Applied Soft Computing 35 (2015) 949–957

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A web tool to support decision making in the housing market using hesitant fuzzy linguistic term sets

R. Montes [a,*], A.M. Sánchez [a], P. Villar [a], F. Herrera [b]

[a] Department of Software Engineering, University of Granada, Spain
[b] Department of Computer Science and Artificial Intelligence, University of Granada, Spain

ARTICLE INFO

Article history:
Received 7 October 2014
Received in revised form 11 January 2015
Accepted 17 January 2015
Available online 4 February 2015

Keywords:
Hesitant fuzzy linguistic term set
Linguistic 2-tuples
Multicriteria decision making
Web services
Housing market

ABSTRACT

In this paper we present a linguistic multiple-expert multi-criteria decision making model and a web tool to support it, that is centred on the housing market. The web tool is integrated with the usual catalogue of resources for rental or for sale, enriched with the possibility of ranking a subset of properties according to the client's preferences and the internal knowledge associated to the properties. Usually the description of a property is quantitative, thought in our case we add qualitative information corresponding to assessments made by housing agents. These agents are considered experts in the market conditions.

We apply the 2-tuple linguistic representation model to keep accuracy in the processes of Computing with Words and the hesitant fuzzy linguistic term sets to qualify in situations of uncertainty and hesitation in the assessments. The software helps the agents in the process of the elicitation of the linguistic expression based on the fuzzy linguistic approach and the use of context-free grammars, and the web clients in the decision of visiting a property.

1. IntroductionIntroduction

Every day people are challenged with multiple acts of decision. It is a natural human activity that is prompt to be subjective in its basis, but also to be uncertain and imprecise. Sometimes we are not aware of the implicit complexity of a problem, except when we try to make decisions with computational models. Problems defined under uncertain conditions are common in real world, but quite challenging to be modelled in a computer program due to the difficulty of dealing with uncertain information. Computing with Words (CW) [30] is a methodology for reasoning and computing with perceptions rather than measurements. CW is able to empower applications that involve people expressing their preferences which happens in linguistic Decision Making (DM): the experts assess the potential of an alternative through qualitative values rather than quantitative ones.

Some models have been proposed to operate with linguistic information [1,6,16]. In this work we operate with the fuzzy linguistic approach, that represents qualitative aspects as linguistics values by means of linguistics variables [29]. These models are preferable because experts are allowed to evaluate closer to natural language and the way people reason. Criteria in different problems may vary, for example, if we describe a car, the criteria would involve price, fuel consumption or comfort. On the other hand, it is sometimes difficult to give an opinion as an exact single label, though we could allow to work with some possible set of values for decision makers.

To manage linguistic information in DM a well known computational model that carries out CW processes without loss of information is the 2-tuple Linguistic Computational Model [8]. This model uses a pair of values called linguistic 2-tuple to represent the linguistic information. Many extensions to the 2-tuple linguistic model have been developed: to deal with unbalanced linguistic information [11] where the linguistic labels in the terms set are not evenly distributed around a central term, or to deal with multi-granular label sets [10]. A generalization of the 2-tuple representation, the proportional 2-tuple models were developed by Wang and Hao in [24]. Also, Dong et al. [3,4] explored the concept of numerical scale, which extends the linguistic 2-tuple and the proportional 2-tuple.

Recently it has been enabled in DM problems the possibility of provide inaccurate rates and comparative linguistic expressions by means of the use of a context-free grammar represented by a hesitant fuzzy linguistic term set (HFLTS) [13,18]. This way to deal with uncertainty and hesitation in the context of fuzzy decision making, derives from the original idea of Torra's hesitant fuzzy sets [22], has been applied in many recent works [2,12–15,17,19,20,26,25,23,31,32].

* Corresponding author. Tel.: +34 615329988.
E-mail addresses: rosana@ugr.es (R. Montes), amlopez@ugr.es (A.M. Sánchez), pvillarc@ugr.es (P. Villar), herrera@decsai.ugr.es (F. Herrera).

http://dx.doi.org/10.1016/j.asoc.2015.01.030
1568-4946/

Particularly, the group decision making (GDM) problem is an area of application of CW to achieve a collective decision, that could benefit from the use of hesitant linguistic assessments [19] or consensus models [5].

In the housing market, a common problem is to describe a property accurately. When the owner fills the description form to upload a new record, the description he/she makes is usually over positive deliberately omitting deficiencies. The price tends to be too high because the owner lacks the knowledge of the area. Additionally the customer could get disappointed when visit the place because it may not fulfil the expectations. The first issue could be resolved with the counselling to the owner. For example, in some Real Estate agencies, a new order of for sale or to rent is taken by a realtor that describes the property after interviewing the owner and advises on the price. In our knowledge for the second issue, there is currently no automated solution to advice a buyer about what property should visit in person according the expectations.

The agents of a Real Estate agency are experts in the housing market and can be involved in a multi-expert multi-criteria decision making model. Criteria are attributes that helps in the description of a house. The benefit from the application of the model could consist in the valuation of candidates, in this case, a list of properties (houses, flats, garages, et.). Usually agencies have an online catalogue, offering properties for sale and to rent to online customers around the world to increase the opportunities of business. Data and customers are online, thus a convenient solution would imply a web tool integrated within the catalogue.

The aim of this paper is to present a practical application in decision making of a 2-tuple linguistic fuzzy model with hesitant information. It represents a Real Estate web site with information of properties for sale or to rent. The novelty of the portal is the possibility of using linguistic expressions to assess a set of qualitative criteria. We have added eight criteria not so commonly used to describe the property, but that we think of relevance to the client following the suggestions of Real Estate agents. The evaluation of the properties is done by the realtors who possess the expertise over the housing market. DM processes that run over the server portal will use this internal information to propose a visitation order to its web clients based on their preferences. Also, a client of a Real Estate web site could assign a preference degree to each criteria in its profile area. The rest of this paper is structured as follows. In Section 2, some preliminaries related with the representation of qualitative data and the grammar used to evaluate are reviewed. In Section 3, a Multi-Criteria Multi-Expert Decision Making problem for the housing market is presented based on 2-tuple fuzzy representation of hesitant expressions. In Section 4, an illustrative example is presented. In Section 5, some conclusions are given.

2. Preliminaries

The fuzzy linguistic approach manipulate qualitative information by using linguistic variables as a representation of those values. Words are not numbers and thus they are more imprecise, but thanks to linguistic models, computations can be carried out using this type of information. In the literature we find a methodology to apply CW in decision making [6], that was introduced by Zadeh in the seminal paper [28] and in the definition of a linguistic variable [29]. We have chosen to apply the 2-tuple linguistic computational model as it represents a transformation of a linguistic variable which is suitable for computations without any lost of information.

In this section we give some general definitions of the 2-tuple representation model and a description of how the HFLTS enables a flexible way of elicit linguistic information.

2.1. A computational 2-tuple fuzzy linguistic model

A linguistic variable can take values only in a finite set of eligible values that are defined by the linguistic term set $S = \{s_0, \ldots, s_g\}$, in which $g + 1$ is called the cardinality of S and usually is an odd number. The more terms in S the more precise an evaluation could be, but on the contrary, it also imposes hesitation to the expert. The linguistic terms $s_k \in S$ are defined by triangular membership functions uniformly distributed. These assumptions guarantee that the 2-tuple linguistic computational model [8] is precise and effective.

Definition 1. [8] Let S be a linguistic term set, and $\beta \in [0, g]$. Then the 2-tuple is defined as:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, i = round(\beta), \\ \alpha = \beta - i \end{cases} \quad (1)$$

The 2-tuple becomes a equivalent representation of any term $s_i \in S$. The inverse function $\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g]$ is defined in [8] by $\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$. The value of $\alpha \in [-0.5, 0.5)$ it is known as the symbolic translation. So, a linguistic term $s_i \in S$ transforms into $(s_i, 0)$ in CW processes.

When rating two alternatives assessed with a linguistic variable represented with a 2-tuple, a comparison rule is needed. We find in [8] a valid comparison rule:

1 if $n < m$, then (s_n, α_1) is smaller than (s_m, α_2)
2 if $n = m$, then

(a) if $\alpha_1 = \alpha_2$, then (s_n, α_1) and (s_m, α_2) are the same
(b) if $\alpha_1 < \alpha_2$, then (s_n, α_1) is smaller than (s_m, α_2)
(c) if $\alpha_1 > \alpha_2$, then (s_n, α_1) is bigger than (s_m, α_2)

To aggregate 2-tuples, the arithmetic mean can be adapted to be applied to the 2-tuple representation. Let $x = \{(s_1, \alpha_1), \ldots, (s_n, \alpha_n)\} = \{\beta_1, \ldots, \beta_n\}$ be a set of linguistic values represented as 2-tuple, W a weighting vector $(\{w_i/i = 1, \ldots, n\})$, and W' its normalised version $(\{w'_i/i = 1, \ldots, n\})$, i.e. $\sum_{i=1}^{n} w'_i = 1$. The arithmetic weighed extended mean $\bar{x}^e$ is defined as:

$$\bar{x}^e(x) = \Delta\left(\frac{\sum_{i=1}^{n} \Delta^{-1}(s_i, \alpha_i) \cdot w_i}{\sum_{i=1}^{n} w_i}\right) = \Delta\left(\frac{1}{n}\sum_{i=1}^{n} \beta_i w'_i\right). \quad (2)$$

Example 1. A linguistic term set with five linguistic terms can be as: $S = \{nothing, little, middle, high, very\ high\}$. A linguistic variable ϑ representing the ambient noise of a neighbourhood could be valued at midnight as *little*. So $\vartheta = s_1$, but in operations the 2-tuple $(s_1, 0)$, or $\beta = 1$, will be used.

2.2. Hesitant fuzzy linguistic term set

In situations where is usual to handle imprecise information, it is needed a solution to model hesitation in the elicitation of linguistic information. In a quantitative setting, the concept of hesitant fuzzy set (HFS) was introduced in [22] to allow decision makers the consideration of several values to determine the membership of an element to a set. The concept of HFS have proved to be applicable to DM, evaluation and clustering techniques [20]. To be used in linguistic fuzzy decision making situations, its extension known as HFLTS was presented in [18]. Afterwards, many researchers have apply the concept of HFLTS [2,12–15,17,19,25,26,31,32].

Definition 2. [18] Let $S = \{s_0, \ldots, s_t\}$ be a fixed set of linguistic term set. A hesitant fuzzy linguistic term set (HFLTS) H, is an ordered finite subset of the consecutive linguistic terms of S.

The HFLTS is a new and flexible tool in representing hesitant qualitative information that can be used to elicit several linguistic values for a linguistic variable. The use of a context-free grammar increase the flexibility of the model by including the possibility of elicit comparative linguistics expressions.

Definition 3. [18] Let G_H be a context-free grammar. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$$
\begin{aligned}
V_N &= \{< \textit{primary term} >, < \textit{composite term} >, < \textit{unary relation} >, \\
&\quad < \textit{binary relation} >, < \textit{conjunction} >\}; \\
V_T &= \{\textit{lower than}, \textit{greater than}, \textit{at least}, \textit{at most}, \textit{between}, \textit{and}, s_{-t}, \ldots, s_0, \ldots, s_t\}; \\
I &\in V_N; \\
P &= \{I ::= < \textit{primary term} > | < \textit{composite term} > \\
&\quad < \textit{composite term} > ::= < \textit{unary relation} > < \textit{primary term} > \\
&\quad | < \textit{binary relation} > < \textit{conjunction} > < \textit{primary term} > \\
&\quad < \textit{primary term} > ::= s_{-t}, \ldots, s_0, \ldots, s_t \\
&\quad < \textit{unary relation} > ::= \textit{lower than} | \textit{greater than} | \textit{at least} | \textit{at most} \\
&\quad < \textit{binary relation} > ::= \textit{between} \\
&\quad < \textit{conjunction} > ::= \textit{and}\}
\end{aligned}
$$

Expressions in this grammar would need to be transformed into something useful to carry out the CW processes. The following transformation functions E_{G_H} [18] are used to generate a HFLTS from a comparative linguistic expression.

$$
\begin{aligned}
E_{G_H}(\textit{greater than}\, s_i) &= \{s_k | s_k \in S \text{ and } s_k > s_i\} \\
E_{G_H}(\textit{lower than}\, s_i) &= \{s_k | s_k \in S \text{ and } s_k < s_i\} \\
E_{G_H}(\textit{at least}\, s_i) &= \{s_k | s_k \in S \text{ and } s_k \geq s_i\} \qquad (3) \\
E_{G_H}(\textit{at most}\, s_i) &= \{s_k | s_k \in S \text{ and } s_k \leq s_i\} \\
E_{G_H}(\textit{between}\, s_i \text{ and } s_j) &= \{s_k | s_k \in S \text{ and } s_i \leq s_k \leq s_j\}
\end{aligned}
$$

Example 2. Following the previous example of S and ϑ, let us give a valuation using the following grammatical expression: the ambient noise of a neighbourhood x is `at least` *little* `and` is `lower than` *very high*. The resulting HFLTS is:

$$
\begin{aligned}
E_{G_H}(\textit{at least}\, s_1) &= \{s_1, s_2, s_3, s_4\} \\
E_{G_H}(\textit{lower than}\, s_4) &= \{s_0, s_1, s_2, s_3\} \\
\{s_1, s_2, s_3, s_4\} \textit{ and } \{s_0, s_1, s_2, s_3\} &= \{s_1, s_2, s_3\} \\
H(\vartheta) = \{\textit{little}, \textit{middle}, \textit{high}\}
\end{aligned}
$$

The previous grammatical expression is equivalent to `between` *little* `and` *high*.

Definition 4. [18] The envelope of a HFLTS, *env*(H), is a linguistic interval whose limits are obtained by means of its upper bound H^+ and lower bound H^-:

$$
\begin{aligned}
H^+ &= \max\{s_i\} = s_j,\ s_i \leq s_j \text{ and } s_i \in H,\ \forall i, \\
H^- &= \min\{s_i\} = s_j,\ s_i \geq s_j \text{ and } s_i \in H,\ \forall i.
\end{aligned} \qquad (4)
$$

The envelope is computed as:

$$env(H) = [H^-, H^+], H_{S^-} \leq H^+,$$

Example 3. Following the previous example of $H(\vartheta)$, the envelope is:

$$env(H(\vartheta)) = [\textit{little}, \textit{high}]$$

Note that when experts give their preferences with grammatical expressions (as in Example 2), the resulting valuation need to be transformed into some computable form. We have considered to operate with linguistic intervals, that allows to propagate the hesitation intrinsic in the assessment. For that reason, our CW processes are carried out using the envelope of a HFLTS. Though we use the original envelope calculation (given in Definition 4), it is possible to compute such linguistic intervals by means of a fuzzy envelope of HFLTS [13].

3. Linguistic multiple-expert multi-criteria decision using the hesitant 2-tuple fuzzy linguistic representation for the housing market

Decision Making problems are common in real life situations, when people have to choose the most desirable alternative under multiple influential attributes and opinions. Since the concepts of fuzzy sets and linguistic variable was introduced by Zadeh [29] many research lines have evolve in the application of linguistic computational models to DM [21,7,16,31]. In this section we describe a fuzzy linguistic multiple-expert multi-criteria decision making (MEMCDM) problem for the housing market and the considerations that should be taken to integrate a fuzzy linguistic DM model with HFLTS in our web tool.

3.1. Problem description

In the housing market, online web sites are commonly used by people met online to buy, for sale or to rent properties. The Real Estate agencies put in their web sites a complete catalogue of house rental and sale offers that can be consulted by web clients around the world. Each house has a record with detailed information about its characteristics, where some of them are private to the realtors and some are public at the website. Basically each property is defined by a set of quantitative values (number of bedrooms, square feet, price, etc.) and a free text field used for an additional description. The description tends to be rich but unstructured and informal. It means that its format does not suit the filters that can be used to search for a property by tags or keywords. This is inherently a situation of having information with a strong qualitative definition.

Linguistic information attracts new possibilities of developments that could benefit from the techniques and proposals of the fuzzy linguistic approach. What we propose in this paper is a new service for a Real Estate agencies. The business model is supported by offering new services to the users under a fee (e.g. free account, pro account, V.I.P. account). This type of web sites could offer a new service that would rely in a fuzzy linguistic decision making model built in. Firstly, we need to provide a method to store houses of interest in the own user space in the website. The key point is to include qualitative information to describe the property to be used internally in the model. These extra information would come by the realtors. It is a reality that the housing market changes continuously, therefore the experts that better describe new acquisitions are the agents of the Real Estate and not the property owners desirous for a good sale. The output of the decision making model are the client's houses of interest sorted in a way that it better reflects the client's preferences.

In our system, to catalogue a new property it is necessary to include qualitative information additionally to the usual fields (number of baths, type of floors, pool yes-no, garage yes-no, etc.). The qualitative description comes in the form of eight criteria (shown in Table 1) assessed by the agent that takes in first place the order of sale or rent. In our case, these set of characteristics were selected by the experts themselves because they have the knowledge of what customers want to know about. For us it is mandatory that at least one agent assess the property, but the main idea of the model proposed is that many other agents (from the same Real Estate agency or from an alliance of agencies) include as well their

Table 1
Criteria used by the experts to assess the benefits of a property.

	Criterion	Description
C_1	Quality / Price	Quality with respect to Price. It represents the sale price of the property as a conditional value to the benefit that would be achieved with the arrangement (rent or purchase) of the property.
C_2	Property Condition	Review the need for housing reform. It represents the time and economic effort invested before you can enjoy the property.
C_3	Luminosity	Combined evaluation of number of exterior rooms, altitude and number of floors, orientation with respect to sun and proximity of other buildings.
C_4	Soundproofing	Evaluates the absence of acoustic discomfort observing the existence of factories or services (such as a fire house) that may increase the recommended threshold for external acoustic noise.
C_5	Public Services	Provision of public services (schools, hospitals, post office, etc.) in the area, accessible within walking distance.
C_6	Shopping &Leisure	Offer related to leisure (cinema, restaurants, gyms), shops (hairdresser, hardware, repair, etc.) and other services within walking distance.
C_7	Transportation	Quality of public transport provision in the area (taxi, bus, metro, etc.).
C_8	Communications	Indicates the proximity to highway access points and roads of the national road network.

assessments in the system. Therefore the problem of suggest a visiting order is transformed into a MEMCDM problem with the houses as the alternatives A ($\{A_i/i = 1, \ldots, N\}$), the realtors as the experts E ($\{E_k/k = 1, \ldots, P\}$), the criteria C ($\{C_j/j = 1 \ldots, M\}$) as the information that is qualified linguistically by realtors. In our proposal, the clients express their preferences over each criterion by means of a percentage value, used as weights W ($\{w_j/j = 1, \ldots, M\}$).

3.2. A hesitant fuzzy linguistic decision making model

The computational linguistic model described in this work allows to elicit evaluations by means of a context-free grammar, that would be managed with HFLTS [18] and the 2-tuple representation. Our general CW scheme is shown in Fig. 1.

Besides aggregation and exploitation our scheme imply other processes. Fig. 2 shows our DM scheme, which combines the two representations.

- *Unification phase*. Valuation matrices are provided by experts by using linguistic expressions for each criterion constructed with the grammar G_H. Some experts would give single term valuations, and others, due to hesitation, would need to elicit comparative preferences values. So, a unification phase is needed to homogenise all the assessments. Transformation functions E_{G_H} (Eq. (3)) are applied to the preference relations getting HFLTS.
- *Interval calculation phase*. Each alternative is valued over a set of criteria by each expert. To operate with linguistic intervals we calculate the envelope of the HFLTS (as in Definition 4). In this stage every single valuation is noted as $[s_i, s_j]$.
- *2-tuple transformation phases*. The linguistic intervals are represented using the 2-tuple fuzzy linguistic computational approach, and are translated to $[(s_i, 0), (s_j, 0)]$. Following an standard scheme of CW processes [27], the translation would also imply the re-translation phase (using Eq. (1)). This backward operation converts 2-tuples results into something understandable by the user, as it is expressed in the original term set S. Other benefits can be derived from the use of the 2-tuple representation as stated in [9].

3.3. Elicitation of linguistic information with HFLTS

To illustrate our MEMCDM problem, let $S = \{nothing, very\ low, low, medium, high, very\ high, perfect\}$ be a linguistic term set, $A = \{A_i/i = 1, \ldots, N\}$ a set of N alternatives or candidates. Each candidate has a common number of M attributes or criteria, $C = \{C_j/j = 1 \ldots, M\}$, that are assessed by experts. The model use the combination of P experts in the subject, $E = \{E_k/k = 1, \ldots, P\}$. Generally, this type of multi-criteria problem is described with an evaluation matrix as the one shown in Table 2 where y_i^{jk} is the assessment given by E_k over the alternative A_i with respect to criterion C_j. Additionally, an input to the computational model is the relevance of each criteria given in percentage $W = \{w_1, \ldots, w_M\}$.

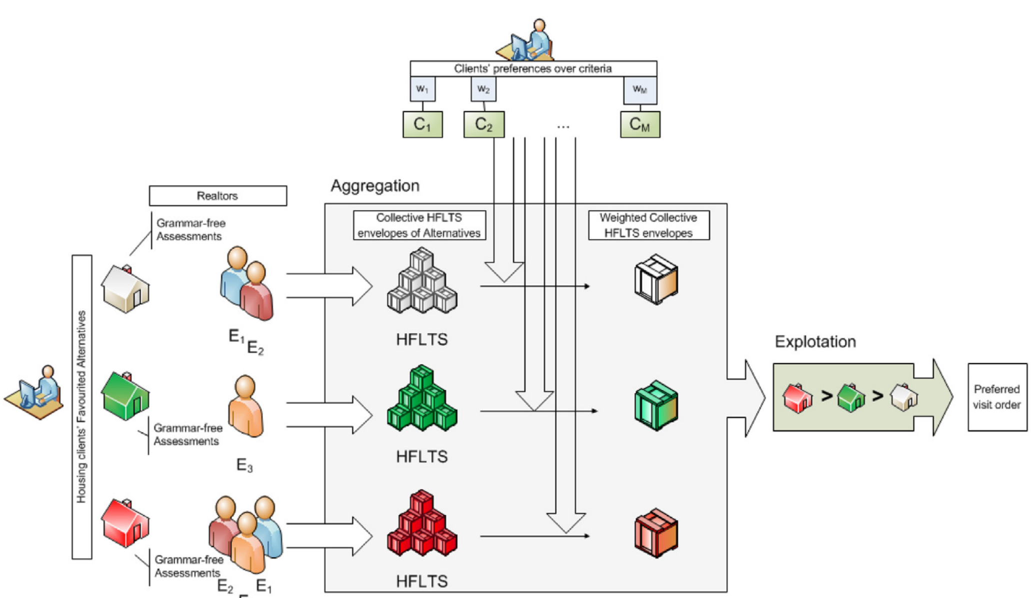

Fig. 1. MEMCDM problem in the case of a Real Estate website.

Fig. 2. Computing with Words scheme using 2-tuples and HFLTS.

Table 2
General case where an expert E_k has given his/her valuations.

		C_1	...	C_M
$E_k =$	A_1	y_1^{1k}	...	y_1^{Mk}
	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	A_N	y_N^{1k}	...	y_N^{Mk}

Later on, the system would use the normalised version of weights $W' = \{w'_1, \ldots, w'_M\}$ with $\sum_{i=0}^{M} w'_i = 1$.

Considering our Real Estate web site problem, for any alternative A_i (a flat, a house, etc.), all the criteria have been mandatory evaluated and at least one expert E_k (a realtor) has reviewed the candidate by means of grammar-free expressions over S. Under this premise, y_i^{jk} is a hesitant expression that is transformed after the application of the function E_{G_H} (see Eq. (3)). Thus we get a HFLTS for each valuation made by the expert.

$$E_{G_H}(y_i^{jk}) = H_i^{jk}$$

Definition 5. A HFLTS fuzzy evaluation matrix F_H is the overall evaluation of experts E, when each criteria C_j is considered as a linguistic variable, and H results after the interpretation of the grammar G_H over S. The matrix F_H collects the opinion of experts over candidates and is given by:

$$F_H(A_i) = \begin{pmatrix} H_i^{11} & H_i^{21} & \cdots & H_i^{M1} \\ H_i^{12} & H_i^{22} & \cdots & H_i^{M2} \\ \vdots & \vdots & \ddots & \vdots \\ H_i^{1p} & H_i^{2p} & \cdots & H_i^{Mp} \end{pmatrix} \text{ where } 0 < p \le P. \quad (5)$$

The condition of which $0 < p \le P$ is going to give us a degree of flexibility in the implementation of the DM model as it would not impose the same number of experts valuations over each alternative, and thus is closer to reality.

Example 4. Let be $N = 3$ the number of alternatives, $M = 3$ the criteria, and $P = 3$ the number of experts. Instead of having an $P \times M$ evaluation matrix we have a situation in which expert E_1 has evaluated all the criteria, E_2 has evaluated A_1 and A_2, and E_3 only gives its opinion about A_3. The three evaluation matrices would be:

$$F_H(A_1) = \begin{pmatrix} H_1^{11} & H_1^{21} & H_1^{31} \\ H_1^{12} & H_1^{22} & H_1^{32} \\ H_1^{13} & H_1^{23} & H_1^{33} \end{pmatrix} \quad \left| \begin{array}{l} F_H(A_2) = \begin{pmatrix} H_2^{11} & H_2^{21} & H_2^{31} \\ H_2^{12} & H_2^{22} & H_2^{32} \end{pmatrix} \\ F_H(A_3) = \begin{pmatrix} H_3^{13} & H_3^{23} & H_3^{33} \end{pmatrix} \end{array} \right.$$

3.4. Choice of an aggregation operator

The aggregation phase works with HFLTS fuzzy evaluation matrices F_H of size $N \times M \times P$, that are transformed into matrices of HFLTS envelopes F_{env}. As we stated before, each envelope $env(H)$ could be consider as linguistic interval $[s_i, s_j]$ and internally as 2-tuple fuzzy linguistic intervals. According to [19], s_i and s_j represent the pessimistic and optimistic perception of the assessments, respectively. When needed, we use in the notation the super index $^-$ or $^+$ to refer to operations performed only with s_i or s_j respectively. Complete operation, shown in Fig. 3, is described in this section.

The aim of the aggregation process is to compute a collective value for each alternative aggregating the experts assessments, and latter to compute a single valuation for each alternative according to the weights of each criterion stated by the customer. In the literature we find that, originally to aggregate HFLTS, Rodriguez et al. [18] defined two symbolic operators: the *min_upper* and *max_lower*, but according to [26] these operators cannot deal with the situation where the importance weights of criteria are considered. In our MEMCDM problem, weights are considered but the fact that each linguistic interval could be converted into a interval of 2-tuples $[(s_i, 0), (s_j, 0)]$ (translation step of Fig. 2) resolves this situation as we could use any 2-tuple weighted aggregation operator. Furthermore, from now on we could use any fuzzy linguistic aggregation operation (φ) and depending of the problem each aggregation operator could be different on each round [31]. Such operator φ should be used separately over the two values of the linguistic interval.

We operate with *min_upper* operator φ^- in two steps [18]: (i) apply the upper bound for each HFLTS envelope, (ii) obtain the minimum linguistic term for each alternative. For some given A_i and C_j we have:

$$env(H(A_i)) = \{env(H_i^{1j})^+, \ldots, env(H_i^{pj})^+\} \text{ with } p \le P,$$

$$\varphi^-(H_i^j) = \min\{env(H_i^{kj})^+ / k \in \{1, \ldots, p\}\}.$$

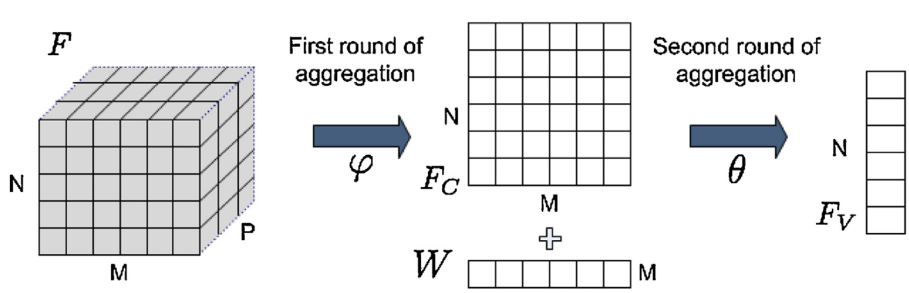

Fig. 3. From F to F_C and F_V by using aggregation operators φ and θ.

The *max_lower* operator, φ^+, also operates in two steps [18]: (i) apply the lower bound for each HFLTS envelope, (ii) obtain the maximum linguistic term for each alternative.

$$env(H(A_i)) = \{env(H_i^{1j})^-, \ldots, env(H_i^{pj})^-\} \text{ with } p \leq P,$$

$$\varphi^+(H_i^j) = \max\{env(H_i^{kj})^- / k \in \{1, \ldots, p\}\}$$

Example 5. Consider a DM problem with $C=5$ and $P=3$, and that we have to aggregate the valuations given by experts for A_1 with HFLTS.

$$F_{env}(A_1) = \begin{pmatrix} [s_3, s_3] & [s_4, s_4] & [s_5, s_5] & [s_5, s_5] & [s_0, s_1] \\ [s_3, s_3] & [s_2, s_2] & [s_3, s_3] & [s_3, s_4] & [s_3, s_4] \\ [s_4, s_5] & [s_5, s_5] & [s_5, s_6] & [s_5, s_6] & [s_0, s_4] \end{pmatrix}$$

We combine φ^- and φ^+ for each criterion to obtain a collective valuation. When C_1, $\varphi^-(1/2\{[s_3, s_3], [s_3, s_3], [s_4, s_5]\}) = s_3$ because $3 = \min(3, 3, 5)$ and because $4 = \max(3, 3, 4)$, $\varphi^+(1/2\{[s_3, s_3], [s_3, s_3], [s_4, s_5]\}) = s_4$. To complete the example:

$$F_C(A_1) = \begin{pmatrix} [s_3, s_4] & [s_2, s_5] & [s_3, s_5] & [s_4, s_5] & [s_1, s_3] \end{pmatrix}$$

The following represents the transformation step on the collective pessimistic evaluation matrix after the first aggregation:

$$(s_r, \alpha)_i^{j-} = \Delta(\varphi(\Delta^{-1}(env(H_i^{jk})))) \forall k \in \{1, \ldots, p\} \text{ with } p \leq P.$$

$$F_C^-(A_i) = \begin{pmatrix} (s_r, \alpha)_i^{1-} & \cdots & (s_r, \alpha)_i^{M-} \\ (s_r, \alpha)_i^{1-} & \cdots & (s_r, \alpha)_i^{M-} \\ \vdots & \ddots & \vdots \\ (s_r, \alpha)_i^{1-} & \cdots & (s_r, \alpha)_i^{M-} \end{pmatrix} \quad (6)$$

The resulting F_C^- and F_C^+, are two $N \times M$ matrices of collective pessimistic and optimistic values respectively. In our MCMEDM problem not every criterion has the same importance, as we allow the web clients to express their criteria preferences with weights. The next aggregation phase represents a weighted combination of collective valuations, that is carried out using the 2-tuple linguistic intervals separately. We choose the aggregation operator θ as the arithmetic weighed extended mean (see Eq. (2)).

We aggregate the valuation for each criterion but under the consideration that not every criterion has equal importance. The result of this aggregation (given in Eq. (7)) is a vector of N collective weighted values one for each alternative, $F_V = (v_1, \ldots, v_N)$. It is applied to the pessimistic and optimistic evaluations respectively as follows:

$$v_i = \Delta(\theta(\Delta^{-1}(s_r, \alpha)_i^j)) \ \forall j \in \{1, \ldots, M\} \quad (7)$$

3.5. Ranking of alternatives

The ranking of alternatives is the last phase to the solution of a MCMEDM problem. With the resulting vector F_V of linguistic 2-tuples for each candidate, we just have to sort it from the best (the greater value) to the worst (the smaller value). It is part of the exploitation phase and it could be performed according several criteria.

So far there are few methods attending specifically the use of HFLTS into decision making problems. In our proposed model we have used five different methods. From the fact that information it is represented by 2-tuple fuzzy linguistic intervals, we could use comparison rule on 2-tuples (see Section 2.1), so the first three cases are straightforward:

1 *Pessimistic case*. We consider only the lower values of the given intervals to arrange elements of the evaluation vector F_V, that it is F_V^-. The higher value is the better option.
2 *Optimistic case*. We apply the comparison rule on 2-tuple values to sort F_V^+. As well, the higher value is the better option.
3 *Average value*. Intervals values of F_V are averaged into single 2-tuple values. The comparison rule is applied to obtain a final rank.
4 *Choice degree*. Is the original procedure for comparison of HFLTS given in [18]. It employs the theory of interval values to rank HFLTS. We have followed the same operations with the 2-tuples linguistic intervals.
5 *Possibility degree*. In [26] a comparison of HFLTS procedure is given based on the probability theory. It calculates the possibility degree matrix P and a preference relation matrix U in an iterative algorithm. We have followed the same operations with the 2-tuples linguistic intervals.

4. A web tool for fuzzy decision making

The model presented in the previous section is applicable to real life decision making problems. We have found a practical application in the housing market and related to it, we have developed a web tool to support the model and the elicitation of linguistic information based on HFLTS. This section gives the details of the web tool and also presents an example.

4.1. Solution with the hesitant 2-tuple fuzzy linguistic representation

The evaluation process in a MEMCDM problem of describe rental housing or for sale opportunities poses some difficulties because valuations could be uncertain or fuzzy, mostly due to the subjectivity of realtors. This could be overcome by the use of a context-free grammar, given by the use of HFLTS, and it is an added value of the website, as it is a flexible procedure for electing linguistic information.

We have implement a linguistic model for decision making applied to offer a recommendation service on a housing market website. The input to the model correspond to a general MEMCDM problem. The web clients are able to browse the catalogue of opportunities for sale or for rent, and include them in a list of favourites (illustrated in Fig. 4). In this way they select a set of alternatives or candidates, which are already assessed by experts. In this case the experts are the realtors and their valuations are internally stored with a combination of HFLTS and 2-tuples representation. In the same client profile area, a customization of the importance of each criteria (a percentage weight used in the aggregation phase) it is permitted.

When the client is ready, he or she will consult the decision making model built in the portal to get a recommendation of the ideal visitation order for those candidates according to the given preferences. The user could modify the list or his/her preferences at any time. Fig. 5 illustrates how this is done in the web site.

4.2. Case use: three customers with same candidates

An unlimited number of users, U, could get logged in our site to store their preferences over the given criteria, and to use our decision making implementation. As a result, they are going to be advised about what candidates are closer to their preferences according to the internal reviews of the realtors. Let consider as an example that three customers are interested on the same zone of one city. Suppose that they have selected the same three flats A_1, A_2 and A_3, after a search is done. What makes the difference in the output of the DM for the three customers is the preferences over the criteria. Their weights are different as could be noted in Table 3.

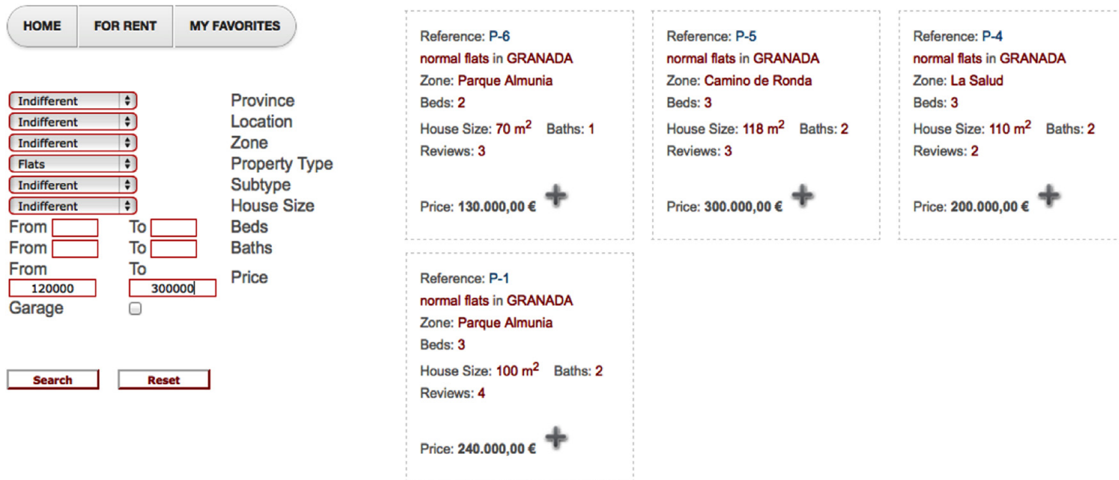

Fig. 4. Candidate selection after a search in the catalogue. Clients add their items of interest into a list called *My favourites*.

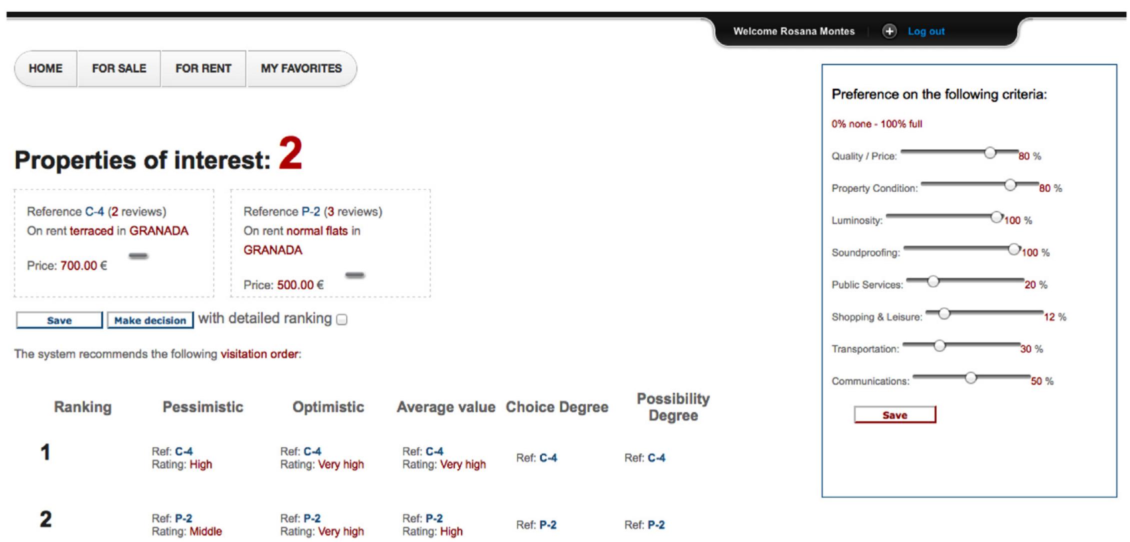

Fig. 5. When the user is logged in, the *My favourites* section includes the sliders used to assign preferences to criteria.

Table 3
Weights W in percentage given by clients U_1, U_2, and U_3. The normalised version W' will be used in the computations.

Client	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
U_1	80	80	100	100	20	12	30	50
U_2	100	85	75	75	80	75	65	50
U_3	81	100	53	81	0	100	71	0
	w'_1	w'_2	w'_3	w'_4	w'_5	w'_6	w'_7	w'_8
U_1	0.1694	0.1694	0.2118	0.2118	0.0423	0.0254	0.0635	0.1059
U_2	0.1652	0.1404	0.1239	0.1239	0.1322	0.1239	0.1074	0.0826
U_3	0.1666	0.2057	0.1090	0.1666	0	0.2057	0.1460	0

The internal linguistic description, which is private, can only be consulted and modified by realtors under identified access in an administrative area. Fig. 6 shows the subarea of the new record form where realtors are allowed to use single linguistic terms or hesitant expressions.

In this case of use, four different realtors has assessed the candidates. We give in Table 4 the original review of one realtor and in Table 5 the complete internal linguistic information retrieved from the database after the unification phase.

After a request for decision making, the portal shows a table that ranks the candidates according five methods of comparison:

Table 4
The realtor E_2 reviews candidate A_1 with hesitant expressions or single terms.

Criteria	Assessment
C_1	At least s_4
C_2	Between s_4 and s_5
C_3	At least s_5
C_4	Between s_3 and s_4
C_5	s_3
C_6	Between s_2 and s_4
C_7	Between s_3 and s_4
C_8	Between s_4 and s_5

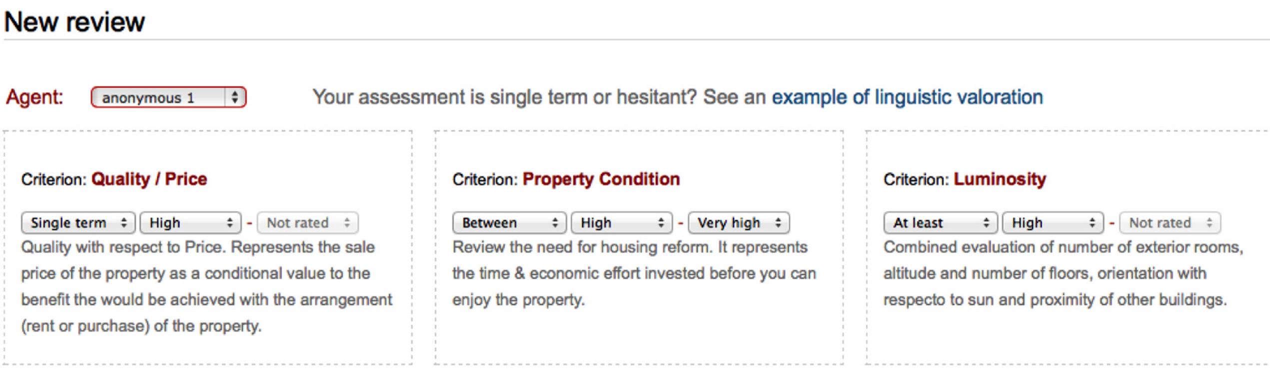

Fig. 6. Area of a web page used to evaluate the given criteria.

Table 5
Internal assessments are stored as linguistic intervals.

Candidate	Expert	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	E_1	[4, 4]	[5, 5]	[6, 6]	[2, 2]	[3, 3]	[2, 3]	[3, 3]	[4, 4]
A_1	E_2	[4, 6]	[4, 5]	[5, 6]	[3, 4]	[3, 3]	[2, 4]	[3, 4]	[4, 5]
A_1	E_4	[4, 4]	[5, 5]	[5, 6]	[3, 3]	[3, 4]	[3, 3]	[2, 3]	[4, 5]
A_1	E_3	[4, 4]	[4, 4]	[3, 4]	[4, 4]	[3, 5]	[3, 4]	[3, 3]	[4, 4]
A_2	E_3	[4, 5]	[6, 6]	[3, 5]	[3, 4]	[3, 4]	[5, 5]	[4, 4]	[5, 5]
A_2	E_1	[3, 3]	[5, 5]	[5, 5]	[3, 4]	[4, 4]	[5, 5]	[5, 5]	[5, 5]
A_2	E_2	[4, 5]	[5, 6]	[3, 5]	[3, 5]	[4, 5]	[5, 6]	[4, 5]	[5, 6]
A_3	E_3	[4, 4]	[3, 4]	[6, 6]	[3, 4]	[3, 3]	[2, 3]	[1, 2]	[4, 4]
A_3	E_1	[3, 5]	[5, 5]	[4, 5]	[3, 3]	[1, 1]	[1, 1]	[1, 2]	[4, 4]
A_3	E_2	[4, 5]	[4, 5]	[4, 5]	[3, 5]	[2, 3]	[2, 3]	[2, 3]	[4, 5]

Table 6
The portal suggest a visitation order adaptive to the customer preferences, and according to different ranking methods.

Client	Ranking	Pessimistic	Optimistic	Average value	Choice degree	Possibility degree
U_1	1	A_2 High (4, 0.13)	A_2 Very high (5, −0.25)	A_2 High (4, 0.44)	A_2 1	A_2 0.694
	2	A_3 High (4, −0.33)	A_1 High (4, 0.46)	A_1 High (4, −0.05)	A_1 0.405	A_3 0.5
	3	A_1 Middle (3, 0.44)	A_3 High (4, 0.16)	A_3 High (4, −0.08)	A_3 0.053	A_1 0.305
U_2	1	A_2 High (4, 0.18)	A_2 Very high (5, −0.28)	A_2 High (4, 0.45)	A_2 1	A_2 0.722
	2	A_1 Middle (3, 0.39)	A_1 High (4, 0.02)	A_1 High (4, −0.29)	A_1 0	A_1 0.388
	3	A_3 Middle (3, 0.02)	A_3 High (4, −0.33)	A_3 Middle (3, 0.34)	A_3 0	A_3 0.388
U_3	1	A_2 High (4, 0.19)	A_2 Very high (5, −0.13)	A_2 Very high (5, −0.47)	A_2 1	A_2 0.722
	2	A_1 Middle (3, 0.31)	A_1 High (4, 0.07)	A_1 High (4, −0.31)	A_3 0	A_3 0.388
	3	A_3 Middle (3, 0.03)	A_3 High (4, −0.45)	A_3 Middle (3, 0.29)	A_1 0	A_1 0.388

pessimistic case, optimistic case, average value, choice degree and possibility degree. Detailed output for this case use is given in Table 6 as labels followed by internal values (2-tuples or a degree value between 0 and 1, depending on the method of comparison). The visiting order suggested vary between web clients:

- Client U_1 is suggested with $A_2 > A_3 > A_1$ with the pessimistic and possibility degree methods, and $A_2 > A_1 > A_3$ with the optimistic, average and choice degree methods. Note that because criterion c_2, c_3 and c_8, candidate A_2 is evaluated the best. This situation match with the preferences of U_1 for these criteria.
- Client U_2 is suggested with $A_2 > A_1 > A_3$ with every method. In this case, criterion c_1 is key for the client, and candidate A_2 is better evaluated with respect to A_1, and A_1 with respect to A_3. This situation is similar if we check the rest of criteria.
- Client U_3 is suggested with $A_2 > A_1 > A_3$ in the three method which ranks 2-tuples. $A_2 > A_3 > A_1$ it is suggested on the choice and possibility degrees methods. This client cancelled with his/her weights criterion c_5 and c_8 in benefit of criterion c_6 which confirms the selection of candidate A_2 as the best choice, the other two candidates are rated quite similarly.

In summary the candidate A_2 is arguably the best in all ranking methods and for every client. In the second and third positions A_3 and A_1 are rated similarly, but according the client's preferences. The perspective of the ranking method (e.g. optimistic vs. pessimistic) is also affecting the final output.

5. Conclusions

In this paper we have presented a web tool to support decision making in the housing market. It is a real application of a multi-expert multi-criteria decision making problem. It benefits from the use of the fuzzy linguistic approach and the techniques available for Computing with Words. We apply the 2-tuple linguistic representation model to keep accuracy in the processes of Computing with Words and the hesitant fuzzy linguistic term sets to qualify for situations of uncertainty and hesitation in the assessments. The housing market valuations are inherently complex because they are under multiple influential attributes and opinions, but with the use of a context-free grammar that concede complex expressions these experts are granted to evaluate each house under hesitation.

The portal has a private area in which agents qualify each property according to the proposed fuzzy linguistic approach with 2-tuples and HFLTS. In our scheme, the first stage comes from the unification and translation of assessments to HFLTS. Then, information is internally managed as linguistic intervals so we apply the envelope of a HFLTS before translation to the 2-tuple linguistic representation. Aggregation comes as two rounds of computations with the application of the user's weights. In the exploitation phase, five different criteria are applied to rank the candidates. Finally, the Real Estate web clients select their alternatives list and their preferences about the characteristics considered (criteria under valuation). The portal is able to rank a candidate list according to the client's preferences and the internal knowledge associated to the properties. Thus, this helps to the web clients in the decision of visiting a property.

We are committed to further improve the online tool on the base of using hesitant context free grammar expressions. Following the line of extensions of the 2-tuple linguistic computational model, we foreseen the application of unbalanced linguistic linguistic term sets. In our context, experts of the housing market could be specialised in a particular city zone. Assessments could be more precise in the zone where he and she develops more intensely their work, and thus the possibility of choice between a larger or an smaller term set is more convenient. The use of a linguistic hierarchy and hence to deal with unbalanced linguistic information, is one of our future lines of work, as well as the inclusion of weights over experts, to consider some opinions over others depending on the zone that is been evaluated. Another improvement to the online tool is in the line to obtain agreement among the decision makers. We know that consensus reaching is a transverse topic in the decision making field, and for this particular application, the customers of a Real Estate agency are the ones that could obtain a benefit. Think in a standard family of four members with two teenage offspring, much in the mood of make their opinion count. The web site should extends the user profile to include not only a single preference set over criteria but a family profile set of preferences, weights over the family roles and the possibility of counting with hesitant information to represent the uncertainty, and to adjust suggestions which help decision makers reach the desired level of consensus [5].

This application is available at the URL http://sci2s.ugr.es/ficticia for demonstrative purposes with the user *demo* / *demo* and agent *expert* / *expert*.

References

[1] S. Alonso, I.J. Pérez, F.J. Cabrerizo, E. Herrera-Viedma, A linguistic consensus model for Web 2.0 communities, Appl. Soft Comput. 13 (1) (2013) 149–157.

[2] I. Beg, T. Rashid, TOPSIS for hesitant fuzzy linguistic term sets, Int. J. Intell. Syst. 28 (12) (2013) 1162–1171.

[3] Y. Dong, Y. Xu, S. Yu, Computing the numerical scale of the linguistic term set for the 2-tuple fuzzy linguistic representation model, IEEE Trans. Fuzzy Syst. 17 (6) (2009) 1366–1378.

[4] Y. Dong, E. Herrera-Viedma, Consistency-driven automatic methodology to set interval numerical scales of 2-tuple linguistic term sets and its use in the linguistic GDM with preference relation, IEEE Trans. Cybern. (99) (2014) (in press), http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=6862017&url=http%3A%2F%2Fieeexplore.ieee.org%2Fxpls%2Fabs_all.jsp%3Farnumber%3D6862017

[5] Y. Dong, X. Chen, F. Herrera, Minimizing adjusted simple terms in the consensus reaching process with hesitant linguistic assessments in group decision making, Inf. Sci. 297 (10) (2015) 95–117.

[6] F. Herrera, S. Alonso, F. Chiclana, E. Herrera-Viedma, Computing with words in decision making: foundations, trends and prospects, Fuzzy Optim. Decis. Mak. 8 (4) (2009) 337–364.

[7] F.F. Herrera, E. Herrera-Viedma, Linguistic decision analysis: steps for solving decision problems under linguistic information, Fuzzy Sets Syst. 115 (2000) 67–82.

[8] F. Herrera, L. Martínez, A 2-tuple fuzzy linguistic representation model for computing with words, IEEE Trans. Fuzzy Syst. 8 (6) (2000) 746–752.

[9] F. Herrera, L. Martínez, The 2-tuple linguistic computational model. Advantages of its linguistic description, accuracy and consistency, Int. J. Uncertain. Fuzziness Knowl.-Based Syst. 9 (2001) 33–48.

[10] F. Herrera, L. Martínez, A model based on linguistic 2-tuples for dealing with multigranular hierarchical linguistic context in multi-expert decision making, IEEE Trans. Syst. Man Cybern. - Part B: Cybern. 31 (2) (2001) 227–234.

[11] F. Herrera, E. Herrera-Viedma, L. Martínez, A fuzzy linguistic methodology to deal with unbalanced linguistic term sets, IEEE Trans. Fuzzy Syst. 16 (2) (2008) 354–370.

[12] H. Liao, Z. Xu, X.J. Zeng, Distance and similarity measures for hesitant fuzzy linguistic term sets and their application in multi-criteria decision making, Inf. Sci. 271 (2014) 125–142.

[13] H. Liu, R.M. Rodríguez, A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making, Inf. Sci. 258 (2014) 220–238.

[14] H. Liu, J. Cai, L. Jiang, On improving the additive consistency of the fuzzy preference relations based on comparative linguistic expressions, Int. J. Intell. Syst. 29 (2014) 544–559.

[15] X. Liu, Y. Ju, S. Yang, Hesitant intuitionistic fuzzy linguistic aggregation operators and their applications to multiple attribute decision making, J. Intell. Fuzzy Syst. 27 (3) (2014) 1187–1201.

[16] L. Martínez, D. Ruan, F. Herrera, Computing with words in decision support systems: an overview on models and applications, Int. J. Comput. Intell. Syst. 3 (4) (2010) 337–364.

[17] F. Meng, X. Chen, Q. Zhang, Multi-attribute decision analysis under a linguistic hesitant fuzzy environment, Inf. Sci. 267 (2014) 287–305.

[18] R.M. Rodríguez, L. Martínez, F. Herrera, Hesitant fuzzy linguistic term sets for decision making, IEEE Trans. Fuzzy Syst. 10 (1) (2012) 109–118.

[19] R.M. Rodríguez, L. Martínez, F. Herrera, A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets, Inf. Sci. 241 (2013) 28–42.

[20] R.M. Rodríguez, L. Martínez, V. Torra, Z.S. Xu, F. Herrera, Hesitant fuzzy sets: state of the art and future directions, Int. J. Intell. Syst. 29 (2014) 495–524.

[21] R.M. Tong, P.P. Bonissone, A linguistic approach to decision making with fuzzy sets, IEEE Trans. Syst. Man Cybern. SMC 10 (11) (1980) 716–723.

[22] V. Torra, Hesitant fuzzy sets, Int. J. Intell. Syst. 25 (6) (2010) 529–539.

[23] I. Truck, M. Abchir, Toward a classification of hesitant operators in the 2-tuple linguistic model, Int. J. Intell. Syst. 29 (2014) 560–578.

[24] J.H. Wang, J. Hao, A new version of 2-tuple fuzzy linguistic representation model for computing with words, IEEE Trans. Fuzzy Syst. 14 (3) (2006) 435–445.

[25] H. Wang, Extended hesitant fuzzy linguistic term sets and their aggregation in group decision making, Int. J. Comput. Intell. Syst. (2014) 1–20.

[26] C. Wei, N. Zhao, X. Tang, Operators and comparisons of hesitant fuzzy linguistic term sets, IEEE Trans. Fuzzy Syst. 22 (3) (2014) 575–585.

[27] R.R. Yager, On the retranslation process in Zadeh's paradigm of computing with words, IEEE Trans. Syst. Man Cybern. - Part B: Cybern. 34 (2) (2004) 1184–1195.

[28] L.A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes., IEEE Trans. Syst. Man Cybern. SMC-3 (1) (1973) 28–44.

[29] L.A. Zadeh, The concept of a linguistic variable and its applications to approximate reasoning, Inf. Sci. Part I 8 (3) (1975) 199–249.

[30] L.A. Zadeh, Fuzzy logic = computing with words, IEEE Trans. Fuzzy Syst. 94 (2) (1996) 103–111.

[31] Z. Zhang, C. Wu, Hesitant fuzzy linguistic aggregation operators and their applications to multiple attribute group decision making, J. Intell. Fuzzy Syst. 26 (2014) 2185–2202.

[32] B. Zhu, Z. Xu, Consistency measures for hesitant fuzzy linguistic preference relations, IEEE Trans. Fuzzy Syst. 2 (1) (2014) 35–45.