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GRASP & Evolutionary Path Relinking for Medical Image Registration based on Point Matching

José Santamaría, Oscar Cordón, Sergio Damas, Rafael Martí, Ricardo J. Palma

***Abstract*— Image registration is a very active research area in computer vision. Image registration methods, aim to find a transformation between two images taken under different conditions. Point matching is an image registration approach based on searching for the right pairing of points between the two images. From this matching, the registration transformation can be inferred by means of numerical methods.**

In this paper, we tackle the medical image registration problem adapting a new advanced hybrid metaheuristic composed by the GRASP and the evolutionary path relinking algorithms, called G&EvPR. The experiments conducted in this work have shown the good performance of G&EvPR compared to similar approaches of the state of the art when dealing with different medical image modalities. In particular, a good trade-off between search space diversification and intensification is achieved.

I. Introduction

Image registration (IR) is a fundamental task in computer vision used to finding a correspondence (or transformation) among two or more images taken under different conditions: at different times, using different sensors, from different viewpoints, or a combination of them [4], [29]. Point matching is an IR approach based on searching for the right pairing of points between two images. From this matching, the registration transformation can be inferred by means of numerical methods. The main advantage of using this IR approach is that it does not require the estimation of the suitable interval ranges of every parameter defining the transformation. Thus, the proposal of outstanding point matching algorithms is important for the IR community.

Evolutionary Computation (EC) [2] uses computational models of evolutionary processes as key elements in the design and implementation of computer-aided problem solving systems. In the last few years, there is an increasing interest on applying evolutionary approaches to IR [6], [7], [8], [25], [26], [28]. In this contribution, we extend our previous work and exploit the benefits of applying the hybridizaton [22] of the *greedy randomized adaptive search procedure* (GRASP) [11] and the *evolutionary path relinking* algorithm [23], called G&EvPR. Our contribution is two-fold. On the one hand, we aim to balance the diversity and intensification components of this hybrid method. On the other hand, we aim to take advantage of heuristic information extracted from the images to guide the process. Such information corresponds to the curvature values of the object under study and it has proved to be useful to achieve high quality solutions [5], [7], [8]. Furthermore, curvature information facilitates a feature-based IR approach characterized by a significant reduction of input data which are represented by the most relevant points (according to this heuristic information) of the object, thus allowing a better matching and speeding up the IR process as well [6], [25]. The performance of the proposed G&EvPR-based IR algorithm is compared to our previous work considering six medical images from two different image datasets.

The structure of the contribution is as follows. In Section II we describe the IR problem and introduce the heuristic information derived from the image. Section III is devoted to briefly describe our previous work on the IR problem based on point matching. Next, we introduce the new hybridization of GRASP and Path relinking algorithms to tackle the IR problem in Section IV. Computational experiments are detailed in Section V. Finally, Section VI presents concluding remarks and new possible designs based on GRASP and path relinking algorithms for tackling point matching in future works.

II. Image registration

In this section the IR problem is formally described. Besides, we detail the heuristic information exploited to guide the search process towards the best solutions for the IR problem.

A. Problem formulation

IR is a difficult optimization problem. It can be stated as finding a mapping between two images: I_1 and I_2 (named scene and model, respectively). The objective is to determine the geometric transformation f that applied to I_1 leads it to I_2. Typically, an image is represented by a huge amount of pixels. Therefore, many IR methods apply a preprocessing step to extract the most relevant geometric primitives (point, lines, etc) in both images. In these *feature-based* approaches, the IR problem consists of finding the transformation between two sets of geometric primitives. In this contribution, we propose a feature-based method thus considering a set of primitives in both images, P_1 and P_2 ($P_1 \subseteq I_1$, $P_2 \subseteq I_2$). Such geometric primitives are points defining a crest line [21]. Crest lines are the locus of points on a surface whose longest curvature (in absolute value) is

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locally maximal in the associated principal direction (see Section II-B). A crest line can be viewed as a generalization of an edge for smooth surfaces in 3D.

Although the final solution of the registration problem consists of the right values for the parameters that determine f, two different approaches arise, each of them working in a different solution space: i) to search for the optimal point matching between the two images and then identify the appropriate transformation parameters —using numerical methods such as least square estimation— to superimpose the scene and the model considering such matching ([3], [5], [8], [19]); and ii) to search in the parameter space directly (usually by means of evolutionary algorithms) to get the estimated transformation ([15], [16], [25], [26], [28]).

Point-matching is the classical approach in feature-based registration. In mathematical terms, it can be described as follows. Given two set of points $P_1 = \{x_1, x_2, ..., x_n\}$ and $P_2 = \{y_1, y_2, ..., y_m\}$, the problem is to find a transformation f such that $y_i = f(x_{\pi(i)})$ for $i = 1, ..., r$ $(r = min(n, m))$, where π is a permutation of size l (with l being the maximum between n and m). Without loss of generality and to simplify the notation, we consider that P_1 is the larger point set, i.e., its dimension n is greater than that of P_2, m.

The problem solving is naturally divided in two phases. In the first one, a permutation of l elements defines the matching between the points in P_1 and P_2 in such a way that the first r elements ($r = m$ in our case) of π are the P_1 points associated to each of the m P_2 points. In the second phase, from this matching of points and using a numerical optimization method (usually least squares estimation), the parameters defining the transformation f are computed. The goal is to find the transformation minimizing the distances between the model points and the corresponding transformed scene points. Therefore, in optimization terms, the value associated with permutation π is given by the expression:

$$g(\pi) = \frac{\sum_{i=1}^{r} \|f_\pi(x_{\pi(i)}) - y_i\|^2}{r}, \qquad (1)$$

i.e., $g(\pi)$ corresponds to the *Mean Square Error (MSE)*. Therefore, the point matching problem can be simply stated as minimizing $g(\pi)$ for any permutation π of l elements and its corresponding transformation f. In this work, we face the IR problem from this point matching approach, adopting a hybrid GRASP & evolutionary path relinking method [22] to find high quality solutions to this combinatorial optimization problem.

B. Heuristic information derived from the object 3D shape

This section is devoted to describe the heuristic information that can be derived from the curvature of the shapes included in the images in order to better address the IR problem. Let us first define the iso-intensity surface of a 3D image, which will be called simply the iso-surface in the rest of this paper. For any continuous function $C(x, y, z)$ of $\mathbb{R}^3$, any value I of $\mathbb{R}$ (called the iso-value) defines a continuous, not self-intersecting surface, without hole, which is called the iso-intensity surface of C [21]. A non ambiguous way to define the iso-surface is to consider it as being the surface which separates the space regions where the intensity of C is greater or equal to I from these regions whose intensity is strictly lower than I. Whether such an iso-surface corresponds to the boundary of the scanned object is another problem, that will not be considered in the current contribution. Because of their good topological properties, iso-surface techniques are the most widely used methods of segmentation for 3D medical images.

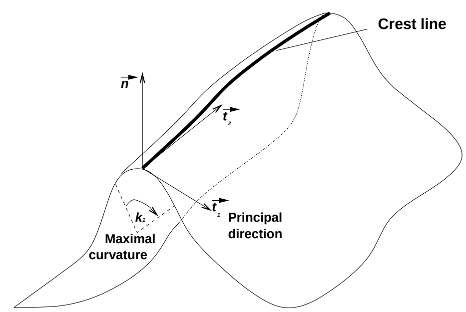

Fig. 1. Differential characteristics of surfaces

Let us see now some properties of the iso-surfaces (see Figure 1). At each point p of those surfaces, there is an infinite number of curvatures but, for each direction $\vec{t}$ in the tangent plane at p, there is only one associated curvature $k_{\vec{t}}$. There are two privileged directions of the surface, called the principal directions ($\vec{t_1}$ and $\vec{t_2}$), which correspond to the two extremal values of the curvature: k_1 and k_2. There are many more parameters to determine the characterization of surfaces, but we have considered the latter two (k_1 and k_2) being enough to be considered as heuristic information to help us the IR problem solving.

III. Previous work

In a previous work [8], we developed an IR method following a point matching approach based on the scatter search (SS) algorithm. SS was firstly introduced in [12] as a heuristic for integer programming, being laterly extended by Laguna and Martí [18]. The key idea of SS is the exploration of the solution space operating on a set of solutions, the reference set, by combining these solutions systematically to create new ones. When the main mechanism for combining solutions is such that a new solution is created from the linear combination of two other solutions, the reference set may evolve. The SS methodology basically consists of five elements (see Figure 2): Diversification Generation Method, Improvement Method, Solution Combination Method, Reference Set Update Method, and Subset Generation Method. The three former are problem dependent, and should be designed specifically for the problem at hand (although it is possible to design "generic" procedures, it is more effective to base the design on specific characteristics of the problem setting). The other two are context independent, and usually have a standard implementation.

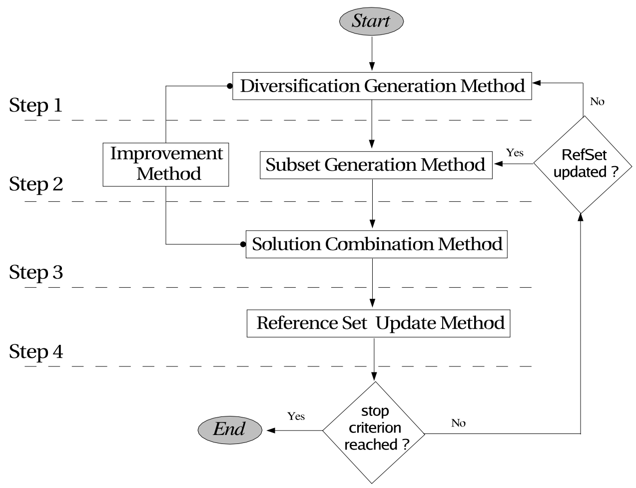

Fig. 2. The SS control diagram

The fact that the mechanisms within SS are not restricted to a single uniform design allowed us the exploration and design of strategic possibilities that demonstrated to be effective tackling the IR problem [8]. One of the main novelties of the SS-based IR method was that it exploited problem dependent information. Moreover, new designs for three of the five SS components – the generator of diverse solutions, the improvement and the combination methods – were proposed to develop a method outperforming the state-of-the-art point matching approaches.

Specifically, the heuristic values of the extracted features (i.e., the curvature of the crest-line points) were used to guide the matching. As said, the method exploited the information relative to local curvature characterizing the set of crest-line points (see Section II-B) extracted as geometric primitives of the scene and model images. Thus, the SS-based IR method extended its capabilities by using an advanced coding scheme introduced in [5] where a given point matching was represented as a permutation. In [8] a function $m_{error}(\cdot)$ was also defined in order to evaluate the quality of the matching stored in a given solution, π, by using the said curvature values:

$$m_{error}(\pi) = \Delta k_1 + \Delta k_2 \quad \text{where}$$
$$\Delta k_j = \sum_{i=1}^{r} (k_j^i - k_j^{\pi_i})^2, \ j = \{1, 2\}$$

where Δk_1 and Δk_2 measure the error associated to the matching of scene and model points with different values for the first and second principal curvatures, respectively.

Meanwhile, the objective function of the previous IR method included both information regarding the usual IR measure $g(\pi)$ (MSE of the registration transformation resulting from the point matching encoded in π) and the previous criterion as follows:

$$\min \ F(\pi) \ = \ w_1 \cdot g(\pi) \ + \ w_2 \cdot m_{error}(\pi) \qquad (2)$$

where w_1, w_2 are weighting coefficients defining the relative importance of each term.

With such a function we defined a more suitable similarity measurement to make a better search process in the space of solutions. Instead of considering a function based on a single registration error criterion, the use of the previous two terms working together to solve the IR problem was an important contribution of those IR methods [8], [5].

IV. GRASP & Evolutionary Path Relinking for the Point Matching-Based IR Problem

In this section, we first present some basics on GRASP and path relinking and their design to tackle the IR problem. Finally, our GRASP & Evolutionary path relinking proposal is detailed.

A. Greedy randomized adaptive search procedure: GRASP

The GRASP methodology was developed in the late 1980s [10], [11]. We refer the reader to [24] for a recent survey of this metaheuristic. Each GRASP iteration consists of constructing a trial solution and then applying local search from the constructed solution. The construction phase is iterative, randomized greedy, and adaptive.

Our adaptation of the GRASP methodology for the point matching problem is as follows. The information extracted from the shape of the object (see Section II-B) can be used to establish a preference order for the assignments between the scene image points and the model image ones. Hence, a point x_i from the scene image is more likely to be assigned to those model points y_j presenting the same or similar curvature values k_1 and k_2. In order to achieve that suitable point assignment, one possible approach consists of considering a greedy heuristic. Such approach is characterized by a strict selection order to assign the closest model point y_i in terms of curvature to every scene point x_i, where y_i was not previously assigned to some other scene point. However, we prefer to follow a different approach by introducing randomness in both processes thus allowing each decision to be taken randomly from the points still stored in the nonempty candidate list. Likewise, the latter procedure behaves similarly to a GRASP construction phase [24].

In the particular case of IR, the construction of a given solution π, starts by creating two candidate lists, CL_1 and CL_2 related to the scene and model images, respectively. At the beginning, every list consists of all the points in the image (i.e., initially $CL_1 = P_1$ and $CL_2 = P_2$). For each element x_i in CL_1, its Euclidean distance to CL_2 in terms of curvature values is:

$$di = \min_{j=1,\ldots,m} \sqrt{(k_1(x_i) - k_1(y_j))^2 + (k_2(x_i) - k_2(y_j))^2} \qquad (3)$$

Thus d_i is the minimum value of the distances from x_i to all the elements in CL_2. Then, we construct the restricted candidate list RCL_1 with a percentage α of the elements in CL_1 with the lowest d_i values. We randomly select one element (say x_k) from RCL_1 for the matching assignment. In order to find an appropriate point in the model to match

x_k, we construct RCL_2 with a percentage α of the elements in CL_2 whose curvature values are closer to those of x_k, i.e., those elements presenting the lowest distance values to x_k. Finally, we randomly select a point (say y_k) in RCL_2 and match it to x_k. We update CL_1 and CL_2 ($CL_1 = CL_1 - \{x_k\}, CL_2 = CL_2 - \{y_k\}$) and perform a new iteration. The construction of the solutions π finishes when $r = min(n, m)$ points have been matched, i.e., when either CL_1 or CL_2 is empty, and the remaining $l-r$ points in the permutation π are taken randomly from the points still stored in the nonempty CL.

Regarding to the local search stage of the GRASP approach, we have used the strategy designed for the *Improvement Method* of the IR proposal designed in our previous work. Therein the "best-first" local search procedure with the swapping neighbor operator is considered. In particular, swappings are used as the primary mechanism to move from one solution to another. Moreover, two improvements were considered in order to speed up the local search procedure. On the one hand, a primary strategy was applied in the neighborhood generation by only considering promising swapping moves taking as a base the curvature information. On the other hand, a selective application of the local optimizer was also considered.

B. Path relinking

Path relinking (PR) [13], [14] was suggested as an approach to integrate intensification and diversification strategies in the context of tabu search. This approach generates new solutions by exploring trajectories that connect high-quality solutions – by starting from one of these solutions, called an *initiating solution*, and generating a path in the neighborhood space that leads toward the other solutions, called *guiding solutions*. This is accomplished by selecting moves that introduce attributes contained in the guiding solutions, and incorporating them in an *intermediate solution* initially originated in the initiating solution.

Let π^1 and π^2 be two solutions of the IR problem, interpreted as the sets of m selected elements Sel_{π^1} and Sel_{π^2}, respectively ($|Sel_{\pi^1}| = |Sel_{\pi^2}| = m$). PR$(\pi^1, \pi^2)$ starts with the first (initiating) solution π^1, and gradually transforms it into the second (guiding) one π^2, by swapping out elements selected in π^1 with elements selected in π^2. The elements selected in both solutions π^1 and π^2, $Sel_{\pi^1\pi^2}$, remain selected in the intermediate solutions generated in the path between them. Let $Sel_{\pi^1-\pi^2}$ be the set of elements selected in π^1 and not selected in π^2 and symmetrically, let $Sel_{\pi^2-\pi^1}$ be the set of elements selected in π^2 and not selected in π^1, i.e.

$$\begin{aligned} Sel_{\pi^1\pi^2} &= Sel_{\pi^1} \cap Sel_{\pi^2}, \\ Sel_{\pi^1-\pi^2} &= Sel_{\pi^1} \setminus Sel_{\pi^1\pi^2}, \\ Sel_{\pi^2-\pi^1} &= Sel_{\pi^2} \setminus Sel_{\pi^1\pi^2}. \end{aligned}$$

Let $p_0(\pi^1, \pi^2) = \pi^1$ be the initiating solution in the path P(π^1, π^2) from π^1 to π^2. To obtain the solution $p_1(\pi^1, \pi^2)$ in this path, we unselect a single element $i \in Sel_{\pi^1-\pi^2}$ in $p_0(\pi^1, \pi^2)$, and select a single element $j \in Sel_{\pi^2-\pi^1}$, thus obtaining:

$$Sel_{p_1(\pi^1,\pi^2)} = Sel_{p_0(\pi^1,\pi^2)} \setminus \{i\} \cup \{j\}.$$

In the *greedy PR* algorithm, the selection of the elements i and j is made in a greedy fashion. To obtain $p_{k+1}(\pi^1, \pi^2)$ from $p_k(\pi^1, \pi^2)$, we evaluate all the possibilities for $i \in Sel_{p_k(\pi^1,\pi^2)-\pi^2}$ to be de-selected and $j \in Sel_{\pi^1-p_k(\pi^1,\pi^2)}$ to be selected, and perform the best swap. In this way, we reach π^2 from π^1 in $r = |Sel_{\pi^1-\pi^2}| = |Sel_{\pi^2-\pi^1}|$ steps, i.e. $p_r(\pi^1, \pi^2) = \pi^2$. The output of the PR algorithm is the best solution, different from π^1 and π^2, found in the P(π^1, π^2) path (among $p_1(\pi^1, \pi^2), p_2(\pi^1, \pi^2), \ldots, p_{r-1}(\pi^1, \pi^2)$).

In our specific design for the IR problem, we adopt a *greedy randomized* variant of PR [9] in which the moves are done in a greedy randomized fashion. This procedure mimics the selection method employed in a GRASP construction. Instead of exploring all the possibilities for $i \in Sel_{p_k(\pi^1,\pi^2)-\pi^2}$ to be de-selected and $j \in Sel_{\pi^2-p_k(\pi^1,\pi^2)}$ to be selected to obtain $p_{k+1}(\pi^1, \pi^2)$ from $p_k(\pi^1, \pi^2)$, we perform a truncated exploration of the 50% of the whole neighborhood in order to speed up the run time of the IR method. Thus, the candidate set C contains all these swaps, i.e.

$$C_k(\pi^1, \pi^2) = \{(i, j) \quad | \quad i \in Sel_{p_k(\pi^1,\pi^2)-\pi^2}, \; j \in Sel_{\pi^2-p_k(\pi^1,\pi^2)}\}.$$

Let $z(i, j)$ be the value of the move associated with de-select i and select j in the current solution $p_k(\pi^1, \pi^2)$ to obtain $p_{k+1}(\pi^1, \pi^2)$. Then,

$$z(i, j) = F(p_{k+1}(\pi^1, \pi^2)) - F(p_k(\pi^1, \pi^2)).$$

In step k of the path from π^1 to π^2, the restricted candidate list $RCL_k(\pi^1, \pi^2)$ of good candidates for swapping is

$$RCL_k(\pi^1, \pi^2) = \{(i, j) \in C_k(\pi^1, \pi^2) \mid z(i, j) \geq \delta z^*\},$$

where z^* is the minimum of $z(i, j)$ in $C_k(\pi^1, \pi^2)$ and δ ($0 \leq \delta \leq 1$) is a search parameter. A pair $(i, j) \in RCL_k(\pi^1, \pi^2)$ is randomly selected and the associated swap is performed.

C. IR proposal based on G&EvPR

Among many of the possible hybridizations of the GRASP and PR algorithms, we follow the one proposed by Resende et.al. [23] in which the sinergy between GRASP and *Evolutionary Path relinking* (EvPR) demostrated to be a promising approach for combinatorial optimization problems. EvPR was introduced as a post-processing phase for GRASP with PR [1]. The GRASP with EvPR algorithm, called G&EvPR here, is detailed in Figure 3. In each iteration of the G&EvPR algorithm, the construction and the improvement phase of GRASP as well as the PR method to obtain the elite set are applied (see steps 5 to 9 in Figure 3). After a number of iterations previously established, the GRASP with greedy path relinking stops. A post-processing phase based on PR is applied to each pair of solutions in *ES* (in steps 18 to 27). The solutions obtained with the latter application of PR are considered candidates to enter *ES*, and PR is again applied to them as long as new solutions enter *ES*. Hence, solutions in *ES* evolve. Figure 3 shows this process is repeated for *GlobalIter* iterations. Instead of using a maximum number of iterations as the stop criterion, our implementation maintains the running time stop criterion considered in our previous works on the IR problem.

```
Begin G&EvPR

1  GlobalIter ← number of global iterations;
2  Apply GRASP (construction and local search) for
   b = |ES| iterations to
   populate ES ← {π^1, π^2, ..., π^b};
3  For iter = 1, ..., GlobalIter Do
4      For i = 1, ..., LocalIter Do
5          π ← GRASP construction phase;
6          π' ← GRASP local search starting at π;
7          Randomly select π^j from ES;
8          Apply PR(π', π^j) and PR(π^j, π') and let π'' be the best
           solution found;
9          π''' ← GRASP local search starting at π'';
10         If (F(π''') < F(π^1) Or F(π''') < F(π^b)) and
             d(π''', ES) ≥ dth) Then
11             π^k ← closest solution to π''' in ES with
               F(π''') < F(π^k);
12             Add π''' to ES and remove π^k;
13             Sort ES from best π^1 to worst π^b;
14         End-If;
15     End-For;
16     NewSol ← 1;
17     While NewSol Do
18         NewSol ← 0;
19         Apply PR(π, π') and PR(π', π) for every pair (π, π') in
           ES not combined before. Let π'' be the best solution found;
20         π''' ← GRASP local search starting at π'';
21         If (F(π''') < F(π^1) Or F(π''') < F(π^b)) and
             d(π''', ES) ≥ dth) Then
22             π^k ← closest solution to π''' in ES with
               F(π''') < F(π^k);
23             Add π''' to ES and remove π^k;
24             Sort ES from best π^1 to worst π^b;
25             NewSol ← 1;
26             π^best ← π^1;
27         End-If;
28     End-While;
29 End-For;
30 Return π^1;

End-G&EvPR
```

Fig. 3. Pseudo-code of the G&EvPR algorithm.

Finally, the proposed IR method based on G&EvPR uses some of the components previously designed for the SS-based method. In particular, it shares the representation of solutions based on permutations, the objective function (Eq. 2) and the shape-derived information as well.

V. Experiments

This section introduces the datasets and IR problem instances considered in our experimental study. The parameter settings established in our implementation are also described. Finally, results achieved are analyzed.

A. Medical image datasets and IR problems considered

Our results correspond to a number of registration problems for six medical images from two different image datasets. The firts dataset is composed by four different magnetic resonance images (MRIs). These images have been obtained from the *BrainWeb* database at McGill University [17]. The purpose of this repository is to provide researchers with ground truth data for image analysis techniques and algorithms. *BrainWeb* has been widely used by the IR research community (see, for example, [27]). One of the most important challenges associated to the current experimentation is that the goal of the IR process is to register *pairs of different images from the same object*. Therefore, we tackle a more realistic problem in medical IR named *intra-subject registration*. The other two images considered belong to a second dataset kindly provided by the Rhode Island Hospital [20], and truly correspond to a real medical IR case study. Both are computerized tomography (CT) images of two different human wrists. In this case, we want to highlight the complexity of the problem to be tackled due to its particular anatomical structure. After preprocessing the six images (I_1 to I_6), 583, 393, 348, 284, 575, and 412 crest line points are obtained, respectively.

TABLE I
Similarity transformations considered.

	T_1	T_2	T_3	T_4
$RotAngle^\circ$	115.0	168.0	235.0	276.9
$RotAxis_x$	-0.863868	0.676716	-0.303046	0.872872
$RotAxis_y$	0.259161	-0.290021	-0.808122	0.436436
$RotAxis_z$	0.431934	0.676716	0.505076	-0.218218
Δ_x	-26.0	6.0	16.0	12.0
Δ_y	15.5	5.5	-5.5	5.5
Δ_z	-4.6	-4.6	-4.6	-24.6
S	1.0	0.8	1.0	1.2

First column of Figures 4 and 5 show the original MRIs and CTs, respectively. Second column of thos figures corresponds to the isosurfaces segmenting the original images to extract the regions of interest of each image, i.e. the brain and the wrist. The third column of the figures shows the crest line points extracted from each 3D object.

In order to be able to evaluate the performance of our proposal, we applied four similarity transformations[1] T_i (see Table I) to four of the input images. Five different IR problem instances are considered (see Table II). The behavior of two IR methods based on G&EvPR and SS tackling those IR problem instances will be analyzed.

B. Parameter settings

All the runs have been performed on a 2.26GHz Intel® Core^{TM}2 Duo P8400. Both G&EvPR and SS methods are run ten times in every problem instance using a different seed of the pseudo-random number generator each time in order to avoid the bias of randomness. Each run takes 600 seconds. After a preliminary study, we consider $\alpha = 0.9$ for the construction phase of GRASP, $\delta = 0.9$ for the greedy

[1]Notice that a Similarity transformation is determined by a 3D rotation (given by a rotation axis and angle), a 3D translation, and a uniform scaling.

TABLE II

DESIGNED IR PROBLEM INSTANCES.

IR problem	Scene image Lesion	Scene image Noise	Model image Lesion	Model image Noise
I_1 vs $T_1(I_2)$	No	No	No	1%
I_1 vs $T_2(I_3)$	No	No	Yes	1%
I_1 vs $T_3(I_4)$	No	No	Yes	5%
I_2 vs $T_4(I_4)$	No	1%	Yes	5%
I_6 vs $T_1(I_5)$	–	–	–	–

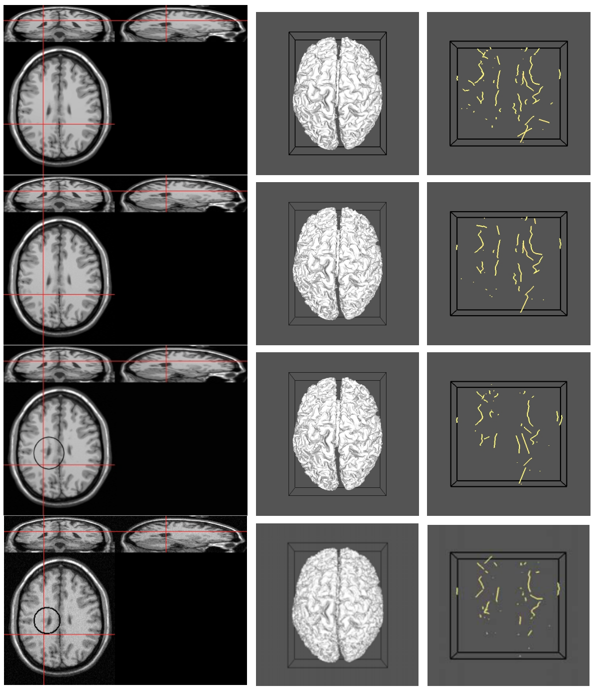

Fig. 4. From left to right, and top to bottom: original MRI images, their respective isosurfaces, and their crest lines points. Note that the second and third MRIs include 1% of Gaussian noise and 5% the fourth one. I_3 and I_4 (last two rows) also considers a multiple sclerosis lesion (see circle).

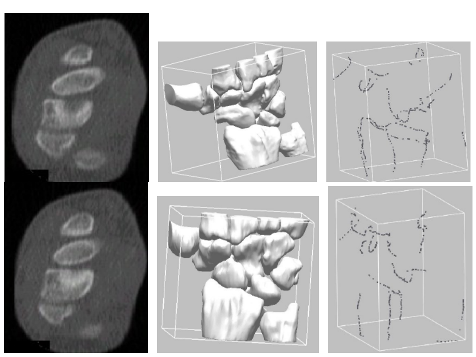

Fig. 5. From left to right: original CT images, their respective isosurfaces, and their crest lines points. First and second rows refers to I_5 and I_6, respectively.

randomized PR scheme, a maximum of 80 iterations for the local search algorithm and six solutions comprise the ES ($b = |ES| = 6$). Regarding to the state-of-the-art IR method based on SS, we maintained the same parameter settings as used in our previous work.

C. Results

We consider the MSE (Eq. 1) of every solution in order to compare the performance of both the G&EvPR and the SS IR methods. Moreover, we have included a pure GRASP variant with the aim of demonstrate that the sinergy between GRASP and PR achieves improved results. Such statistical values are presented in Table III.

TABLE III

STATISTICAL RESULTS COMPUTED FROM TEN RUNS OF EACH OF THE FIVE IR PROBLEMS CONSIDERED. IT IS SHOWN THE MINIMUM (M), MAXIMUM (M), MEAN (μ), AND STANDARD DEVIATION (σ) VALUE OF MSE. BEST RESULTS ACCORDING TO MEAN AND STANDARD DEVIATION VALUES ARE HIGHLIGHTED.

		GRASP	G&EvPR	SS
I_1 vs $T_1(I_2)$	m	44.09	41.79	**39.70**
	M	58.27	47.26	45.53
	μ	50.39	44.43	**42.85**
	σ	4.84	1.86	1.88
I_1 vs $T_2(I_3)$	m	44.16	**39.17**	40.77
	M	91.74	45.05	49.71
	μ	60.44	**43.22**	44.62
	σ	13.59	1.76	2.23
I_1 vs $T_3(I_4)$	m	83.25	57.41	**57.37**
	M	141.04	62.73	64.90
	μ	108.60	**60.88**	61.18
	σ	18.62	1.57	1.88
I_2 vs $T_4(I_4)$	m	136.10	81.00	**78.70**
	M	228.06	91.89	95.02
	μ	156.38	88.51	**87.80**
	σ	26.49	3.59	5.19
I_6 vs $T_1(I_5)$	m	2.26	1.68	**1.63**
	M	3.57	1.95	3.99
	μ	2.88	**1.80**	2.19
	σ	0.44	0.10	0.83

On the one hand, it is proven that G&EvPR outperforms pure GRASP in all the considered IR problem instances. On the other hand, notice that the IR method based on G&EvPR-based achieves a competitive performance compared to the state-of-the-art IR algorithm based on SS. Specifically, G&EvPR achieves the best mean results for three of the five IR problems. Moreover, it obtains the lowest standard deviation values in all the cases. Therefore, G&EvPR provides a good trade-off between search space diversification and intensification, thus showing a more robust behavior than the IR method based on SS.

Regarding to the accuracy of the approaches (i.e. minimum MSE), both IR methods behave in a really similar way obtaining precise results when tackling IR problems involving MRIs, being SS the IR method achieving the overall best solution in four of the five IR problems. Figure 6 shows the said similar behavior. For the sake of visual interpretation, different colors are used to represent the scene and model images. First column in Figure 6 corresponds to the initial configurations of the different problem instances. The results achieved by G&EvPR and SS are presented in the second and third columns of Figure 6, respectively. Notice that the initial configurations considered correspond to important similarity transformations leading to substantial changes in the geometry of the images. Hence, the IR problem instances tackled are really complex. Even dealing with such complex scenarios, both G&EvPR and SS methods achieve outstanding best solutions. That is visually shown by the almost perfect overlapping of the colors of the objects in the second and third column of Figure 6. The visual results corresponding to the IR of CTs, i.e. I_6 $vs.$ $T_1(I_5)$ (see Figure 7) show the high complexity of this real-world case study, mainly originated by the nature of the anatomical structure of the human wrist.

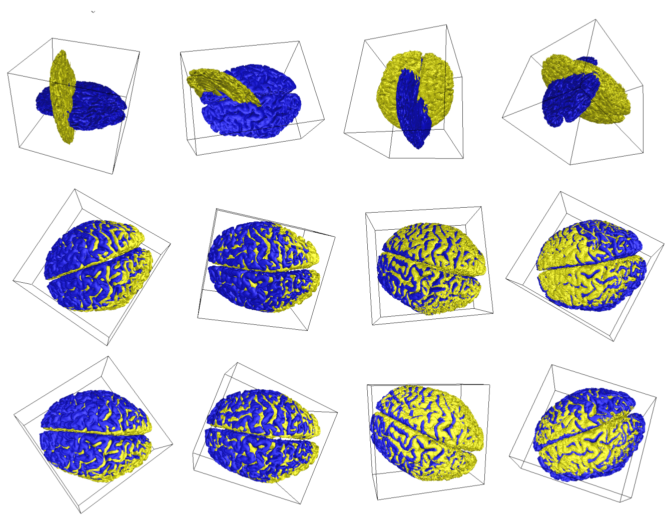

Fig. 6. First row refers to the four IR problems using MRIs (I_1 vs $T_1(I_2)$, I_1 vs $T_2(I_3)$, I_1 vs $T_3(I_4)$, and I_2 vs $T_4(I_4)$). Second and third rows show the best IR results achieved by the G&EvPR and the SS IR methods facing each IR problem, respectively.

VI. Conclusions and Future Works

We presented a contribution facing the medical IR problem from the point matching approach. Our evolutionary proposal is an adaptation of an advanced hybrid method composed by the GRASP and the evolutionary path relinking algorithms. Specifically, we studied the performance of the method in both realistic and real-world medical applications. We proved how the sinergy between the single and multiple trajectory approaches and the evolutionary scheme of PR provided more robust results than the IR method based on SS considered as one of the state-of-the-art algorithms in point matching approaches. In particular, a good trade-off

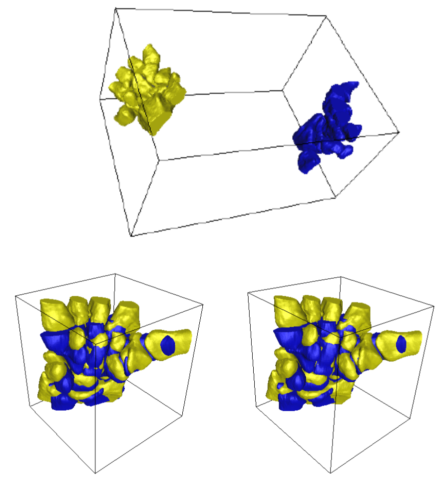

Fig. 7. At the top: the IR problem using CTs (I_6 vs $T_1(I_5)$). At the bottom and from left to right: the best IR results achieved by the G&EvPR and the SS IR methods, respectively.

between search space diversification and intensification have been achieved by our proposal.

Finally, new possible designs based on GRASP and path relinking algorithms for tackling IR problem could be considered in order to go one step further in this challenging problem. In particular, a global study on the different designs of the construction phase of GRASP, the combination scheme with PR, and new shape-derived information are open research directions for future work.

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