Strategies to Manage Ignorance Situations in Multiperson Decision Making Problems

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Abstract. Multiperson decision making problems involve using the preferences of some experts about a set of alternatives in order to find the best of those alternatives. However, sometimes experts cannot give all the information that they are required. Particularly, when dealing with fuzzy preference relations they can avoid giving some of the preference values of the relation. In the literature these incomplete information situations have been faced giving procedures which are able to compute missing information from the preference relations. However, these approaches usually need at least a piece of information about every alternative in the problem. In this paper, several strategies to manage *total ignorance* situations, that is, situations where an expert does not provide *any* information on at least one alternative are presented, and their advantages and disadvantages analised.

Keywords: Ignorance, Incomplete Information, Consistency, Multiperson Decision Making, Fuzzy Preference Relations.

1 Introduction

Multiperson decision-making (MPDM) consists of multiple individuals (usually experts) $E = \{e_1, ..., e_m\}$ interacting to reach a decision. Each expert may have unique motivations or goals and may approach the decision process from a different angle, but have a common interest in reaching eventual agreement on selecting the *best* solution(s) to the problem to be solved [4, 12]. Fuzzy preference relations are commonly used to represent decision makers' preferences over the set of possible alternative solutions $X = \{x_1, ..., x_n\}$, $(n \geq 2)$ [2, 5, 6, 7, 16, 17].

In many cases, some experts may not have a "perfect" knowledge of the problem to be solved [3, 9, 10, 11, 18]. For example, an expert might not possess a precise or sufficient level of knowledge of part of the problem or might be unable to discriminate the degree to which some options are better than others. In such cases, an expert would not be able to efficiently express any kind of preference degree between two or more of the available options, an therefore the fuzzy preference relation provided is *incomplete* [3, 18]. Therefore, it would be

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of great importance to provide these experts with appropriate tools that allow them to overcome this lack of knowledge in their opinions.

Two different kinds of incomplete information in a MPDM can be identified:

- *Partial incomplete information.* In this case at least one expert does not provide all possible preference degrees over the set of alternatives, but provides information on his/her preferences in which every alternative is at least compared once against one of the rest of alternatives.
- *Total incomplete information.* In this case at least one expert does not provide all possible preference degrees over the set of alternatives, and provides information on his/her preferences in which at least one alternative is not compared against any one of the rest of alternatives. We call this an *ignorance situation.*

Some attention has been paid to the case of partial incomplete information [1, 3, 18]. However, as far as we know, no study has been yet published on MPDM problem with total incomplete information. This paper presents several possible strategies to manage *ignorance* situations in MPDM problems: *ad-hoc strategies* and *consistency guided strategies*. We analyse both their advantages and disadvantages and illustrate their application by examples. To model the consistency property we use the additive transitivity property proposed by Tanino in [16].

The rest of the paper is set out as follows. Section 2 presents notation and concepts needed throughout the papers. In section 3 we present a general consistency based procedure to estimate unknown preferences values in an incomplete fuzzy preference relation. Section 4 presents several strategies to manage ignorance situations in MPDM problems. Advantages and disadvantages associated to each one of these strategies are discussed in section 5. Finally, our concluding remarks will be pointed out in Section 6.

2 Preliminaries

Fuzzy preference relations are commonly used to represent decision makers' preferences over the set of possible alternative solutions $X = \{x_1, ..., x_n\}$, $(n \geq 2)$ [2, 5, 6, 7, 8, 13, 16, 17].

Definition 1. A Fuzzy Preference Relation (FPR) P on a set of alternatives X is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P \colon X \times X \longrightarrow [0, 1]$.

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ik})$, being $p_{ik} = \mu_P(x_i, x_k)$ $(\forall i, k \in \{1, \ldots, n\})$ interpreted as the preference degree or intensity of the alternative x_i over x_k: $p_{ik} = 1/2$ indicates indifference between x_i and x_k $(x_i \sim x_k)$, $p_{ik} = 1$ indicates that x_i is absolutely preferred to x_k, and $p_{ik} > 1/2$ indicates that x_i is preferred to x_k $(x_i \succ x_k)$. Based on this interpretation we have that $p_{ii} = 1/2 \ \ \forall i \in \{1, \ldots, n\}$ $(x_i \sim x_i)$.

Since each expert is characterized by his/her own personal background and experience of the problem to be solved, experts' opinions may differ substantially (there are plenty of educational and cultural factors that influence an expert's preferences). This diversity of experts could lead to situations where some of them would not be able to efficiently express any kind of preference degree between two or more of the available options. Indeed, this may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, or because that expert is unable to discriminate the degree to which some options are better than others. We must note that an expert which is not able to provide a particular preference value p_{ik} does not necessarily imply that he/she is indifferent between both x_i and x_k alternatives, that is, we cannot directly suppose that $p_{ik} = 0.5$.

2.1 Incomplete Fuzzy Preference Relations

Usually, we assume that experts are always able to provide all the preferences required, that is, to provide all p_{ik} values. However, this may not always be the case, and experts end providing an *incomplete fuzzy preference relations* [1, 18]. In the following definitions we express the concept of an incomplete fuzzy preference relation:

Definition 2. A function $f\colon X \longrightarrow Y$ is *partial* when not every element in the set X necessarily maps onto an element in the set Y. When every element from the set X maps onto one element of the set Y then we have a *total* function.

Definition 3. [1] An *Incomplete Fuzzy Preference Relation* P on a set of alternatives X is a fuzzy set on the product set $X \times X$ that is characterized by a *partial* membership function.

When a particular preference value p_{ik} is not given by an expert we will note $p_{ik} = x$ and we will call it a *missing value*.

From a particular incomplete fuzzy preference relation P_h we define the following sets [1]:

$$A = \{(i,j) \mid i,j \in \{1,\ldots,n\} \wedge i \neq j\}$$

$$MV_h = \left\{(i,j) \in A \mid p_{ij}^h = x\right\}$$

$$EV_h = A \setminus MV_h$$

$$EV_h^i = \{(a,b) \mid (a,b) \in EV_h \ \wedge \ (a = i \vee b = i)\}$$

where MV_h is the set of pairs of alternatives for which the preference degree of the first alternative over the second one is not given by expert e_h, that is, the set of *missing values* of the expert e_h, EV_h is the set of pairs of alternatives for which the expert e_h provides preference values (we call it the *expert values* for e_h) and EV_h^i is the set of preferences about pairs of alternatives given by an expert e_h involving alternative x_i.

2.2 Consistency Property

The definition of a preference relation does not imply any kind of consistency property. In fact, the values of a preference relation may be contradictory. Consistency is usually characterised by *transitivity*, which represents the idea that the preference value obtained by directly comparing two alternatives should be equal to or greater than the preference value between these two alternatives obtained using an indirect chain of alternatives.

One of the properties suggested to model the concept of transitivity in the case of fuzzy preference relations is the *additive transitivity* property [16]:

$$(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5) \quad \forall i, j, k \in \{1, \ldots, n\}$$

or equivalently:

$$p_{ik} = p_{ij} + p_{jk} - 0.5 \quad \forall i, j, k \in \{1, \ldots, n\} \tag{1}$$

As shown in [6], additive transitivity for fuzzy preference relations can be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations [14].

This kind of transitivity has the following interpretation: suppose we want to establish a ranking between three alternatives x_i, x_j and x_k, and that the information available about these alternatives suggests that we are in an indifference situation, i.e. $x_i \sim x_j \sim x_k$. When giving preferences this situation would be represented by $p_{ij} = p_{jk} = p_{ik} = 0.5$. Suppose now that we have a piece of information that says $x_i \prec x_j$, i.e. $p_{ij} < 0.5$. This means that p_{jk} or p_{ik} have to change, otherwise there would be a contradiction, because we would have $x_i \prec x_j \sim x_k \sim x_i$. If we suppose that $p_{jk} = 0.5$ then we have the situation: x_j is preferred to x_i and there is no difference in preferring x_j to x_k. We must then conclude that x_k has to be preferred to x_i. Furthermore, as $x_j \sim x_k$ then $p_{ij} = p_{ik}$, and so $(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ij} - 0.5) = (p_{ik} - 0.5)$. We have the same conclusion if $p_{ik} = 0.5$. In the case of $p_{jk} < 0.5$, then we have that x_k is preferred to x_j and this to x_i, so x_k should be preferred to x_i. On the other hand, the value p_{ik} has to be equal to or lower than p_{ij}, being equal only in the case of $p_{jk} = 0.5$ as we have already shown. Interpreting the value $p_{ji} - 0.5$ as the intensity of preference of alternative x_j over x_i, then it seems reasonable to suppose that the intensity of preference of x_i over x_k should be equal to the sum of the intensities of preferences when using an intermediate alternative x_j, that is, $p_{ik} - 0.5 = (p_{ij} - 0.5) + (p_{jk} - 0.5)$. The same reasoning can be applied in the case of $p_{jk} > 0.5$.

3 Consistency Based Procedure to Estimate Missing Values in Incomplete Fuzzy Preference Relations

Given a complete fuzzy preference relation, expression 1 can be used to calculate an estimated value cp_{ik} for every p_{ik} as follows:

$$cp_{ik} = \frac{\sum_{j=1; i \neq k \neq j}^{n} cp_{ik}^{j1} + cp_{ik}^{j2} + cp_{ik}^{j3}}{3(n-2)} \tag{2}$$

where cp_{ik}^{j1}, cp_{ik}^{j2}, cp_{ik}^{j3} are directly obtained from expression 1, and the fact that additive transitivity implies reciprocity ($p_{ik} = 1 - p_{ki} \quad \forall i, k$):

$$cp_{ik}^{j1} = p_{ij} + p_{jk} - 0.5, \tag{3}$$

$$cp_{ik}^{j2} = p_{jk} - p_{ji} + 0.5, \quad (4)$$

$$cp_{ik}^{j3} = p_{ij} - p_{kj} + 0.5 \quad (5)$$

When working with an incomplete fuzzy preference relation, the previous expressions cannot be directly applied, as some of the preference values used in the expressions may be unknown. However, an iterative procedure to estimate these unknown or missing values can be derived from the above expressions. The following two different tasks have to be carried out:

A) Establish the elements that can be estimated in each step of the procedure, and
B) produce the particular expression that will be used to estimate a particular missing value.

A) Elements to be estimated in step h. The subset of missing values MV that can be estimated in step h of our procedure is denoted by EMV_h (*estimated missing values*) and defined as follows:

$$EMV_h = \left\{ (i,k) \in MV \setminus \bigcup_{l=0}^{h-1} EMV_l \mid i \neq k \ \wedge \ \exists j \in \{H_{ik}^1 \cup H_{ik}^2 \cup H_{ik}^3\} \right\}$$

with

$$H_{ik}^1 = \left\{ j \mid (i,j),(j,k) \in \{EV \bigcup_{l=0}^{h-1} EMV_l\} \right\}$$

$$H_{ik}^2 = \left\{ j \mid (j,i),(j,k) \in \{EV \bigcup_{l=0}^{h-1} EMV_l\} \right\}$$

$$H_{ik}^3 = \left\{ j \mid (i,j),(k,j) \in \{EV \bigcup_{l=0}^{h-1} EMV_l\} \right\}$$

and $EMV_0 = \emptyset$ (by definition). When $EMV_{maxIter} = \emptyset$ with $maxIter > 0$ the procedure will stop as there will not be any more missing values to be estimated. Moreover, if $\bigcup_{l=0}^{maxIter} EMV_l = MV$ then all missing values are estimated, and consequently, the procedure is said to be successful in the completion of the incomplete fuzzy preference relation.

B) Expression to estimate a particular value p_{ik} in step h. In order to estimate a particular value p_{ik} with $(i,k) \in EMV_h$, we propose the application of the following function:

function estimate_p(i,k)

1. $cp^1_{ik} = 0,\ cp^2_{ik} = 0,\ cp^3_{ik} = 0,\ \mathcal{K} = 0$
2. $if\ \#H^1_{ik} \neq 0 \ \Rightarrow\ cp^1_{ik} = \frac{\sum_{j \in H^1_{ik}} cp^{j1}_{ik}}{\#H^1_{ik}}\ ;\ \mathcal{K}++.$
3. $if\ \#H^2_{ik} \neq 0 \ \Rightarrow\ cp^2_{ik} = \frac{\sum_{j \in H^2_{ik}} cp^{j2}_{ik}}{\#H^2_{ik}}\ ;\ \mathcal{K}++.$
4. $if\ \#H^3_{ik} \neq 0 \ \Rightarrow\ cp^3_{ik} = \frac{\sum_{j \in H^3_{ik}} cp^{j3}_{ik}}{\#H^3_{ik}}\ ;\ \mathcal{K}++.$
5. Calculate $cp_{ik} = \frac{1}{\mathcal{K}} \left(cp^1_{ik} + cp^2_{ik} + cp^3_{ik} \right)$

end function

The function $estimate\_p(i,k)$ computes the final estimated value of the missing value, cp_{ik}, as the average of all estimated values that can be calculated using all the possible intermediate alternatives x_j and using the three possible expressions (3–5).

Then, the *iterative estimation procedure pseudo-code* is as follows:

```
   ITERATIVE ESTIMATION PROCEDURE
0. EMV_0 = ∅
1. h = 1
2. while EMV_h ≠ ∅ {
3.     for every (i,k) ∈ EMV_h {
4.         estimate_p(i,k)
5.     }
6.     h++
7. }
```

This procedure is able to estimate all the missing values for a given incomplete fuzzy preference relation if a set of $n-1$ non-leading diagonal preference values where each one of the alternatives is compared at least once is known [1]. That means that partial incomplete MPDM problems can be successfully solved using this procedure. However, the only application of this procedure does not solve MPDM problem with total incomplete information. The rest of the paper is devoted to the study of some possible strategies to tackle these situations.

4 Strategies to Manage Ignorance Situations in Decision Making Problems

As per the notation introduced in section 2, an ignorance situation in MPDM problems is defined as follows:

Definition 4. In a MPDM problem with a set of alternatives $X = \{x_1, ..., x_n\}$ and a group of experts $E = \{e_1, ..., e_m\}$ which provide a set of incomplete fuzzy preference relations $\{P_1, ..., P_m\}$, we have a *ignorance situation* if

$$\exists \ (h, i) \mid EV_h^i = \emptyset,$$

that is, at least one of the experts (e_h) does not provide any preference value involving a particular alternative (x_i). We will call x_i the "unknown alternative" for the expert e_h.

4.1 Ad-Hoc Strategies to Manage Ignorance Situations

These strategies estimate missing values in ignorance situations by ad-hoc procedures which are not based in any particular basic principle or property associated to the set of alternatives, experts or relations. Two simple examples of this kind of strategies are the following:

Strategy 1: Assume Indifference Values in the Missing Values

In this case, because an expert does not provide information on an alternative relating it to the rest of alternatives, we may model this situation as a total indifference one and therefore each missing values for the ignored alternative can be replace with a value of 0.5. In this case, the estimation procedure of missing values is as follows:

Estimation Procedure 1: If an incomplete fuzzy preference relation P_h has an ignored alternative x_i, this strategy will compute all its associated missing value as:

$$p_{ik}^h = 0.5 \ ; \ p_{ki}^h = 0.5 \ \ \forall k \in \{1, ..., n\}, k \neq i.$$

Example 1: We have to solve a decision making problem to find the best of 4 different alternatives: $X = \{x_1, x_2, x_3, x_4\}$. An expert gives the following incomplete fuzzy preference relation

$$P = \begin{pmatrix} - & 0.7 & x & 0.68 \\ 0.4 & - & x & 0.7 \\ x & x & - & x \\ 0.6 & 0.75 & x & - \end{pmatrix},$$

that is, he gives no information about alternative x_3, and thus, we are in a *ignorance situation*. The first estimation procedure assumes that the expert is indifferent with respect to x_3, and the reconstructed fuzzy preference relation is:

$$P = \begin{pmatrix} - & 0.7 & 0.5 & 0.68 \\ 0.4 & - & 0.5 & 0.7 \\ 0.5 & 0.5 & - & 0.5 \\ 0.6 & 0.75 & 0.5 & - \end{pmatrix}.$$

Strategy 2: Assume Random Values in the Missing Values

This strategy estimates the missing values for an ignored alternative as random values within the range of preference values provided by that particular experts, i.e, an unknown preference value will be computed randomly between the maximum and minimum preference degrees of its corresponding column and row. In this case, the estimation procedure of missing values is as follows:

Estimation Procedure 2: If an incomplete fuzzy preference relation P_h has an ignored alternative x_i, this strategy will compute every missing value as:

$$p^h_{ik} = rand(min(\{p^h_{jk}\}), max(\{p^h_{jk}\}))\ ;\ p^h_{ki} = rand(min(\{p^h_{kj}\}), max(\{p^h_{kj}\}))$$
$$\forall j, k \in \{1, ..., n\}, j \neq k \neq i$$

where $rand(a, b)$ means a random value between a and b and $max(...)$ and $min(...)$ are the usual maximum and minimum operators.

Example 2: We part from the previously presented problem (in example 1). In this case, the estimation procedure reconstructs the missing values with random values between the maximum and minimum preference degrees provided by the expert. For example, $p_{13} \in [0.68, 0.7]$ and $p_{32} \in [0.7, 0.75]$. An example of a possible reconstructed preference relation is:

$$P = \begin{pmatrix} - & 0.7 & 0.69 & 0.68 \\ 0.4 & - & 0.47 & 0.7 \\ 0.53 & 0.71 & - & 0.7 \\ 0.6 & 0.75 & 0.72 & - \end{pmatrix}.$$

4.2 Consistency Based Strategies

These strategies are guided by a basic principle, the consistency property of the incomplete fuzzy preference relations represented by the additive transitivity property. To do so, these strategies use the estimation procedure presented in section 3.

As aforementioned, that procedure needs at least a preference value involving the ignored alternative to be able to estimate the rest of missing preference values of the ignored alternative. Therefore, we need a 'seed' value to initiate the estimation procedure. Depending on the computation of that seed value we can define the following two consistency based strategies:

Strategy 3: Consistency Based Strategies with Indifference Seed Values

Similarly, as in the first strategy, we can start by assuming indifference on the preference values for the ignored alternative, followed by the application of the estimation procedure to complete the rest of missing values of the alternative. Thus, in this case the estimation procedure of missing values is as follows:

Estimation Procedure 3: Suppose an incomplete fuzzy preference relation P with an ignored alternative x_i, and assume $p_{ij} = 0.5$ for a particular $j \in \{1, \ldots, n\}$

(initial indifference). The preference degrees $\{p_{ik}\}, \forall k \neq i \neq j$ can be estimated via the alternative x_j by means of two of the three possible estimation equations (3–5): $cp_{ik}^{j1} = p_{ij} + p_{jk} - 0.5$ and $p_{ij} = cp_{ik}^{j3} + p_{kj} - 0.5$, which result in $cp_{ik}^{j1} = p_{jk}$ and $cp_{ik}^{j3} = 1 - p_{kj}$, respectively. Because the indifference of a preference value can be assumed for any of the possible values of $j \in \{1, \ldots, n\}$ with $j \neq i \neq k$, then the final estimated values for the i-th row of the incomplete fuzzy preference relation are:

$$\begin{aligned} cp_{ik} &= \frac{1}{2}\left(\frac{\sum_{j=1;j\neq i\neq k}^{n} cp_{ik}^{j1}}{n-2} + \frac{\sum_{j=1;j\neq i\neq k}^{n} cp_{ik}^{j3}}{n-2}\right) \\ &= \frac{1}{2}\left(\frac{\sum_{j=1\ ;\ j\neq i\neq k}^{n} p_{jk}}{n-2} + \frac{\sum_{j=1\ ;\ j\neq i\neq k}^{n} (1-p_{kj})}{n-2}\right) \\ &= 0.5 + \frac{SC_k - SR_k}{2} \end{aligned}$$

with SC_k and SR_k representing the average of the k-th column and k-th row of the complete $(n-1) \times (n-1)$ fuzzy preference relation that is obtained without taking into account the alternative x_i. The parallel application of the above assumption for the preference values p_{ki} provides the following estimation of the values of the i-th column:

$$cp_{ki} = 0.5 + \frac{SR_k - SC_k}{2}. \tag{6}$$

Example 3: If we apply this strategy to the previously mentioned problem (examples 1 and 2), we obtain the following values for p_{13} and p_{32}:

$$p_{13} = 0.5 + \frac{(0.7 + 0.68)/2 - (0.4 + 0.6)/2}{2} = 0.6$$

and

$$p_{32} = 0.5 + \frac{(0.7 + 0.75)/2 - (0.4 + 0.7)/2}{2} = 0.59$$

In this case, the complete reconstructed preference relation is:

$$P = \begin{pmatrix} - & 0.7 & 0.6 & 0.68 \\ 0.4 & - & 0.41 & 0.7 \\ 0.4 & 0.59 & - & 0.51 \\ 0.6 & 0.75 & 0.49 & - \end{pmatrix}.$$

Strategy 4: Consistency Based Strategies with Random Seed Values

This strategy, similarly as in the second strategy, is based on obtaining just one 'seed' random value followed by the application of the procedure to estimate the rest of missing values for the ignored alternative. Thus, in this case the estimation procedure of missing values is as follows:

Estimation procedure 4: Suppose an incomplete fuzzy preference relation P_h with an ignored alternative x_i. The estimation procedure is drawn in the following scheme:

1. *do* {
2. $\quad k = irand(1, n)$ // Choose random k
3. } $while(k \neq i)$
4. $if\ (rand(0,1) < 0.5)$ { // Place it in missing row
5. $\quad p_{ik}^{h} = rand(min(\{p_{jk}^{h}\}), max(\{p_{jk}^{h}\}))$
   $\quad \forall j \in \{1, ..., n\}, j \neq k \neq i$
6. } *else* { // Place it in missing column
7. $\quad p_{ki}^{h} = rand(min(\{p_{kj}^{h}\}), max(\{p_{kj}^{h}\}))$
   $\quad \forall j \in \{1, ..., n\}, j \neq k \neq i$
8. }
9. Apply the estimation procedure

where $irand(a, b)$ means an integer random value between a and b.

Example 4: From the problem presented in the previous examples, we are going to apply this strategy to reconstruct the missing values. First of all, we obtain a random $k \neq i$. For example $k = 2$. We obtain a random value between $[0, 1]$ to determine if we are going to calculate a seed value for p_{32} or p_{23}. Suppose that the random value is 0.34, so we are going to obtain a random value for $p_{32} \in [0.7, 0.75]$, for example, $p_{32} = 0.74$. Then, we apply the estimation procedure:

$$\begin{pmatrix} - & 0.7 & x & 0.68 \\ 0.4 & - & x & 0.7 \\ x & 0.74 & - & x \\ 0.6 & 0.75 & x & - \end{pmatrix} \rightarrow \begin{pmatrix} - & 0.7 & 0.46 & 0.68 \\ 0.4 & - & x & 0.7 \\ 0.59 & 0.74 & - & 0.61 \\ 0.6 & 0.75 & 0.51 & - \end{pmatrix} \rightarrow \begin{pmatrix} - & 0.7 & 0.46 & 0.68 \\ 0.4 & - & 0.42 & 0.7 \\ 0.59 & 0.74 & - & 0.61 \\ 0.6 & 0.75 & 0.51 & - \end{pmatrix}$$

5 Analysis of the Advantages and Disadvantages of Each Strategy

In this section we analyze some advantages and disadvantages of the proposed strategies, identifying situations where some of the strategies may be more adequate than the others.

- *Strategy 1* is a very simple approach to solve ignorance situations. Although it is not always adequate to assume that not giving preference values for one alternative implies indifference between the unknown alternative and the rest of them, in some situations could be an acceptable option. In fact, its easiness application can be a very appealing factor for its use, specially in problems where there are no other sources of information (neither information about the alternatives or other experts). Particularly, decision making problems with only one expert or criterion are good candidates to apply this strategy.
- *Strategy 2* is also a simple approach, but it can produce a higher level of diversity in the opinions given by the experts. However, it is important to remark that this strategy can produce a decrease in the consistency of the

fuzzy preference relations, because the random values will not usually comply with any kind of transitivity property. This strategy can be a good one to apply in decision problems with a high number of experts or criteria which do not differ too much between them (because it can introduce some diversity in the problem).

- *Strategy 3*: This strategy improves strategy 1, as it adjusts the estimated preference values to make the preference relation more consistent with the previously existing information. Moreover, the initial indifference supposed for every preference value for the unknown alternative is softened according to the existing information in the preference relation. This approach is interesting when there are no external sources of information about the problem and when a high consistency level is required in the experts' preference relations.
- *Strategy 4* tries to unify the advantages of strategies 2 and 3: it tries to maintain a high consistency degree in the fuzzy preference relations (with the application of the estimation procedure) whilst it gives a slightly higher level of diversity than strategy 3 (with the generation of the random seed for the estimation procedure).

6 Conclusions

In this paper we have presented several different strategies to solve ignorance situations in Decision Making problems. We have presented some ad-hoc strategies and some consistency guided strategies and have analysed their advantages and disadvantages.

In the future, we will study other possibilities to deal with ignorance situations using different criteria to the consistency one as it could be the use of consensus and/or proximity measures to provide a management system of ignorance situations.

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