system: $\gamma = 0.8$, $\delta = 0.5$ and *maxIters* = 5.

4.1. First consensus round

We will describe in detail how the expert e_1 provides his fuzzy preference relation to the system. A similar procedure applies to the rest of experts.

As soon as expert e_1 access to the system, he provides the following preference values: $p^1_{12} = 0.6$ and $p^1_{13} = 0.6$, and the system replies by providing the following preference values $p^1_{23} = 0.5$ and, $p^1_{32} = 0.5$ (see Fig. 13). At this point, his consistency level $cl = 1.0$ (maximum), and the completeness level $C = 0.17$.

Expert e_1 accepts the values p^1_{23} and p^1_{32} estimated by the system, and the system reacts by providing new estimated values $p^1_{21} = 0.4$ and $p^1_{31} = 0.4$, updates the completeness level $C = 0.33$ (increases) while the consistency level is not affected ($cl = 1.0$) (see Fig. 14).

Expert e_1 considers the estimated value p^1_{21} does not reflect his real preference value $p^1_{21} = 0.8$, which is inserted instead. The applet detects a high inconsistency associated to this new value, and consequently, it is highlighted in red and the global consistency level is decreased to $cl = 0.85$ (see Fig. 15). Expert e_1 realizes that the system advice is correct and that he is introducing a contradiction in his preference relation ($p^1_{12} = 0.6 \Rightarrow x_1 \succ x_2$ and $p^1_{21} = 0.8 \Rightarrow x_2 \succ x_1$). In order to avoid this situation, expert e_1 changes p^1_{21} to the value that the system initially suggested ($p^1_{21} = 0.4$).

We assume that expert e_1 completes his preference relation taking into account the advice given by the applet. The experts' starting preference relations are therefore the following, with experts e_2 and e_4 providing incomplete preference relations (the values marked with a x are missing values):

$$P^1 = \begin{pmatrix} - & 0.60 & 0.60 & 0.30 \\ 0.40 & - & 0.50 & 0.20 \\ 0.40 & 0.50 & - & 0.30 \\ 0.62 & 0.72 & 0.70 & - \end{pmatrix}; \quad P^2 = \begin{pmatrix} - & 0.20 & 0.20 & 0.80 \\ 0.70 & - & x & x \\ x & x & - & x \\ 0.20 & x & x & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & L & M & M \\ L & - & H & M \\ H & L & - & M \\ H & M & H & - \end{pmatrix}; \quad P^4 = \begin{pmatrix} - & VH & H & x \\ x & - & L & x \\ M & H & - & x \\ x & VH & H & - \end{pmatrix}.$$

The computing module returns the current consensus/consistency level $CCL = 0.73$ (see Fig. 16), and because it is lower than the minimum threshold value was $\gamma = 0.80$ the system applies the feedback mechanism to generate advice rules as Fig. 16 illustrates. In this case, the system does not provide any recommendation to expert e_1 but many to e_2 to complete the corresponding preference relation and to increase its associated consensus/consistency level. A similar comment applies to experts e_3 and e_4.

As it can be seen, the system automatizes most of the tasks that are usually assumed by the moderator, and therefore the moderator can actually be replaced by it.

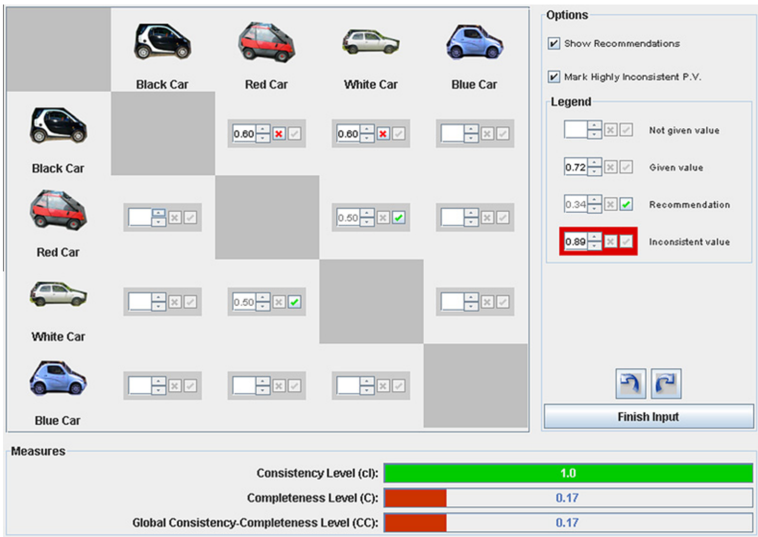

Fig. 13. The expert e_1 gives his first preference values.

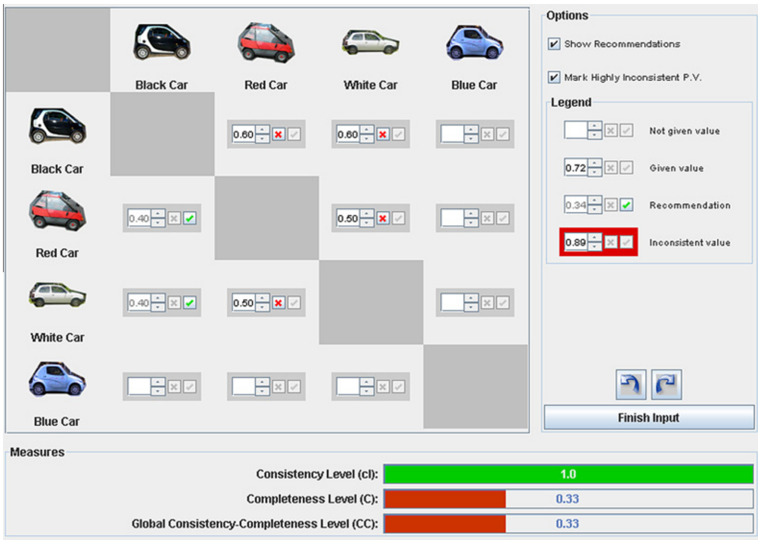

Fig. 14. Expert e_1 accepts the values that the system suggested.

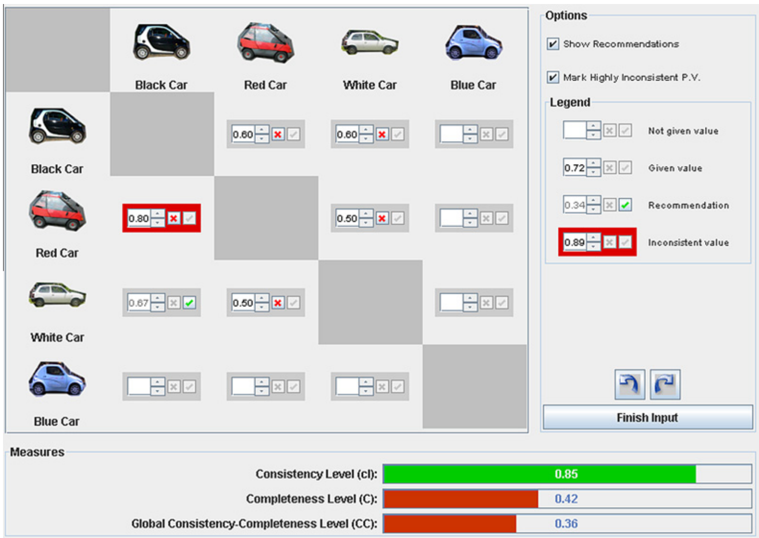

Fig. 15. Expert e_1 introduces some inconsistency in his preferences.

4.2. Second consensus round

We assume that the experts follow the advice and use the values given by the consensus system, and thus, in the new consensus round the experts preference relations are:

$$P^1 = \begin{pmatrix} - & 0.60 & 0.60 & 0.30 \\ 0.40 & - & 0.50 & 0.20 \\ 0.40 & 0.50 & - & 0.30 \\ 0.62 & 0.72 & 0.70 & - \end{pmatrix}; \quad P^2 = \begin{pmatrix} - & 0.20 & 0.51 & 0.49 \\ 0.37 & - & 0.43 & 0.36 \\ 0.53 & 0.56 & - & 0.53 \\ 0.57 & 0.52 & 0.58 & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & L & M & M \\ L & - & H & M \\ H & L & - & M \\ H & M & H & - \end{pmatrix}; \quad P^4 = \begin{pmatrix} - & VH & H & M \\ L & - & L & VL \\ M & H & - & L \\ H & VH & H & - \end{pmatrix}.$$

Current CCL: 0.73

Recommendations given to Expert e1 (60):

Recommendations given to Expert e2 (61):

You should provide a value for (1,3) near to 0.51
You should provide a value for (1,4) near to 0.49
You should provide a value for (2,1) near to 0.37
You should provide a value for (2,3) near to 0.43
You should provide a value for (2,4) near to 0.36
You should provide a value for (3,1) near to 0.53
You should provide a value for (3,2) near to 0.56
You should provide a value for (3,4) near to 0.53
You should provide a value for (4,1) near to 0.57
You should provide a value for (4,2) near to 0.52
You should provide a value for (4,3) near to 0.58

Fig. 16. Current CCL and Recommendations generated.

Current PC:

-	0.25	0.52	0.48
0.38	-	0.39	0.27
0.46	0.61	-	0.5
0.63	0.53	0.71	-

Current CCL: 0.86

Final solution: Alternative Blue Car (94)

Fig. 17. CCL at the second iteration and the final solution.

The computing module returns the current consensus/consistency level $CCL = 0.86$, which is greater than the minimum threshold value and therefore the selection process is applied leading to the *Blue Car* as the final collective consensus choice (see Fig. 17).

5. Conclusions and future works

We have presented a web consensus support system to deal with GDM problems with different kinds of incomplete preference relations (fuzzy, linguistic and multi-granular linguistic preference relations). The consensus reaching process is guided by both consistency and consensus measures. Consistency has been modelled via the multiplicative consistency property also known as the Cross Ratio uninorm, and it has been used to estimate unknown values of incomplete preference relations as well as to compute the needed consistency measures.

The system aims were to facilitate experts to express their preferences on the alternatives in the problem while maintaining their consistency and to provide easy to understand recommendations in form of simple rules to help them to converge to a solution for the problem with a high level of consensus. Because the system automatizes most of the tasks that are usually assumed by the moderator, the moderator can actually be replaced by it.

In future works we will improve the system by incorporating informative graphs to visually represent the consensus and consistency state at every step of the model and will adapt it to allow experts to use the system from almost any mobile device.

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