

Multiobjective Genetic Fuzzy Systems

**- Accurate and Interpretable Fuzzy Rule-Based
Classifier Design -**

Hisao Ishibuchi

Osaka Prefecture University, Japan

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Accurate and Interpretable Fuzzy Rule-Based Classifier Design

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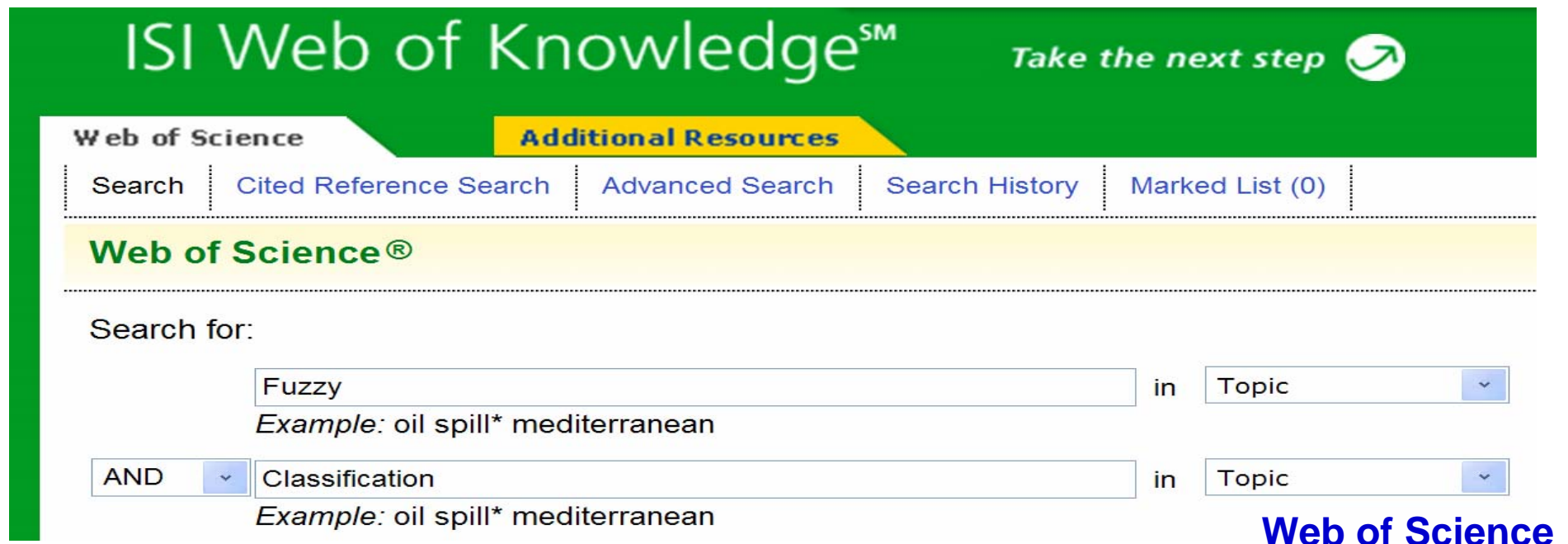
Application Areas of Fuzzy Systems


Application Areas of Fuzzy Systems

- Fuzzy Control
- Fuzzy Clustering
- Fuzzy Classification

Control and clustering are well-known application areas !

Question: Is “fuzzy classification” popular ?



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Control, Clustering, Classification

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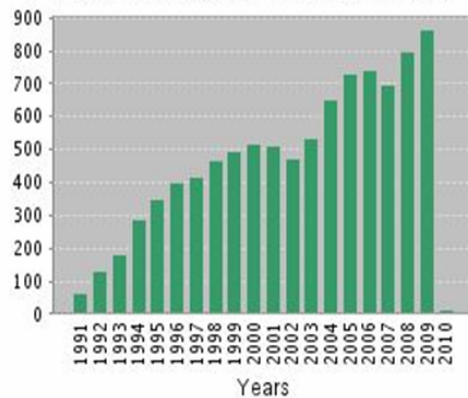
Application Areas of Fuzzy Systems

Fuzzy Control: Well-Known Application Area

Citation Report Topic=(Fuzzy) AND Topic=(Control)
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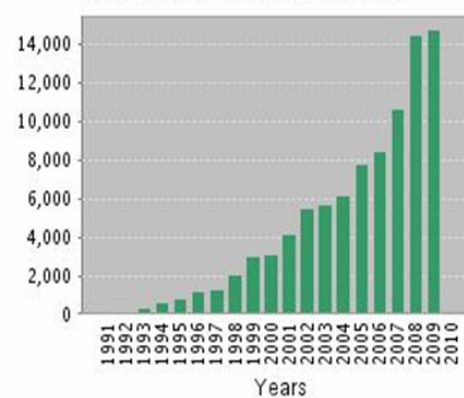
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9421 Fuzzy Control Papers

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<input type="checkbox"/> 1. Title: FUZZY IDENTIFICATION OF SYSTEMS AND ITS APPLICATIONS TO MODELING AND CONTROL Author(s): TAKAGI T, SUGENO M Source: IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS Volume: 15 Issue: 1 Pages: 116-132 Published: 1985	336	409	476	433	0	3,801	152.04

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Application Areas of Fuzzy Systems

Fuzzy Control: Well-Known Application Area

<p>□ 1. Title: FUZZY IDENTIFICATION OF SYSTEMS AND ITS APPLICATIONS TO MODELING AND CONTROL Author(s): TAKAGI T, SUGENO M Takagi-Sugeno Model (1985) Source: IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS Volume: 15 Issue: 1 Pages: 116-132 Published: 1985</p>	336	409	476	433	0	3,801
						3801
<p>□ 2. Title: ANFIS - ADAPTIVE-NETWORK-BASED FUZZY INFERENCE SYSTEM ANFIS (1993) Author(s): JANG JSR Source: IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS Volume: 23 Issue: 3 Pages: 665-685 Published: MAY-JUN 1993</p>	150	166	256	289	0	1,983
						1983
<p>□ 3. Title: FUZZY-LOGIC IN CONTROL-SYSTEMS - FUZZY-LOGIC CONTROLLER .1. CC Lee: Fuzzy Logic Controller (1990) Author(s): LEE CC Source: IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS Volume: 20 Issue: 2 Pages: 404-418 Published: MAR-APR 1990</p>	92	125	137	84	0	1,896
						1896
<p>□ 4. Title: GENERATING FUZZY RULES BY LEARNING FROM EXAMPLES Author(s): WANG LX, MENDEL JM Source: IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS Volume: 22 Issue: 6 Pages: 1414-1427 Published: NOV-DEC 1992</p>	79	98	94	73	0	899
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<p>□ 5. Title: STABILITY ANALYSIS AND DESIGN OF FUZZY CONTROL-SYSTEMS Author(s): TANAKA K, SUGENO M Source: FUZZY SETS AND SYSTEMS Volume: 45 Issue: 2 Pages: 135-156 Published: JAN 24 1992</p>	68	88	112	79	0	817
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<p>□ 6. Title: An approach to fuzzy control of nonlinear systems: Stability and design issues Author(s): Wang HO, Tanaka K, Griffin MF Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS Volume: 4 Issue: 1 Pages: 14-23 Published: FEB 1996</p>	81	85	123	81	0	811
						811
<p>□ 7. Title: FUZZY BASIS FUNCTIONS, UNIVERSAL APPROXIMATION, AND ORTHOGONAL LEAST-SQUARES LEARNING Author(s): WANG LX, MENDEL JM Source: IEEE TRANSACTIONS ON NEURAL NETWORKS Volume: 3 Issue: 5 Pages: 807-814 Published: SEP 1992</p>	55	63	56	40	0	737
						737

Application Areas of Fuzzy Systems

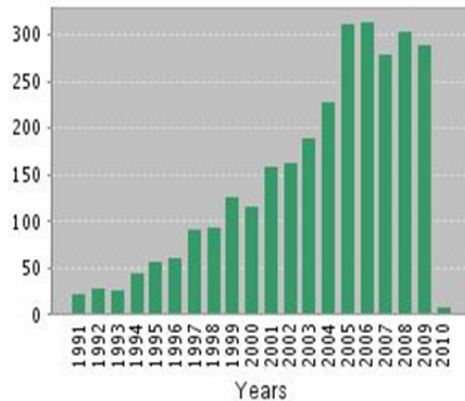
Fuzzy Clustering: Well-Known Application Area

Citation Report **Topic=(Fuzzy) AND Topic=(Clustering)**

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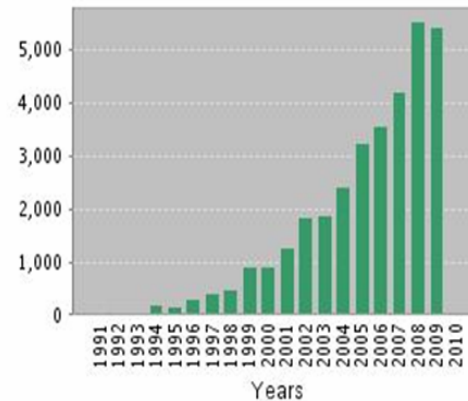
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<input type="checkbox"/> 1. Title: A REVIEW ON IMAGE SEGMENTATION TECHNIQUES Author(s): PAL NR, PAL SK Source: PATTERN RECOGNITION Volume: 26 Issue: 9 Pages: 1277-1294 Published: SEP 1993	87	84	105	67	0	819	48.18

Application Areas of Fuzzy Systems

Fuzzy Clustering: Well-Known Application Area

<p>1. Title: A REVIEW ON IMAGE SEGMENTATION TECHNIQUES Pal NR, Pal SK Author(s): PAL NR, PAL SK Source: PATTERN RECOGNITION Volume: 26 Issue: 9 Pages: 1277-1294 Published: SEP 1993</p>	87	84	105	67	0	819 819
<p>2. Title: UNSUPERVISED OPTIMAL FUZZY CLUSTERING Author(s): GATH I, GEVA AB Source: IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE Volume: 11 Issue: 7 Pages: 773-781 Published: JUL 1989</p>	53	48	56	42	0	513
<p>3. Title: A VALIDITY MEASURE FOR FUZZY CLUSTERING Author(s): XIE XLL, BENI G Source: IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE Volume: 13 Issue: 8 Pages: 841-847 Published: AUG 1991</p>	61	57	80	91	0	506
<p>4. Title: FCM - THE FUZZY C-MEANS CLUSTERING-ALGORITHM Bezdek JC Author(s): BEZDEK JC, EHRlich R, FULL W Source: COMPUTERS & GEOSCIENCES Volume: 10 Issue: 2-3 Pages: 191-203 Published: 1984</p>	19	42	47	39	0	376
<p>5. Title: ON CLUSTER VALIDITY FOR THE FUZZY C-MEANS MODEL Pal NR, Bezdek JC Author(s): PAL NR, BEZDEK JC Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS Volume: 3 Issue: 3 Pages: 370-379 Published: AUG 1995</p>	35	52	55	48	0	347
<p>6. Title: An on-line self-constructing neural fuzzy inference network and its applications Author(s): Juang CF, Lin CT Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS Volume: 6 Issue: 1 Pages: 12-32 Published: FEB 1998</p>	27	37	40	41	0	284
<p>7. Title: Color image segmentation: advances and prospects Author(s): Cheng HD, Jiang XH, Sun Y, et al. Source: PATTERN RECOGNITION Volume: 34 Issue: 12 Pages: 2259-2281 Published: DEC 2001</p>	32	43	55	60	0	283

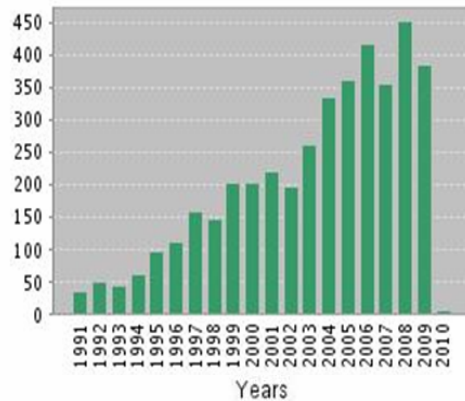
Application Areas of Fuzzy Systems

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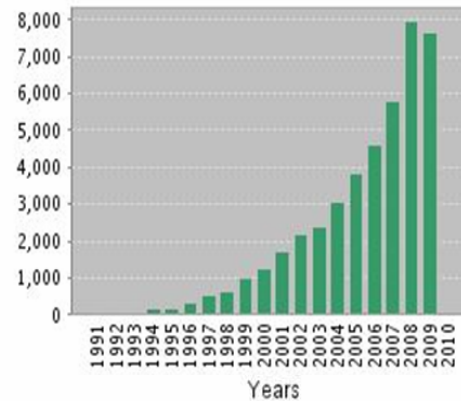
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<input type="checkbox"/> 1. Title: Status of land cover classification accuracy assessment Author(s): Foody GM Source: REMOTE SENSING OF ENVIRONMENT Volume: 80 Issue: 1 Pages: 185-201 Published: APR 2002	68	56	85	81	0	389	48.62

Application Areas of Fuzzy Systems

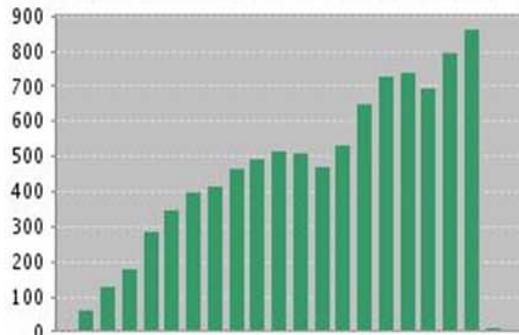
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<p>2. Title: ROUGH FUZZY-SETS AND FUZZY ROUGH SETS Author(s): DUBOIS D, PRADE H Source: INTERNATIONAL JOURNAL OF GENERAL SYSTEMS Volume: 17 Issue: 2-3 Pages: 191-209 Published: 1990</p>	46	43	67	54	0	343
<p>3. Title: FUZZY-SETS IN APPROXIMATE REASONING .1. INFERENCE WITH POSSIBILITY DISTRIBUTIONS Author(s): DUBOIS D, PRADE H Source: FUZZY SETS AND SYSTEMS Volume: 40 Issue: 1 Pages: 143-202 Published: MAR 5 1991</p>	21	30	22	15	0	333
<p>4. Title: FUZZY MIN MAX NEURAL NETWORKS .1. CLASSIFICATION Author(s): SIMPSON PK Source: IEEE TRANSACTIONS ON NEURAL NETWORKS Volume: 3 Issue: 5 Pages: 776-786 Published: SEP 1992</p>	36	35	24	20	0	330
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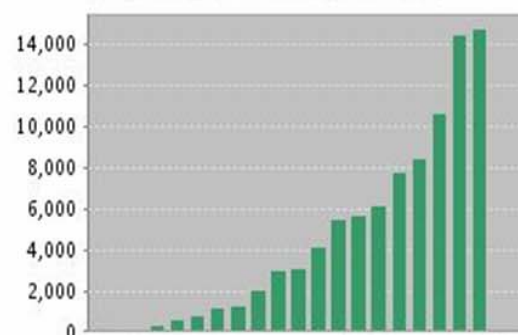
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Control, Clustering, and Classification

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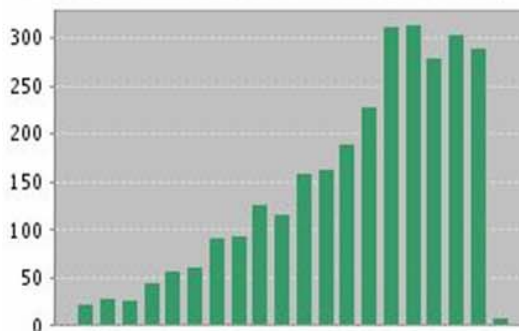
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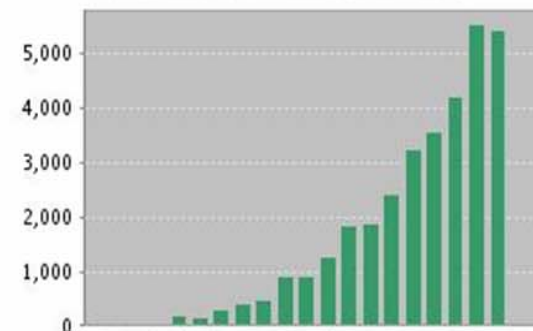
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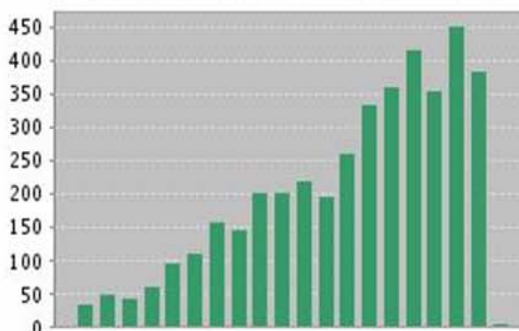
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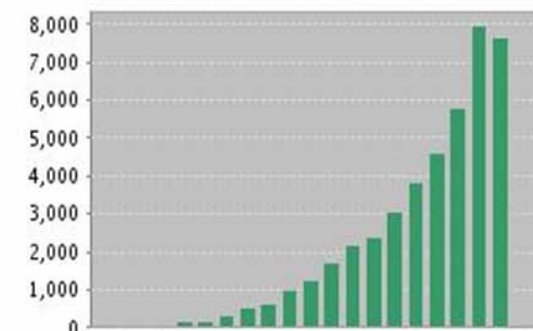
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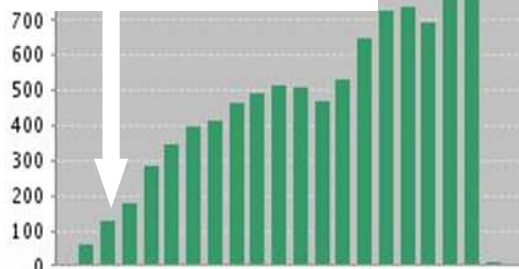
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Application Areas of Fuzzy Systems

Control, Clustering, and Classification

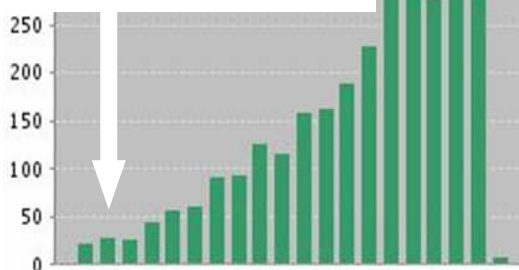
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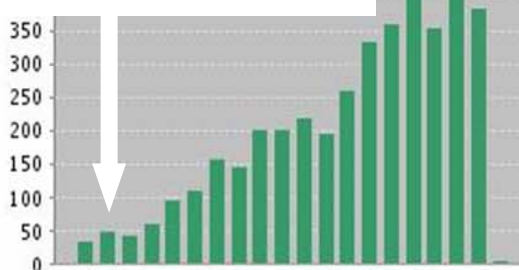
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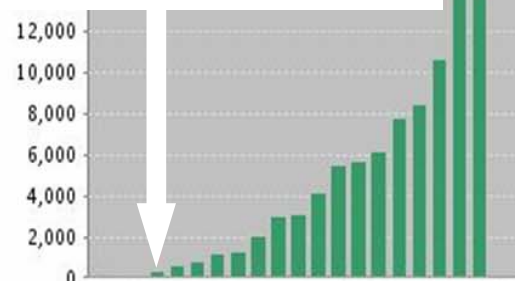
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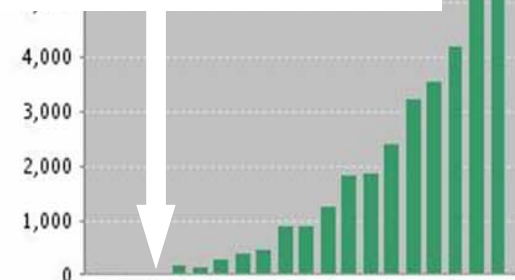
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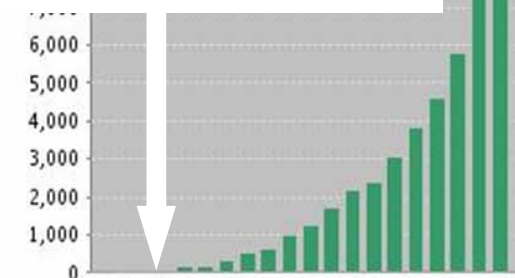
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Author(s): ISHIBUCHI H, NOZAKI K, YAMAMOTO N, et al.

Source: **IEEE TRANSACTIONS ON FUZZY SYSTEMS** Volume: **3** Issue: **3** Pages: **260-270**

Published: **AUG 1995**

Fitness Function

$$w_1 \textit{Accuracy}(S) - w_2 \textit{Complexity}(S)$$

Accuracy Maximization and Complexity Minimization

This Presentation: **Accurate and Interpretable Fuzzy Rule-Based Classifier Design**

Title: **SELECTING FUZZY IF-THEN RULES FOR CLASSIFICATION PROBLEMS USING GENETIC ALGORITHMS**

Author(s): ISHIBUCHI H, NOZAKI K, YAMAMOTO N, et al.

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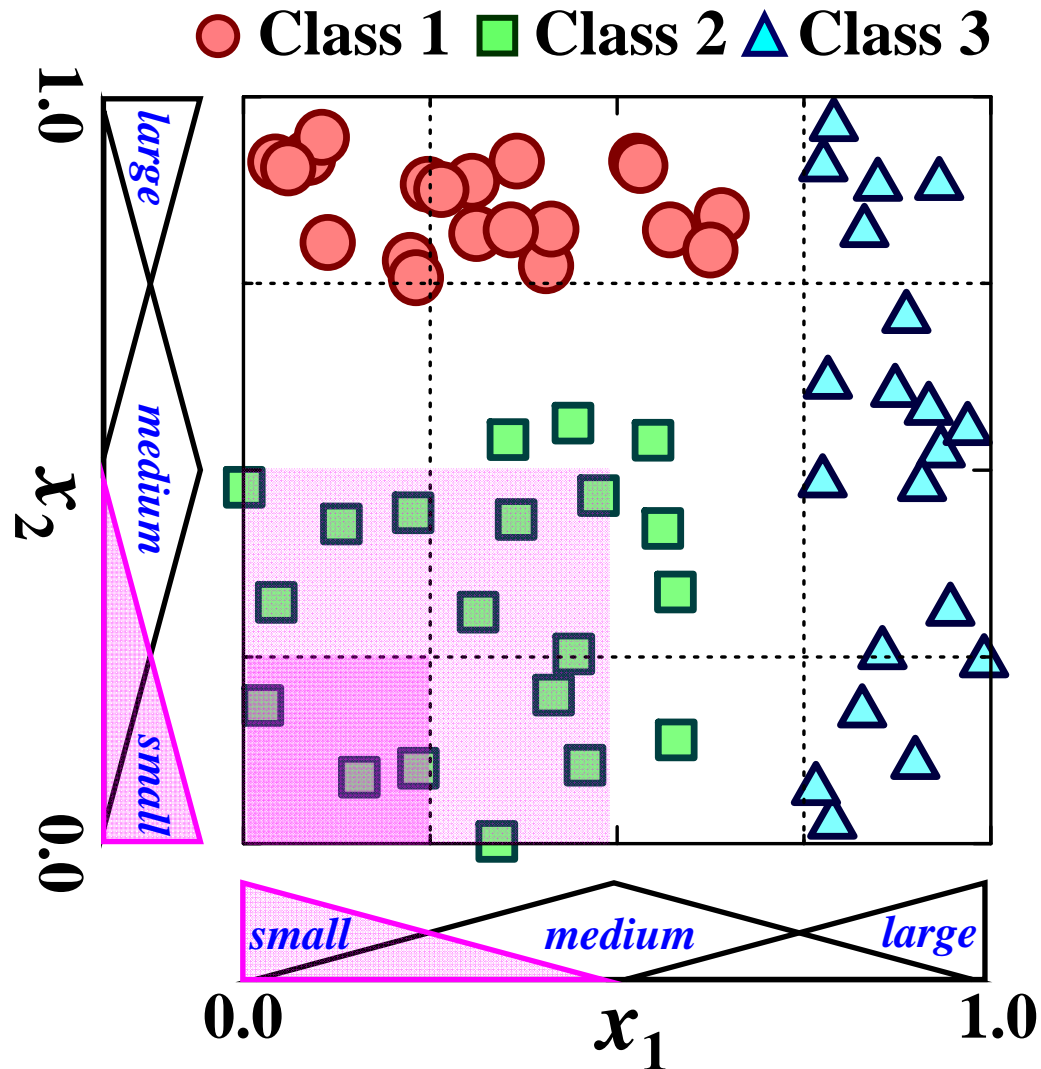
The number of selected fuzzy rules

$$w_1 \text{Accuracy}(S) - w_2 \text{Complexity}(S)$$

The number of correctly classified training patterns

Fuzzy Rules for Classification

Accurate and Interpretable Fuzzy Rule-Based Classifier Design

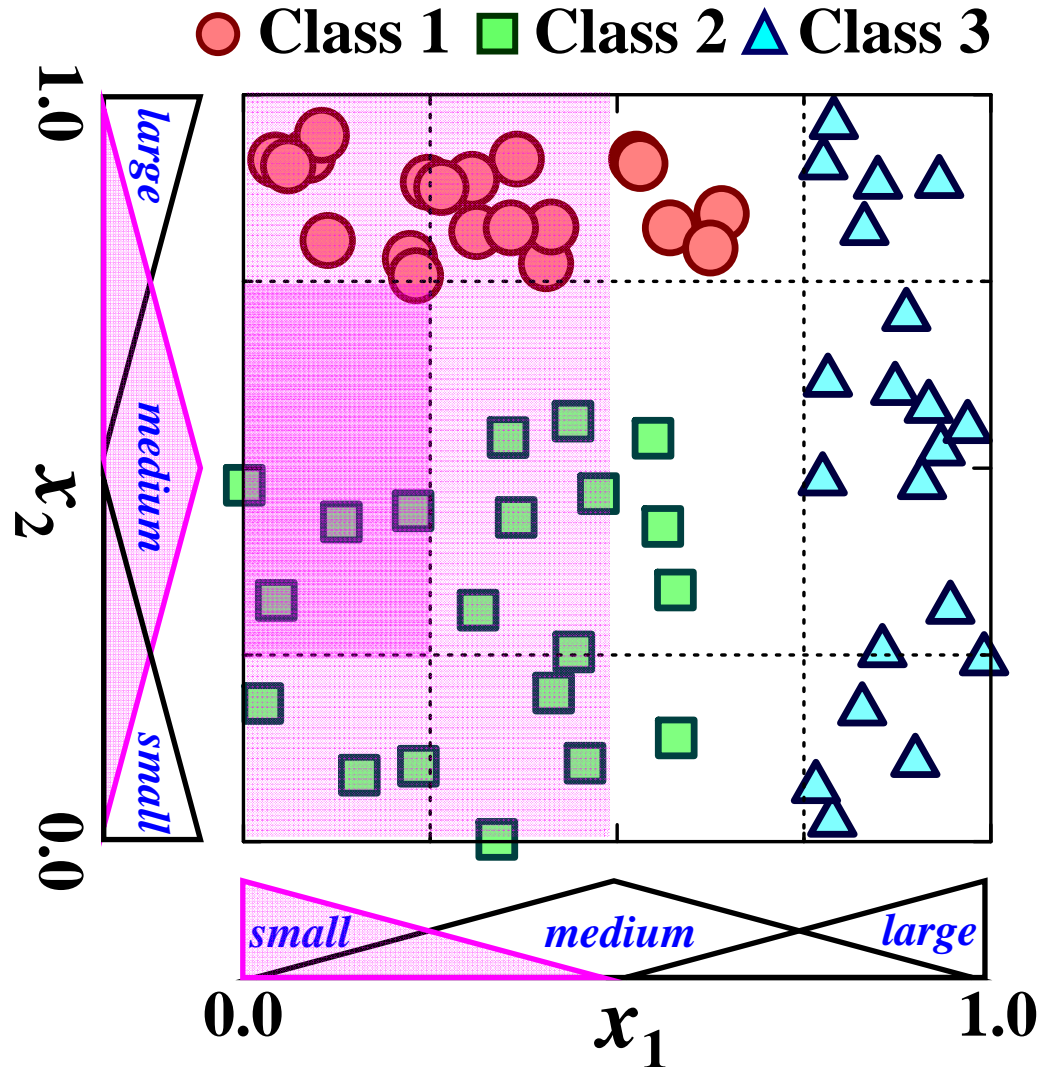


Basic Form

If x_1 is *small* and x_2 is *small*
then Class 2

Fuzzy Rules for Classification

Accurate and Interpretable Fuzzy Rule-Based Classifier Design



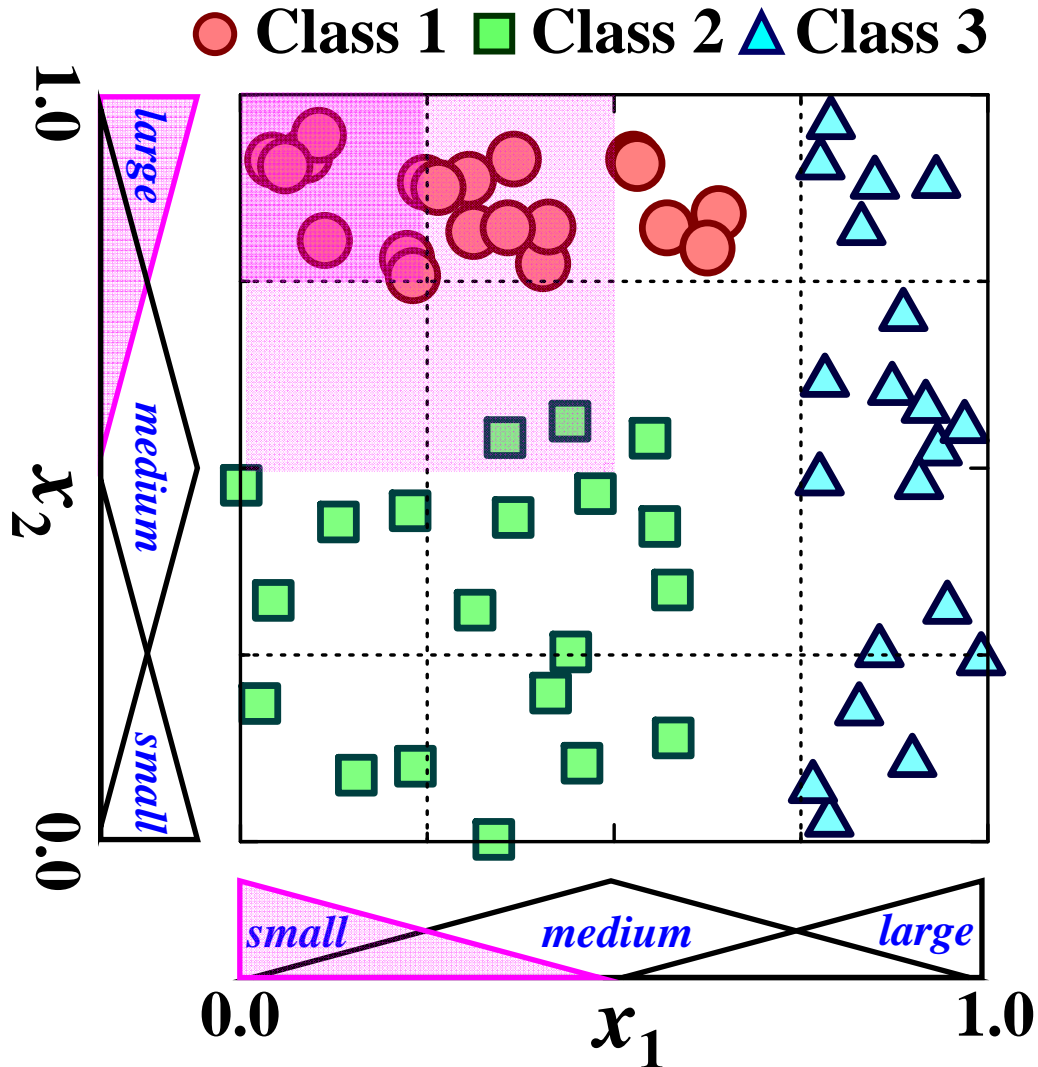
Basic Form

If x_1 is *small* and x_2 is *small*
then Class 2

If x_1 is *small* and x_2 is *medium*
then Class 2

Fuzzy Rules for Classification

Accurate and Interpretable Fuzzy Rule-Based Classifier Design



Basic Form

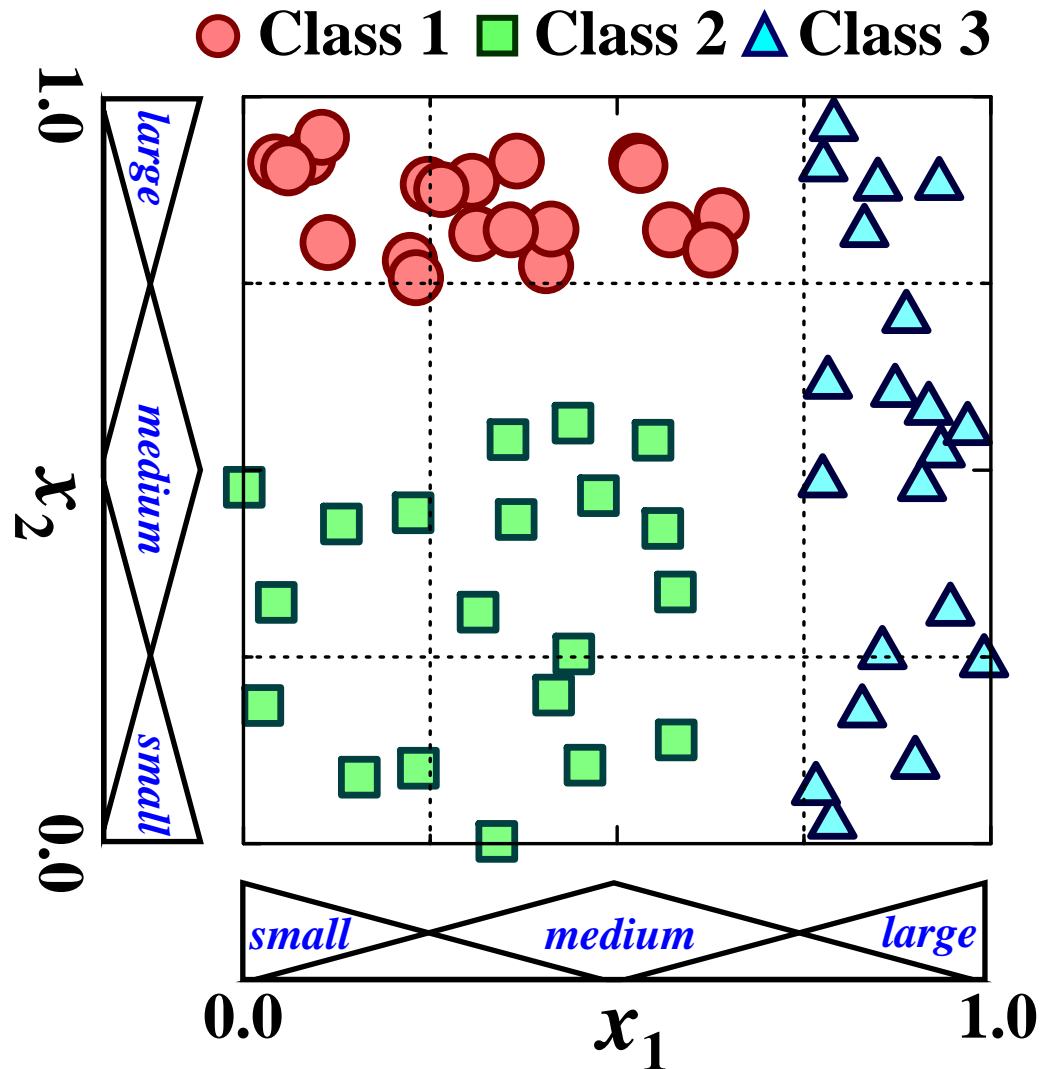
If x_1 is *small* and x_2 is *small*
then Class 2

If x_1 is *small* and x_2 is *medium*
then Class 2

If x_1 is *small* and x_2 is *large*
then Class 1

Fuzzy Rules for Classification

Accurate and Interpretable Fuzzy Rule-Based Classifier Design



Basic Form

If x_1 is *small* and x_2 is *small*
then Class 2

If x_1 is *small* and x_2 is *medium*
then Class 2

If x_1 is *small* and x_2 is *large*
then Class 1

...

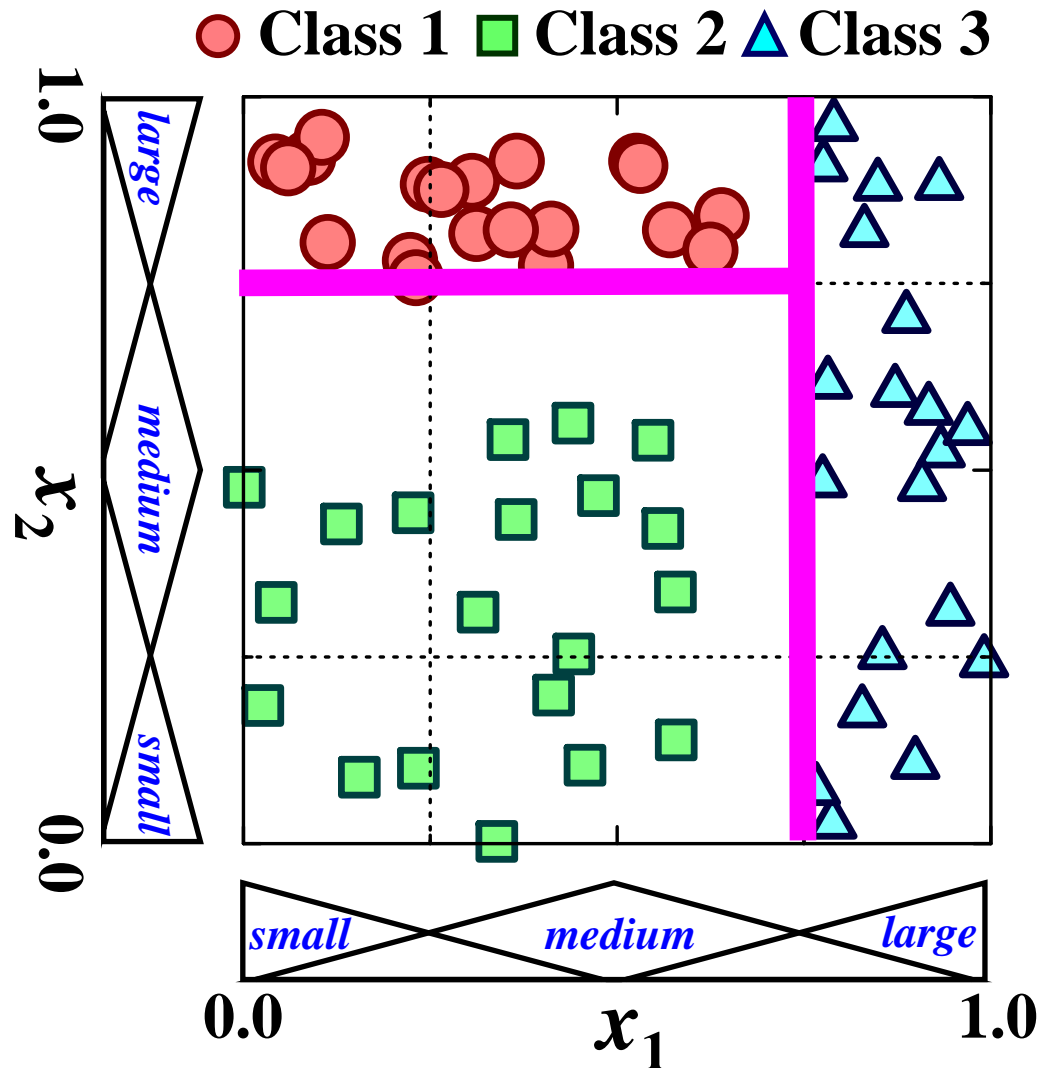
If x_1 is *large* and x_2 is *large*
then Class 3

High Interpretability

Easy to Understand !

Classification Boundary

Accurate and Interpretable Fuzzy Rule-Based Classifier Design



Basic Form

If x_1 is *small* and x_2 is *small*
then Class 2

If x_1 is *small* and x_2 is *medium*
then Class 2

If x_1 is *small* and x_2 is *large*
then Class 1

...

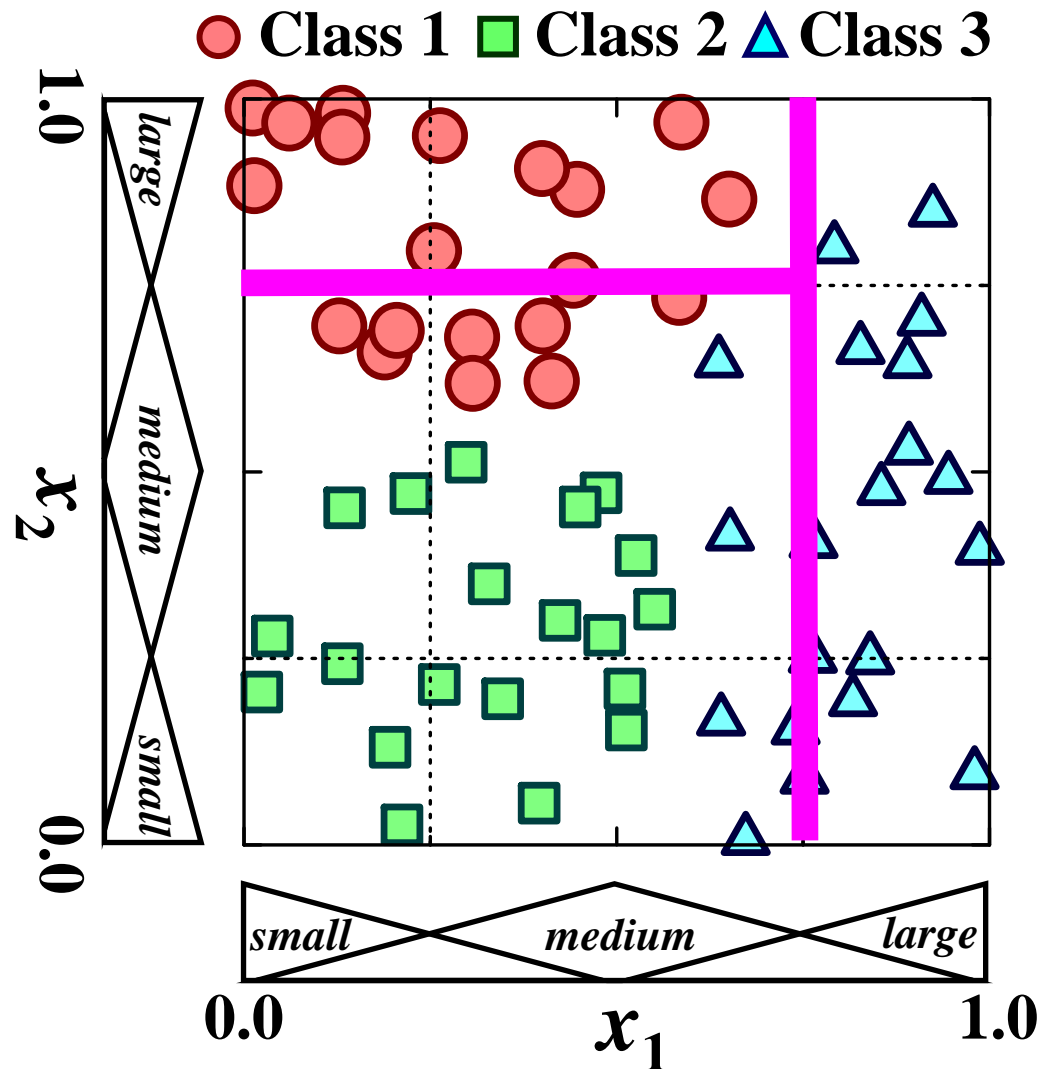
If x_1 is *large* and x_2 is *large*
then Class 3

High Interpretability

Easy to Understand !

Fuzzy Rules for Classification

Basic form does not always have high accuracy



Basic Form

If x_1 is *small* and x_2 is *small*
then Class 2

If x_1 is *small* and x_2 is *medium*
then Class 2

If x_1 is *small* and x_2 is *large*
then Class 1

...

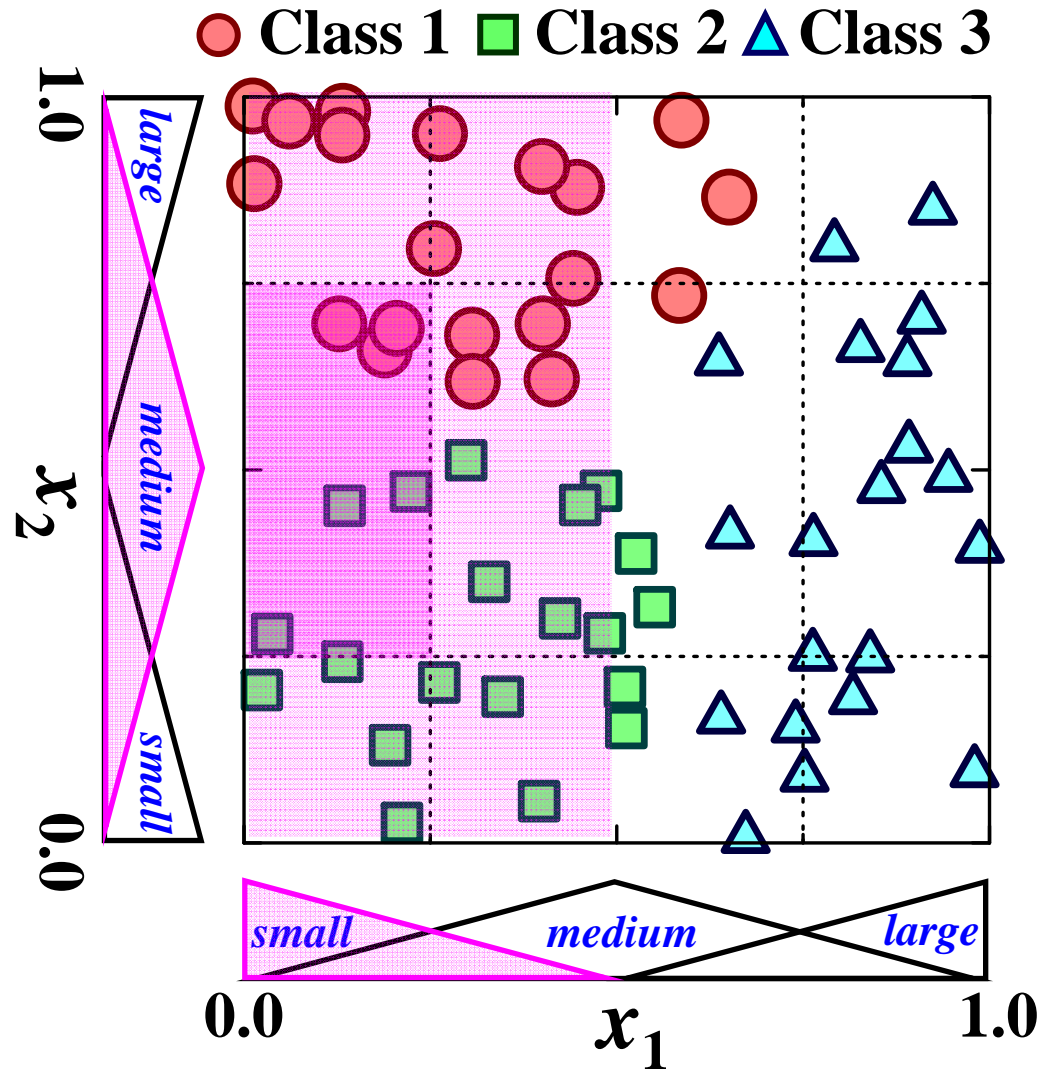
If x_1 is *large* and x_2 is *large*
then Class 3

High Interpretability

Low Accuracy

Fuzzy Rules for Classification

Another form has a rule weight (certainty)



Basic Form

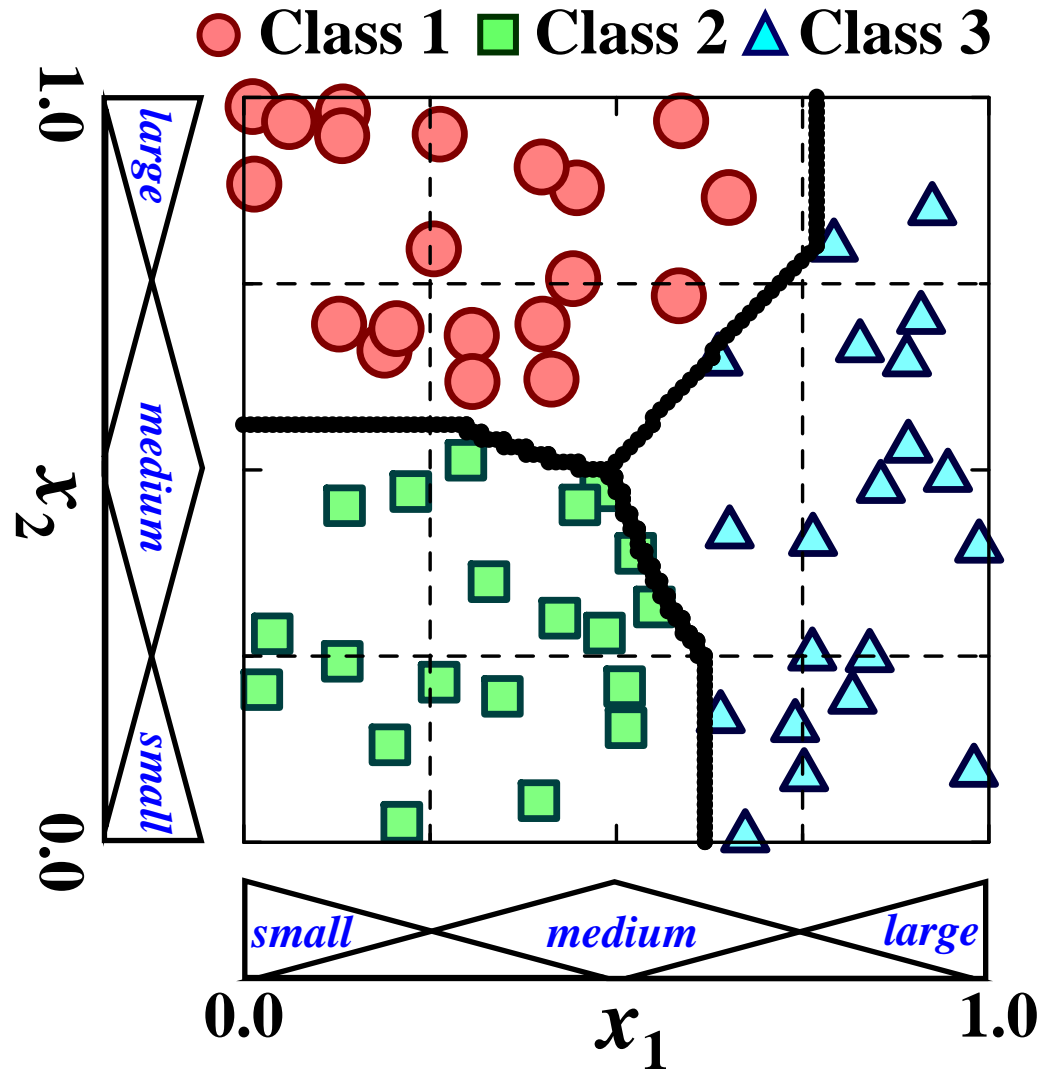
If x_1 is *small* and x_2 is *medium*
then Class 2

Rule Weight Version

If x_1 is *small* and x_2 is *medium*
then Class 2 with 0.158

Classification Boundary

Accurate and Interpretable Fuzzy Rule-Based Classifier Design



Basic Form

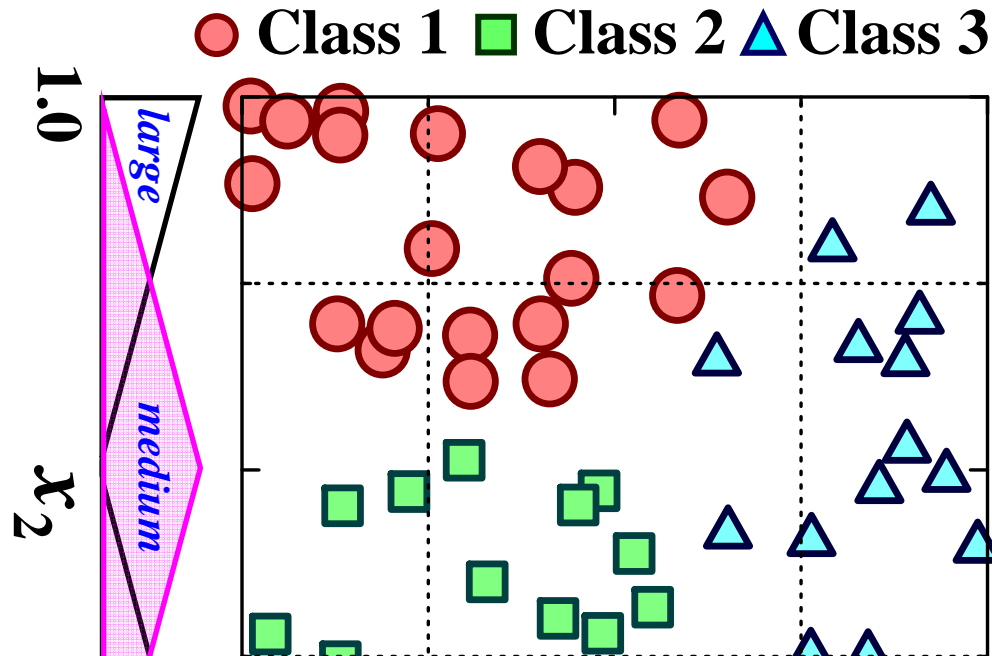
If x_1 is *small* and x_2 is *medium*
then Class 2

Rule Weight Version

If x_1 is *small* and x_2 is *medium*
then Class 2 with 0.158

Fuzzy Rules with Rule Weights

Accurate and Interpretable Fuzzy Rule-Based Classifier Design



Basic Form

If x_1 is *small* and x_2 is *medium*
then Class 2

Rule Weight Version

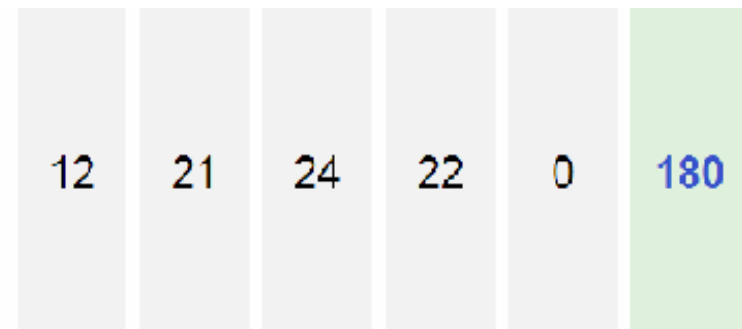
If x_1 is *small* and x_2 is *medium*
then Class 2 with 0.158

Title: DISTRIBUTED REPRESENTATION OF FUZZY RULES AND ITS APPLICATION TO PATTERN-CLASSIFICATION

Author(s): ISHIBUCHI H, NOZAKI K, TANAKA H

Source: **FUZZY SETS AND SYSTEMS** Volume: 52

Issue: 1 Pages: 21-32 Published: NOV 25 1992

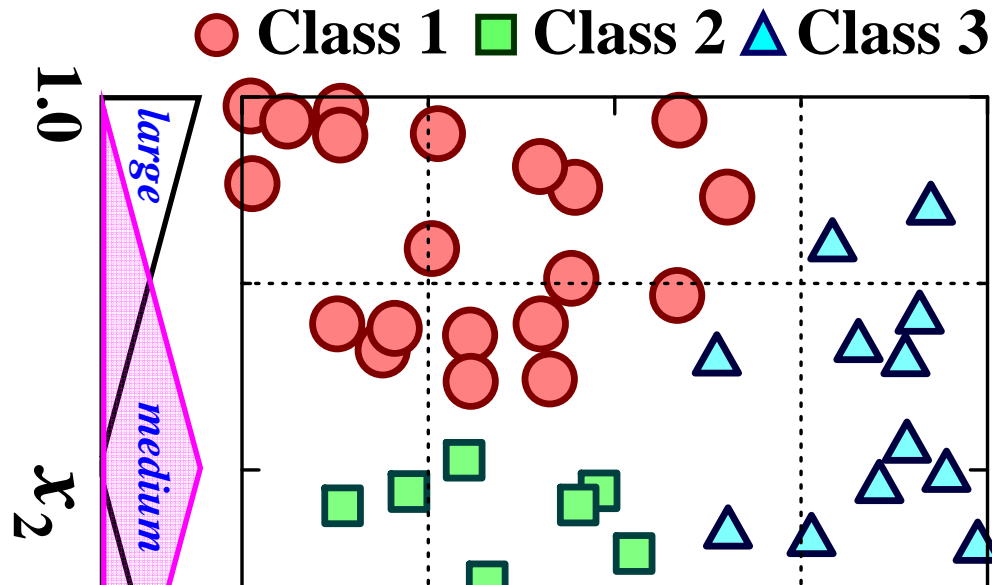


H. Ishibuchi et al., *Fuzzy Sets and Systems* (1992)

Web of Science

Fuzzy Rules in This Presentation

Fuzzy Rules with Rule Weights



Basic Form

If x_1 is *small* and x_2 is *medium*
then Class 2

Rule Weight Version

If x_1 is *small* and x_2 is *medium*
then Class 2 with 0.158

Use of Rule Weights: Controversial Issue

(1) Rule weight adjustment can be replaced with membership learning.

[PDF] ► [How the learning of rule weights affects the interpretability of fuzzy systems](#)

D Nauck, R Kruse - Proc. IEEE International Conference on Fuzzy Systems, 1998 - Citeseer

Neuro-fuzzy systems have recently gained a lot of interest in research and application. These are approaches that learn fuzzy systems from data. Many of them use rule weights for this task. In this paper we discuss the influence ...

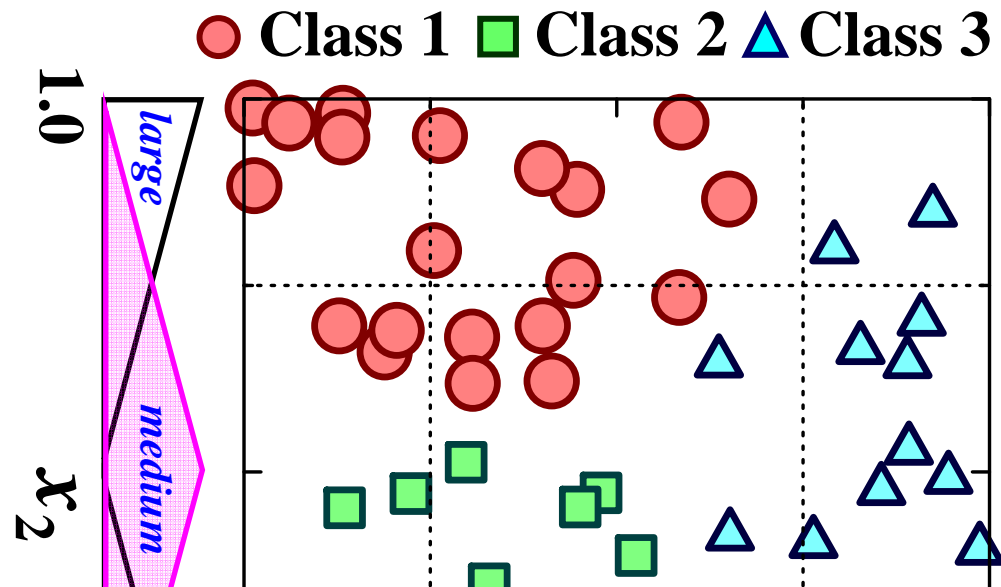
Google Scholar

Cited by 56 -

D. Nauck and R. Kruse, *Proc. of FUZZ-IEEE 1998 (1998)*.

Fuzzy Rules in This Presentation

Fuzzy Rules with Rule Weights



Basic Form

If x_1 is *small* and x_2 is *medium*
then Class 2

Rule Weight Version

If x_1 is *small* and x_2 is *medium*
then Class 2 with 0.158

Use of Rule Weights: Controversial Issue

(2) Membership learning can be partially replaced with weight adjustment.

[PDF] ► [Effect of rule weights in fuzzy rule-based classification systems](#)

H Ishibuchi, T Nakashima - algorithms - Citeseer

... Hisao Ishibuchi, Member, IEEE, and Tomoharu Nakashima, Member, IEEE ... Hisao

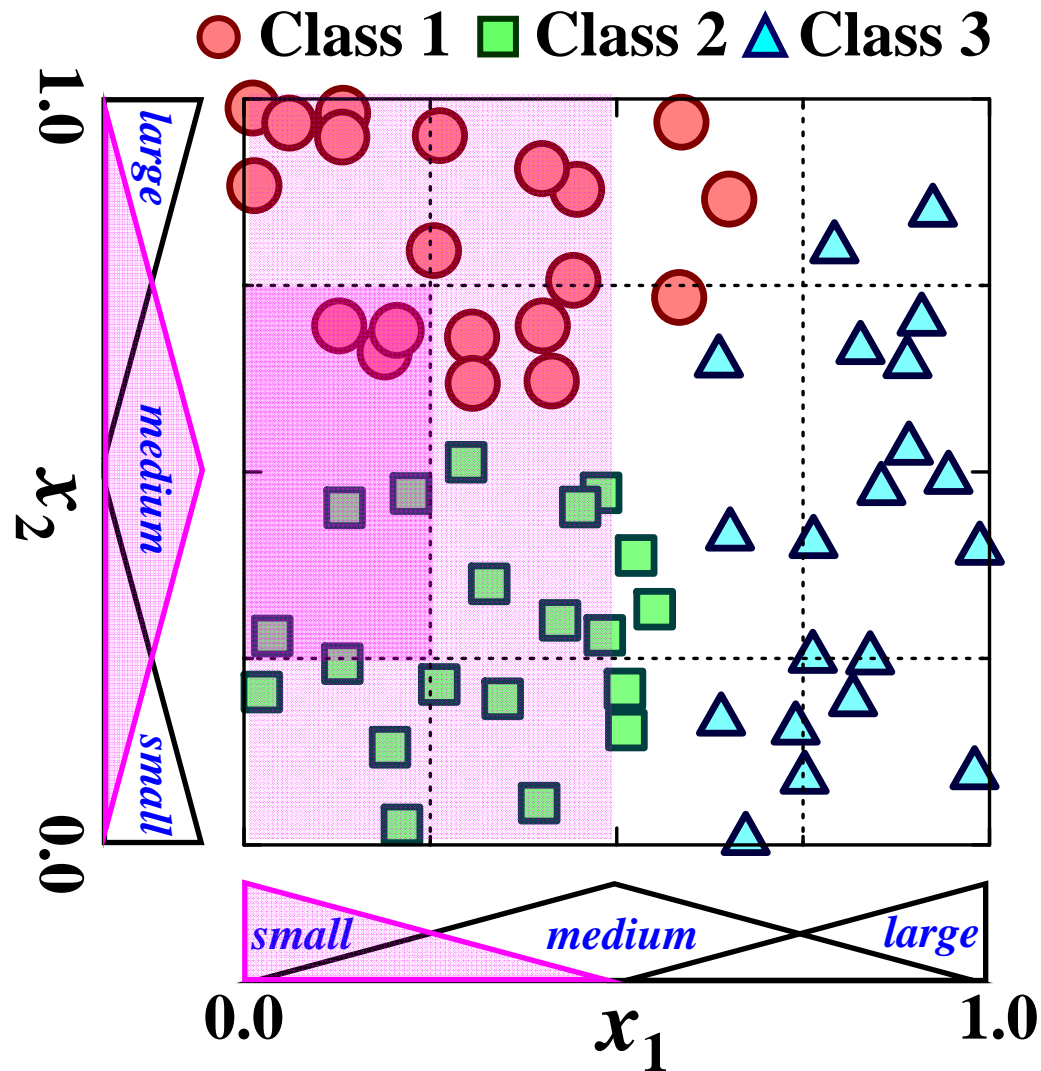
Ishibuchi, Member, IEEE, and Tomoharu Nakashima, Member, IEEE ... [Google Scholar](#)

[Cited by 170](#)

H. Ishibuchi and T. Nakashima, *IEEE Trans. FS* (2001)

Fuzzy Rules for Classification

Another Form: Multiple Consequents



Basic Form

If x_1 is *small* and x_2 is *medium*
then Class 2

Rule Weight Version

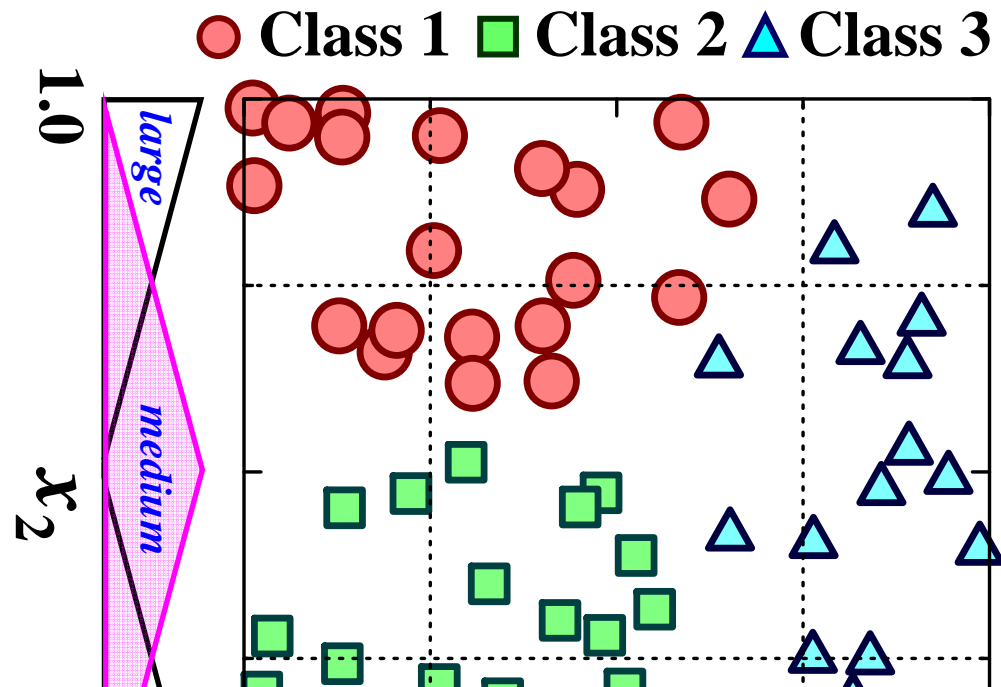
If x_1 is *small* and x_2 is *medium*
then Class 2 with 0.158

Multiple Consequents

If x_1 is *small* and x_2 is *medium*
then Class 1 with 0.579,
Class 2 with 0.421,
Class 3 with 0.000

Fuzzy Rules for Classification

Another Form: Multiple Consequents



Basic Form

If x_1 is *small* and x_2 is *medium*
then Class 2

Rule Weight Version

If x_1 is *small* and x_2 is *medium*
then Class 2 with 0.158

Multiple Consequents

[A proposal on reasoning methods in fuzzy rule-based classification systems-](#)

[O Cordon, MJ del Jesus, F Herrera - International Journal of Approximate Reasoning, 1999](#)

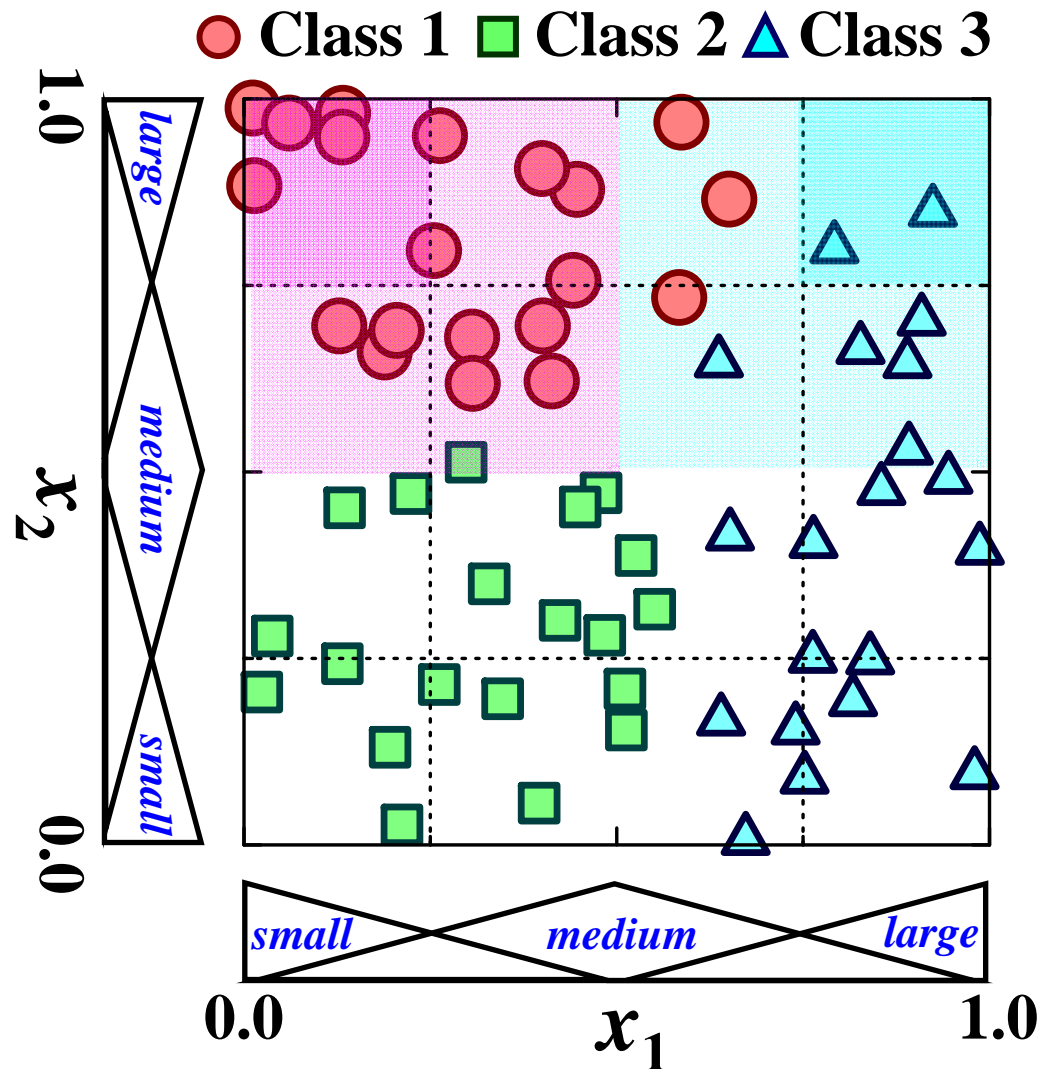
Fuzzy Rule-Based Systems have been successfully applied to pattern classification problems. In this type of classification systems, the classical Fuzzy Reasoning Method (FRM) classifies a new example with the consequent of the rule with ...

[Cited by 127 - Related articles - BL Direct](#) [O. Cordon et al., IJAR \(2001\)](#)

[Google Scholar](#)

Other Forms of Fuzzy Rules

Handling of Classification as Function Approximation



Integer Consequent

If x_1 is *small* and x_2 is *large*
then $y = 1$

If x_1 is *large* and x_2 is *large*
then $y = 3$

Binary Consequent

If x_1 is *small* and x_1 is *large*
then $(y_1, y_2, y_3) = (1, 0, 0)$

If x_1 is *large* and x_2 is *large*
then $(y_1, y_2, y_3) = (0, 0, 1)$

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2. Fuzzy Rule-Based Classifier Design

- **Accuracy Improvement**
- Scalability to High-Dimensional Problems
- Complexity Minimization

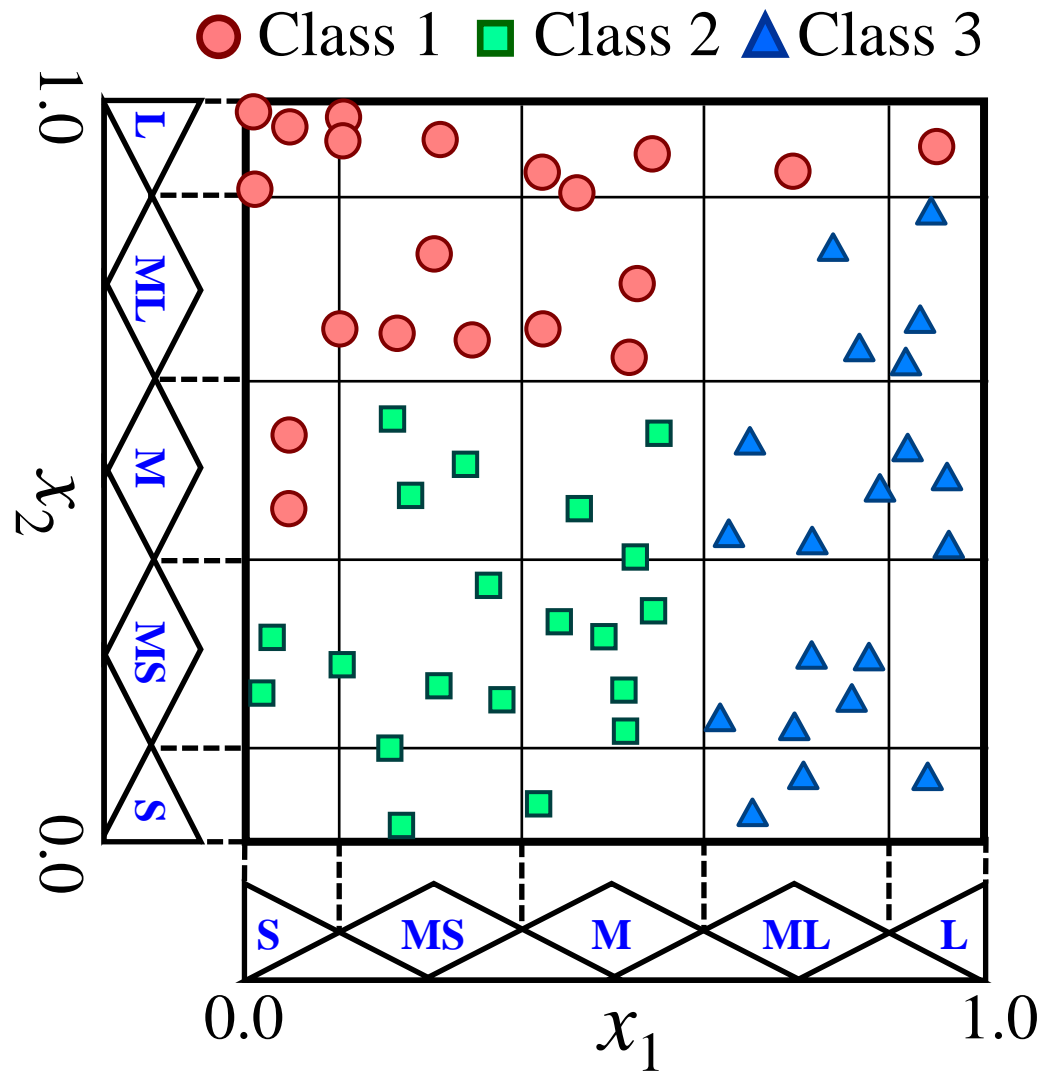
3. Multiobjective Fuzzy Rule-Based Classifier Design

- Formulation of Multi-objective Problems
- Accuracy-Complexity Tradeoff Analysis
- Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions

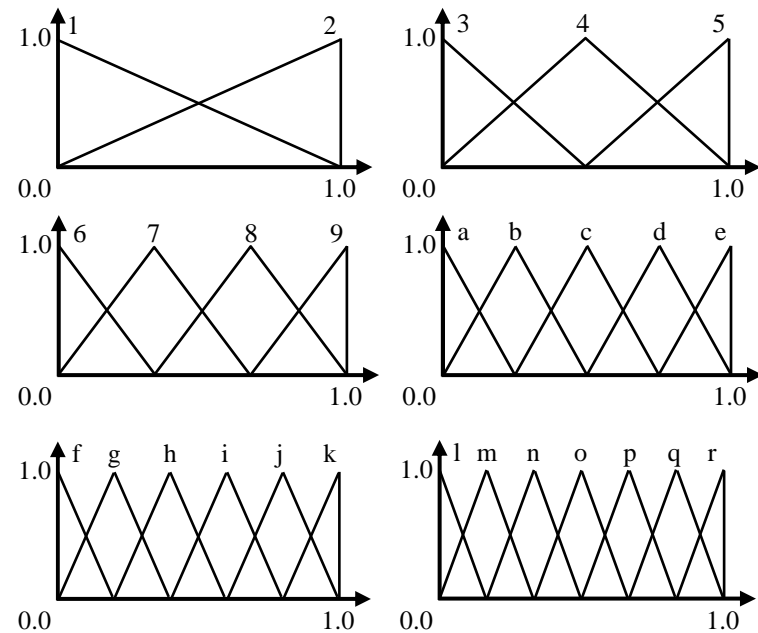
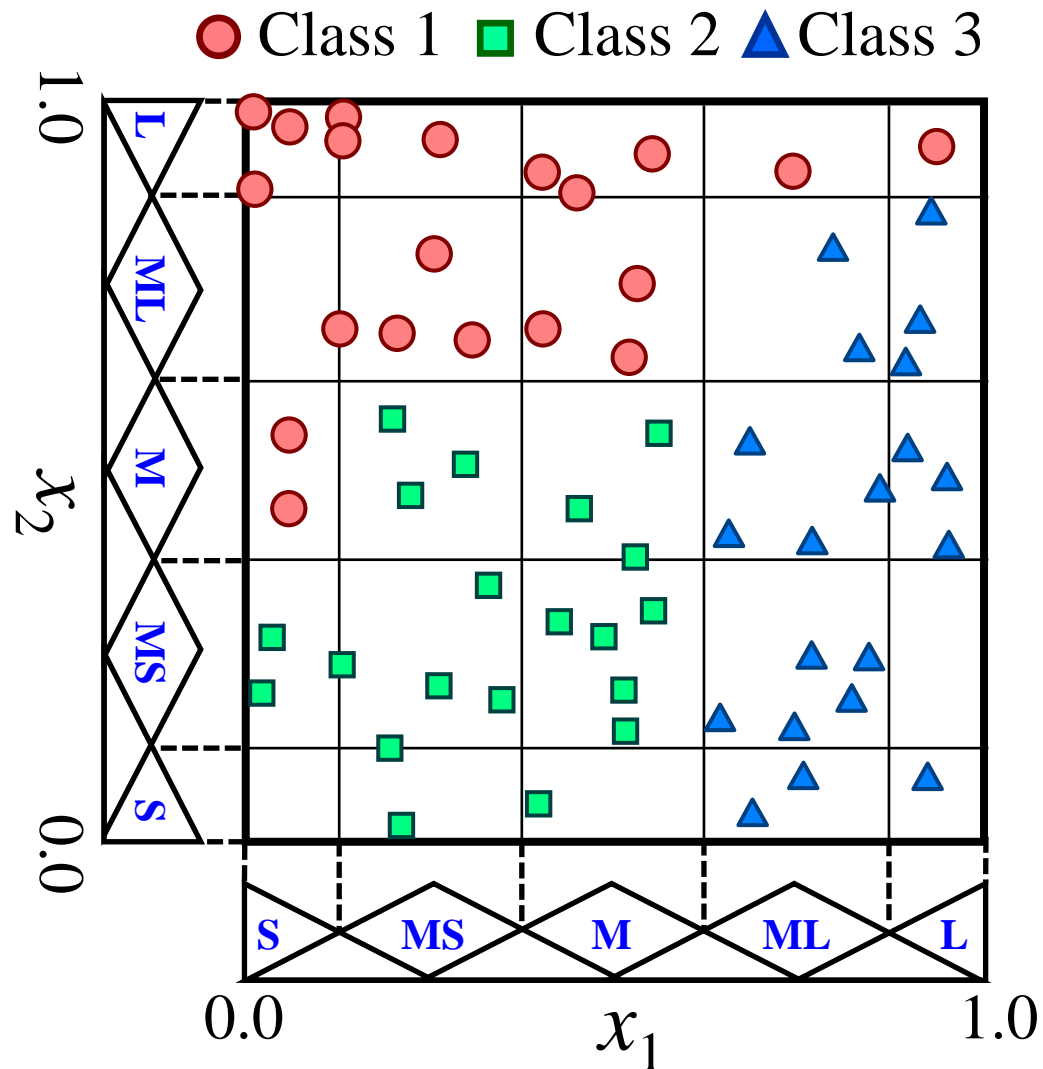
- Search Ability of EMO for Fuzzy System Design
- Definition of Interpretability of Fuzzy Systems
- Explanation Ability of Fuzzy Rule-Based Systems
- Various Classification Problems: Imbalanced, Online, ...

Accuracy Improvement Use of Fine Fuzzy Partition



Accuracy Improvement

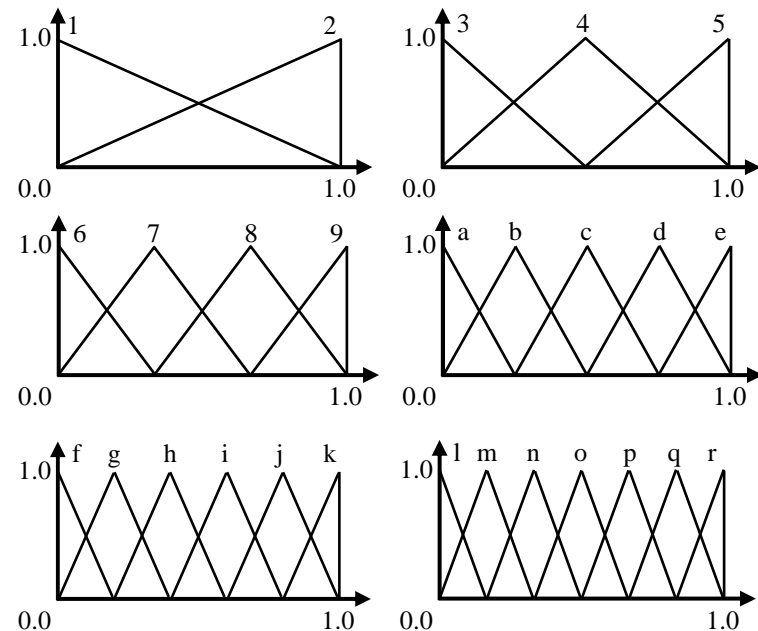
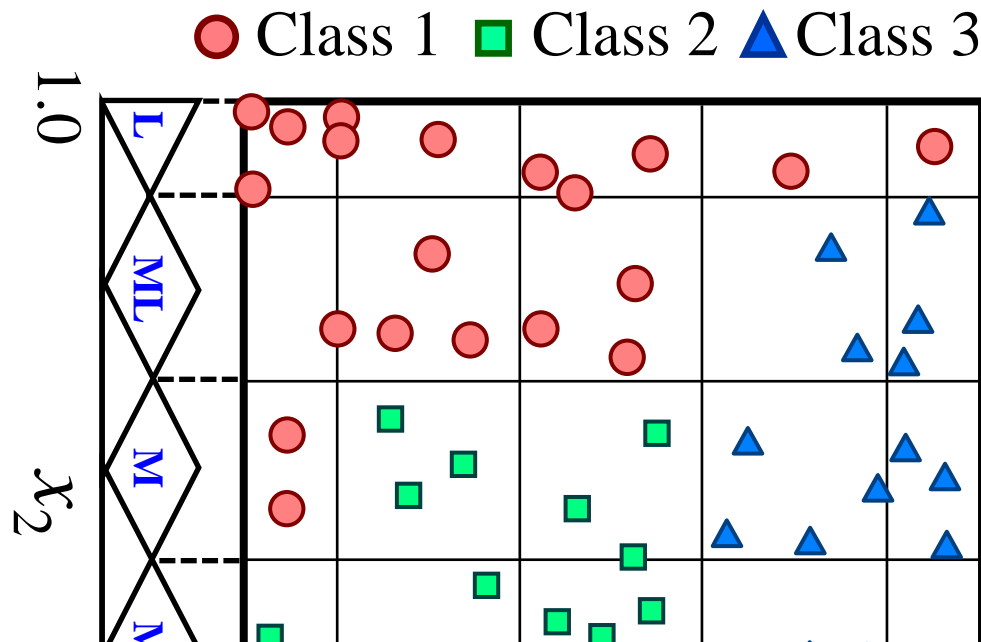
How to choose an appropriate partition ?



**Too Fine Fuzzy Partition
==> Over-Fitting
(Poor Generalization Ability)**

Accuracy Improvement

How to choose an appropriate partition ?



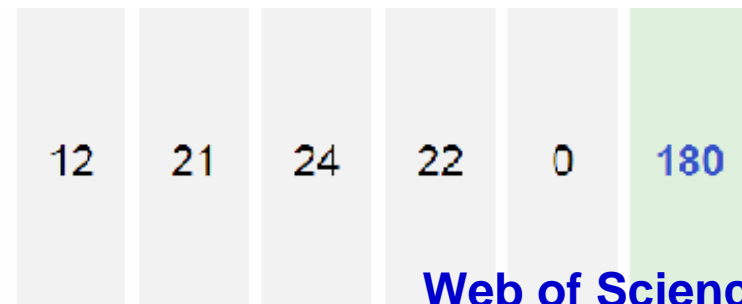
One Idea: Use of All Partitions (Multiple Fuzzy Grid Approach)

Title: DISTRIBUTED REPRESENTATION OF FUZZY RULES AND ITS APPLICATION TO PATTERN-CLASSIFICATION

Author(s): ISHIBUCHI H, NOZAKI K, TANAKA H

Source: **FUZZY SETS AND SYSTEMS** Volume: 52

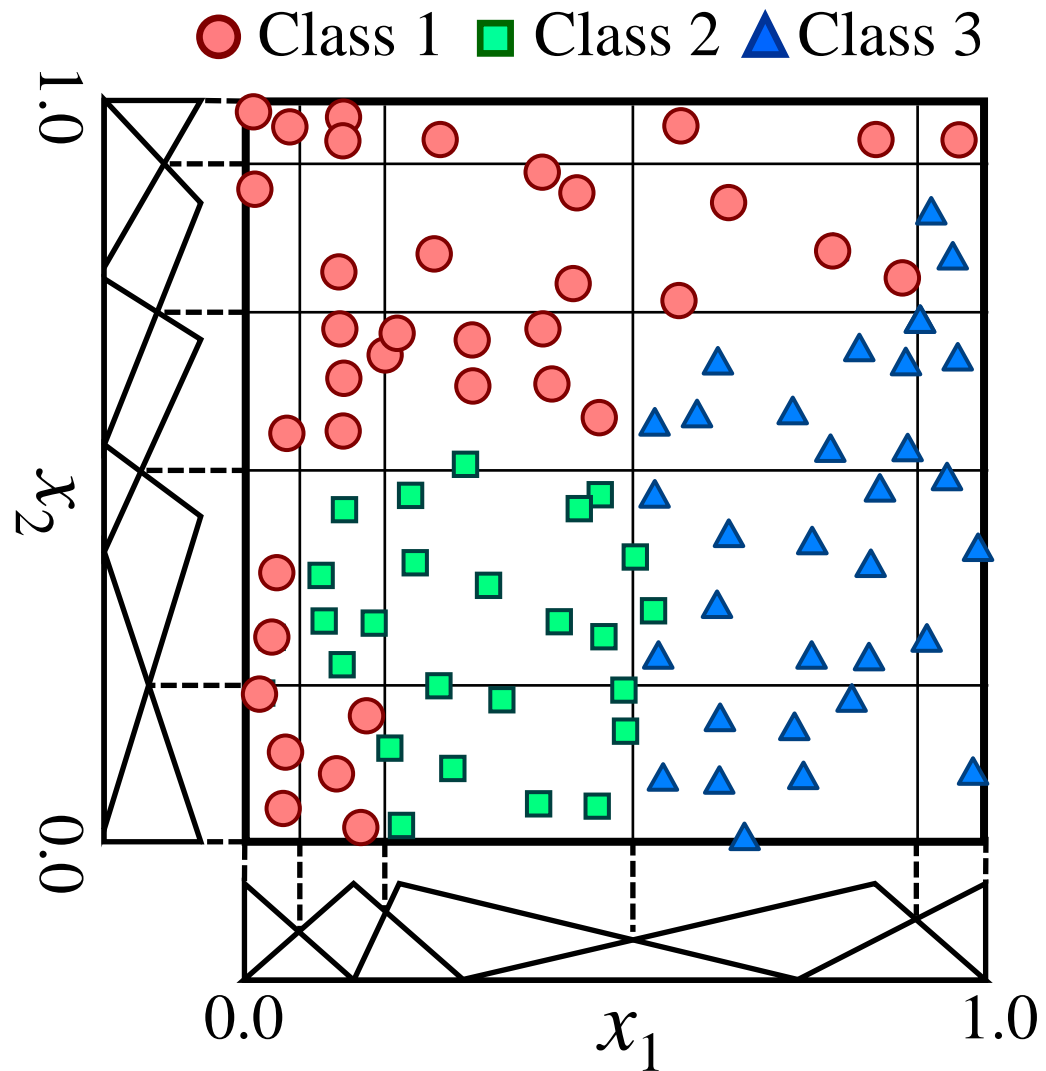
Issue: 1 Pages: 21-32 Published: NOV 25 1992



Web of Science

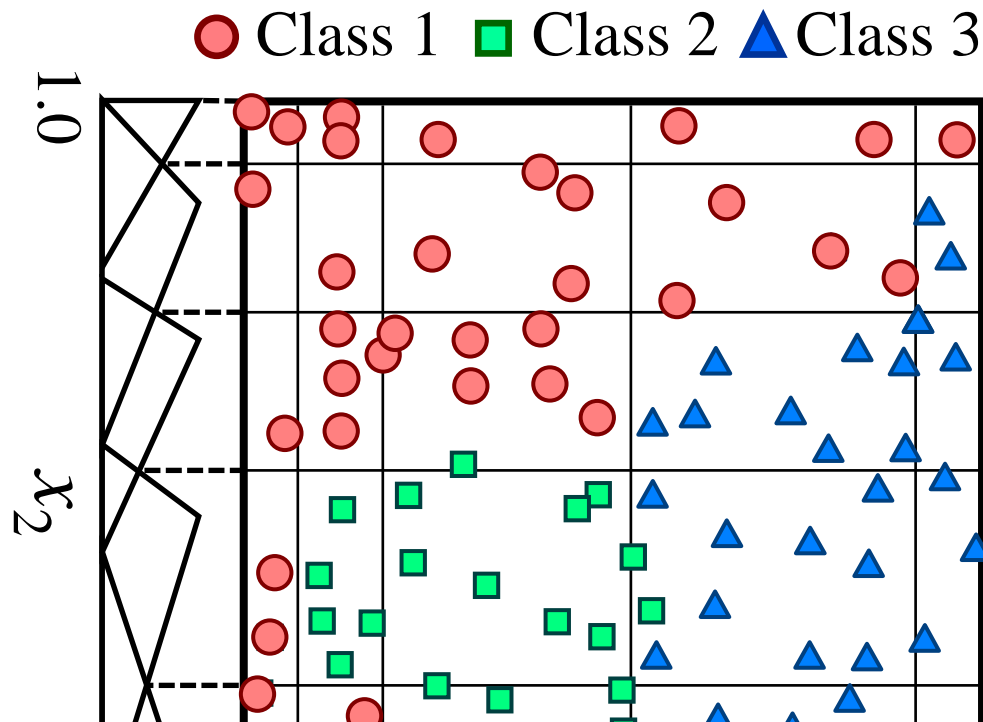
Ishibuchi et al., *Fuzzy Sets and Systems* (1992)

Accuracy Improvement Learning of Membership Functions



Accuracy Improvement

Learning of Membership Functions



Various learning methods such as neuro-fuzzy and genetic-fuzzy methods are available.

Title: A neuro-fuzzy method to learn fuzzy classification rules from data

Author(s): Nauck D, Kruse R

Source: **FUZZY SETS AND SYSTEMS** Volume: 89

Issue: 3 Pages: 277-288 Published: **AUG 1 1997**

14

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16

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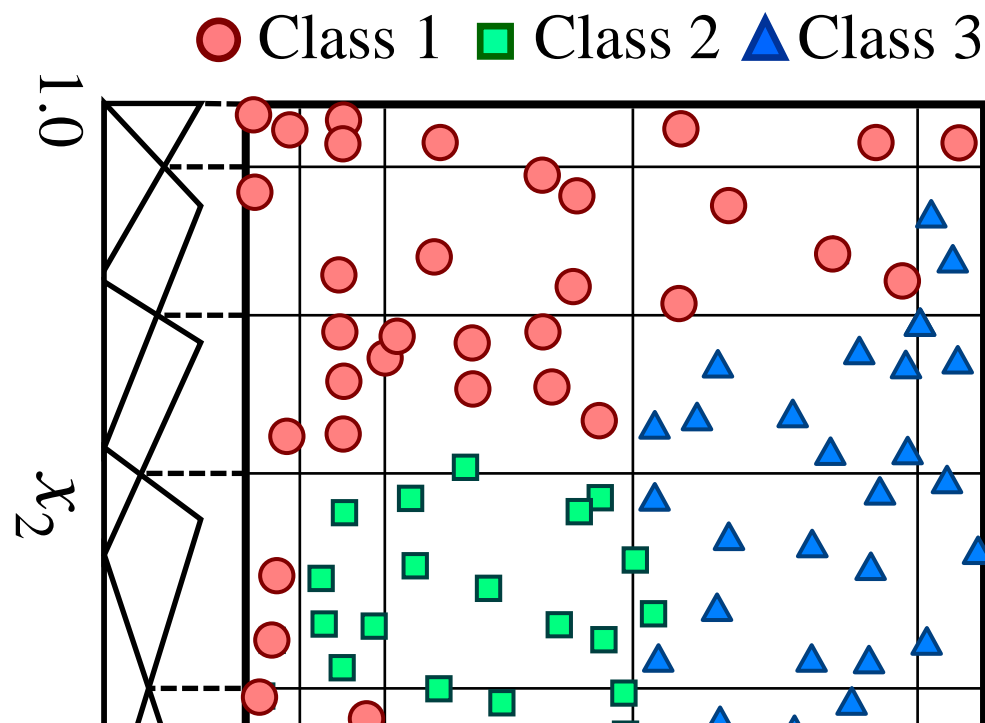
137

Web of Science

D. Nauck and R. Kruse, *Fuzzy Sets and Systems* (1997).

Accuracy Improvement

Learning of Membership Functions



Various learning methods such as neuro-fuzzy and genetic-fuzzy methods are available.

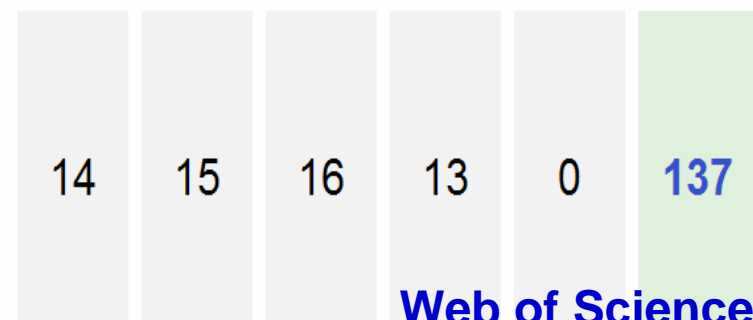
Interpretability is degraded.

Title: A neuro-fuzzy method to learn fuzzy classification rules from data

Author(s): Nauck D, Kruse R

Source: **FUZZY SETS AND SYSTEMS** Volume: 89

Issue: 3 Pages: 277-288 Published: **AUG 1 1997**

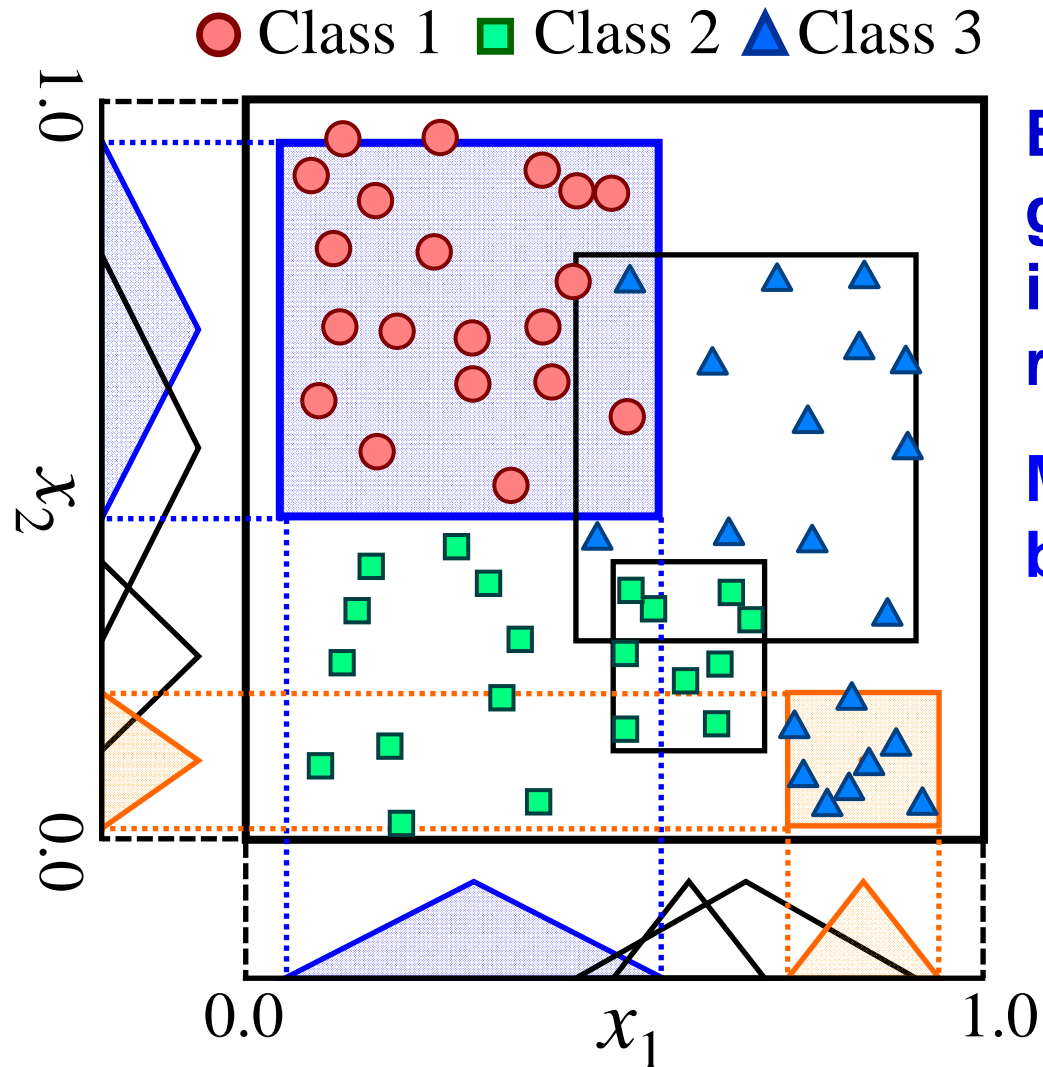


Web of Science

D. Nauck and R. Kruse, *Fuzzy Sets and Systems* (1997).

Accuracy Improvement

Use of Independent Membership Functions



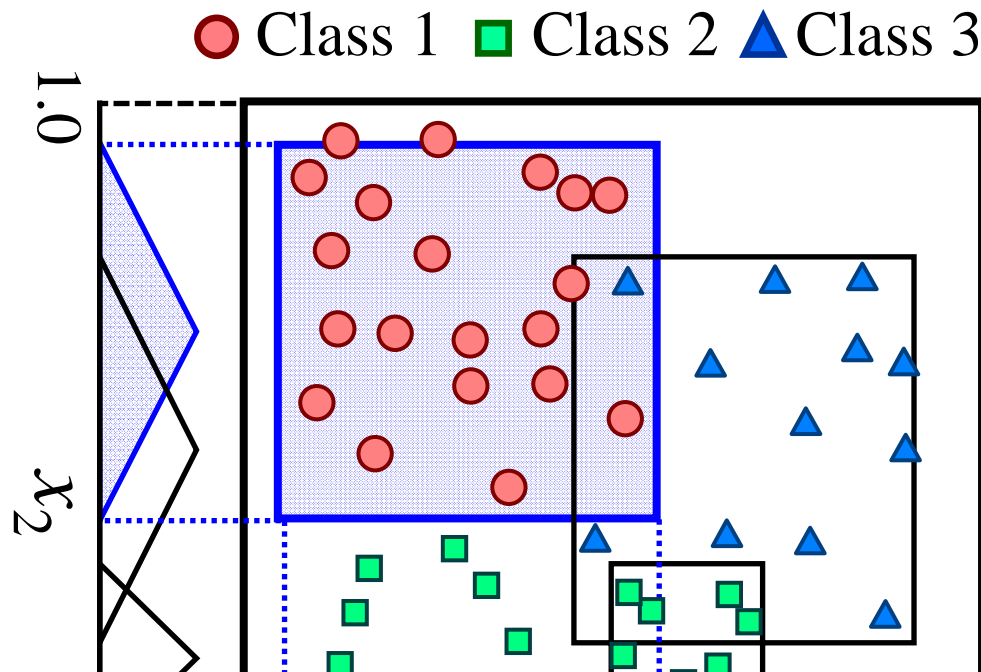
Each fuzzy rule can be generated and adjusted independently from other rules. **==> High Accuracy**

Membership functions can be heavily overlapping.

==> Poor Interpretability

Accuracy Improvement

Use of Independent Membership Functions



Each fuzzy rule can be generated and adjusted independently from other rules. ==> **High Accuracy**

Membership functions can be heavily overlapping.

==> **Poor Interpretability**

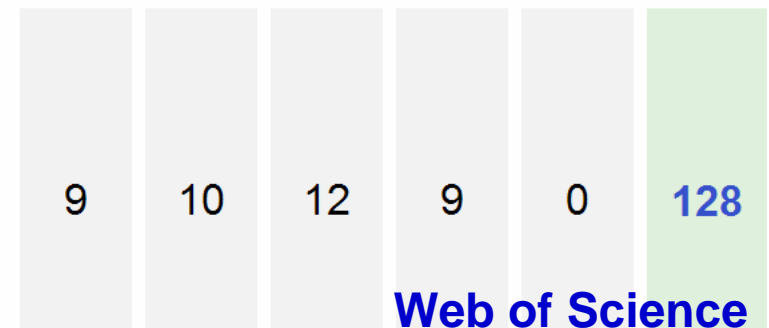
Title: A METHOD FOR FUZZY RULES EXTRACTION DIRECTLY FROM NUMERICAL DATA AND ITS APPLICATION TO PATTERN-CLASSIFICATION

Author(s): ABE S, LAN MS

Source: **IEEE TRANSACTIONS ON FUZZY SYSTEMS**

Volume: 3 Issue: 1 Pages: 18-28 Published: FEB

1995

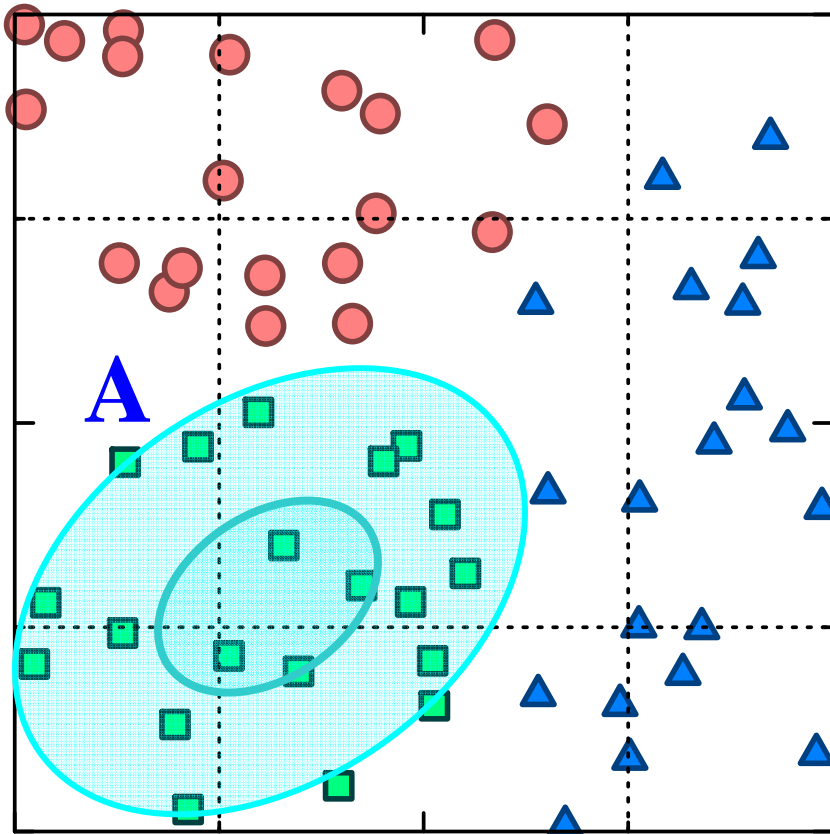


S. Abe and M. S. Lan, *IEEE Tras. on FS* (1995)

Accuracy Improvement

Use of Multi-Dimensional Membership Functions

● Class 1 ■ Class 2 ▲ Class 3



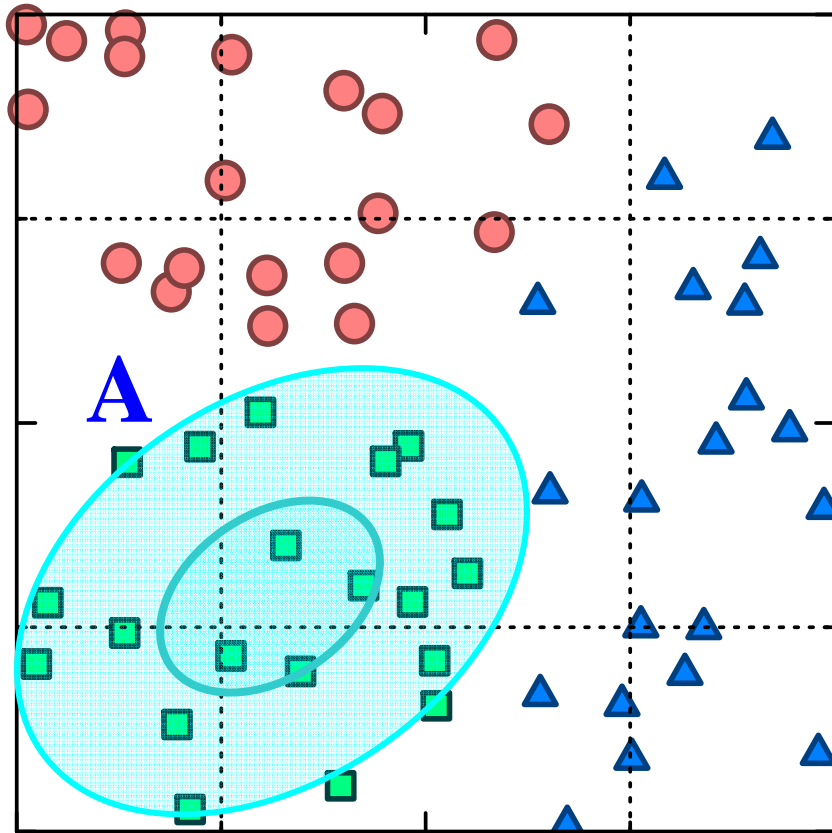
If x is A then Class 2

**A : Multi-dimensional Fuzzy Set
(Membership Function)**

Accuracy Improvement

Use of Multi-Dimensional Membership Functions

● Class 1 ■ Class 2 ▲ Class 3



If x is A then Class 2

**A: Multi-dimensional Fuzzy Set
(Membership Function)**

Fuzzy rules are flexibility.

==> High Accuracy

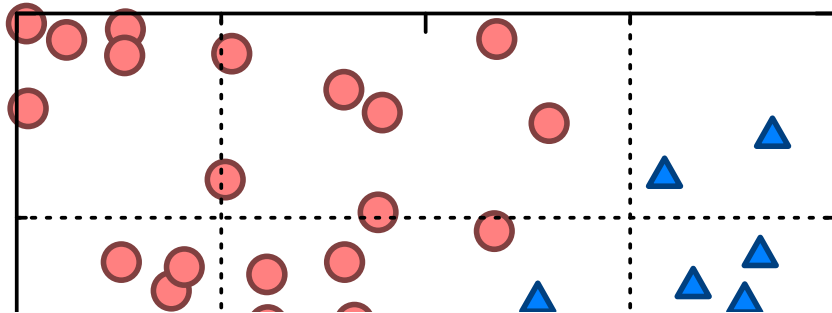
**Each membership function
is multi-dimensional.**

==> Poor Interpretability

Accuracy Improvement

Use of Multi-Dimensional Membership Functions

● Class 1 ■ Class 2 ▲ Class 3



If x is A then Class 2

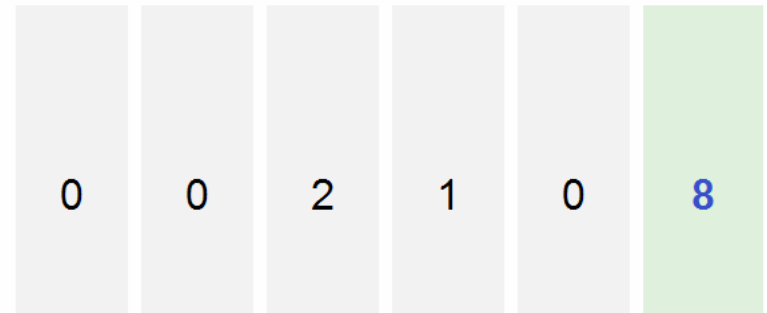
A : Multi-dimensional Fuzzy Set
(Membership Function)

Title: Feature selection by analyzing class regions approximated by ellipsoids

Author(s): Abe S, Thawonmas R, Kobayashi Y

Source: **IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS PART C-APPLICATIONS AND REVIEWS**

Volume: 28 Issue: 2 Pages: 282-287 Published: MAY 1998



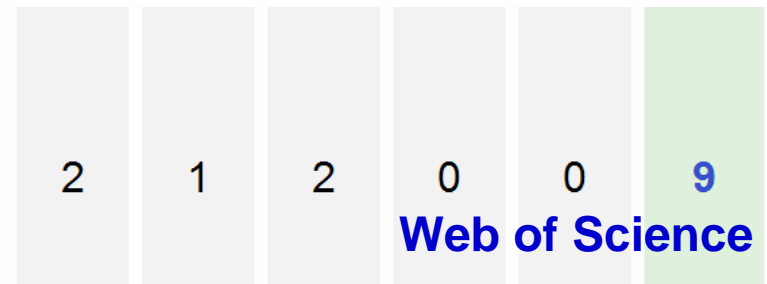
S. Abe et al., *IEEE TSMC-C* (1998)

Title: A fuzzy classifier with ellipsoidal regions for diagnosis problems

Author(s): Abe S, Thawonmas R, Kayama M

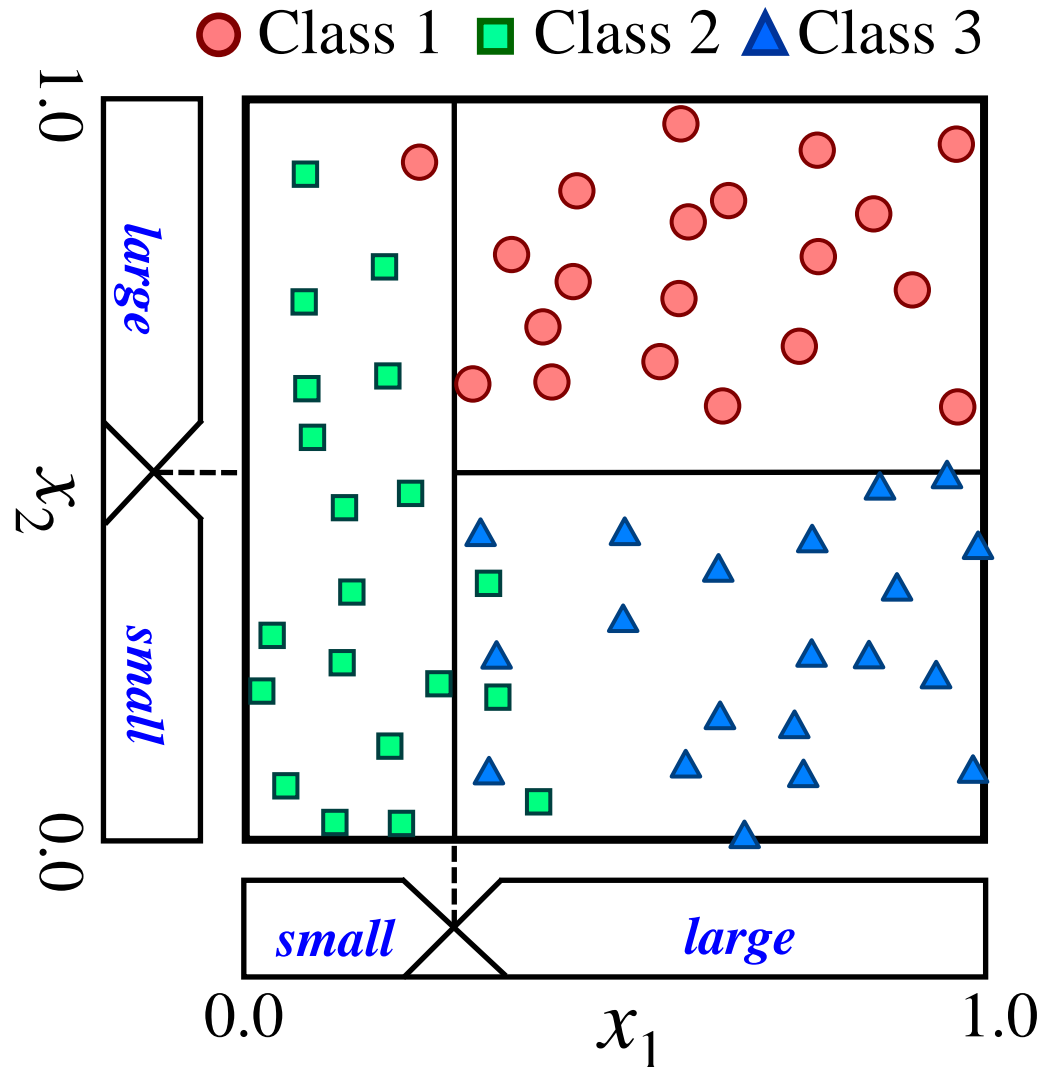
Source: **IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS PART C-APPLICATIONS AND REVIEWS**

Volume: 29 Issue: 1 Pages: 140-149 Published: FEB 1999



S. Abe et al., *IEEE TSMC-C* (1999)

Accuracy Improvement Use of Tree-Type Fuzzy Partitions



If x_1 is *small* then Class 2.

If x_1 is *large* and x_2 is *small*
then Class 3.

If x_1 is *large* and x_2 is *large*
then Class 1.

Accuracy Improvement Use of Tree-Type Fuzzy Partitions

Title: **INDUCTION OF FUZZY DECISION TREES**

Author(s): YUAN YF, SHAW MJ

Source: **FUZZY SETS AND SYSTEMS** Volume: **69**

Issue: **2** Pages: **125-139** Published: **JAN 27 1995**

27	28	35	25	0	232
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Title: **Fuzzy decision trees: Issues and methods**

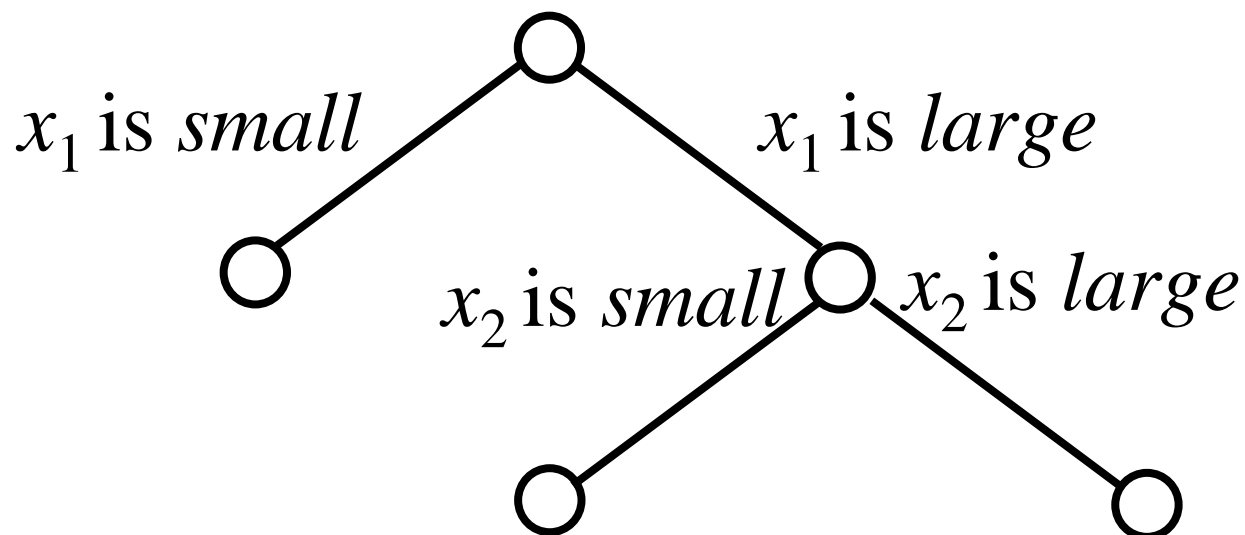
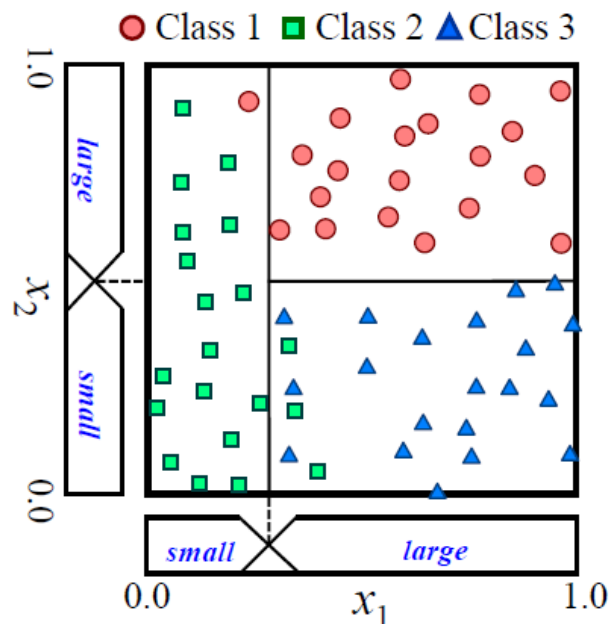
Author(s): Janikow CZ

Source: **IEEE TRANSACTIONS ON SYSTEMS MAN AND CYBERNETICS PART B-CYBERNETICS** Volume: **28**

Issue: **1** Pages: **1-14** Published: **FEB 1998**

21	24	25	23	0	179
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3. Multiobjective Fuzzy Rule-Based Classifier Design

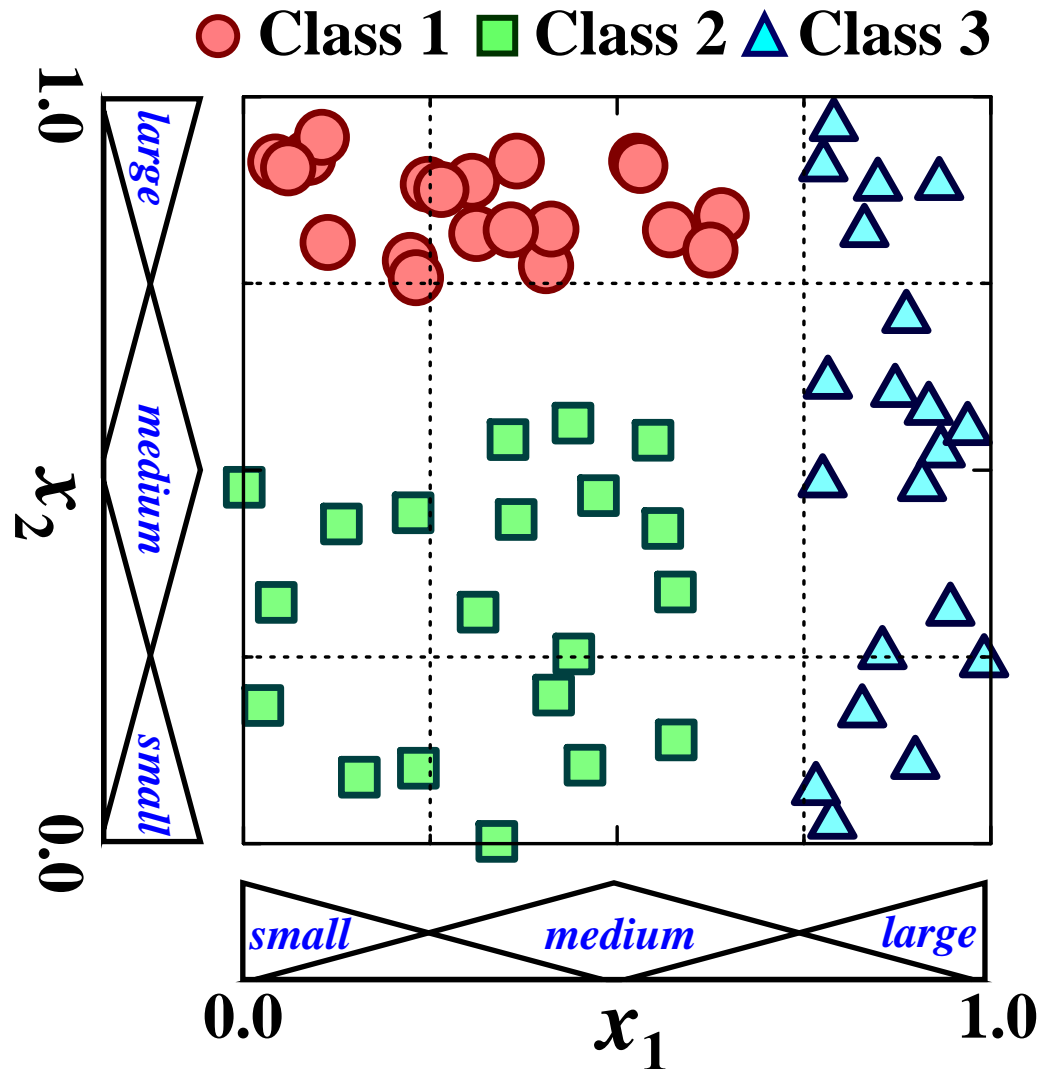
- Formulation of Multi-objective Problems
- Accuracy-Complexity Tradeoff Analysis
- Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions

- Search Ability of EMO for Fuzzy System Design
- Definition of Interpretability of Fuzzy Systems
- Explanation Ability of Fuzzy Rule-Based Systems
- Various Classification Problems: Imbalanced, Online, ...

Difficulty of High-Dimensional Problems

Exponential Increase of Fuzzy Rules



Basic Form

If x_1 is *small* and x_2 is *small*
then Class 2

If x_1 is *small* and x_2 is *medium*
then Class 2

...

If x_1 is *large* and x_2 is *large*
then Class 3

Number of Fuzzy Rules:

2-D Problem: 3×3

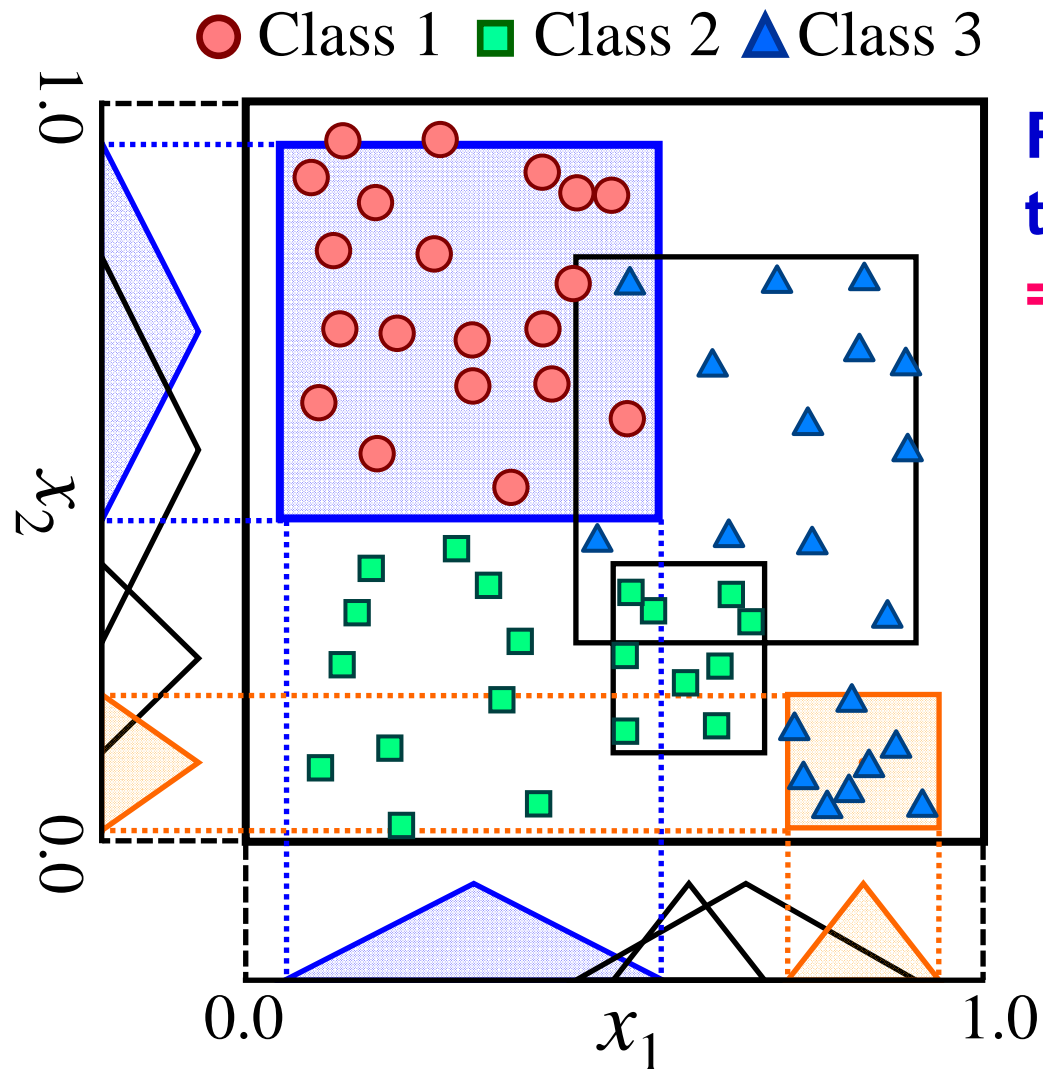
3-D Problem: $3 \times 3 \times 3$

4-D Problem: $3 \times 3 \times 3 \times 3$

5-D Problem: $3 \times 3 \times 3 \times 3 \times 3$

Scalability Improvement

Use of Independent Membership Functions

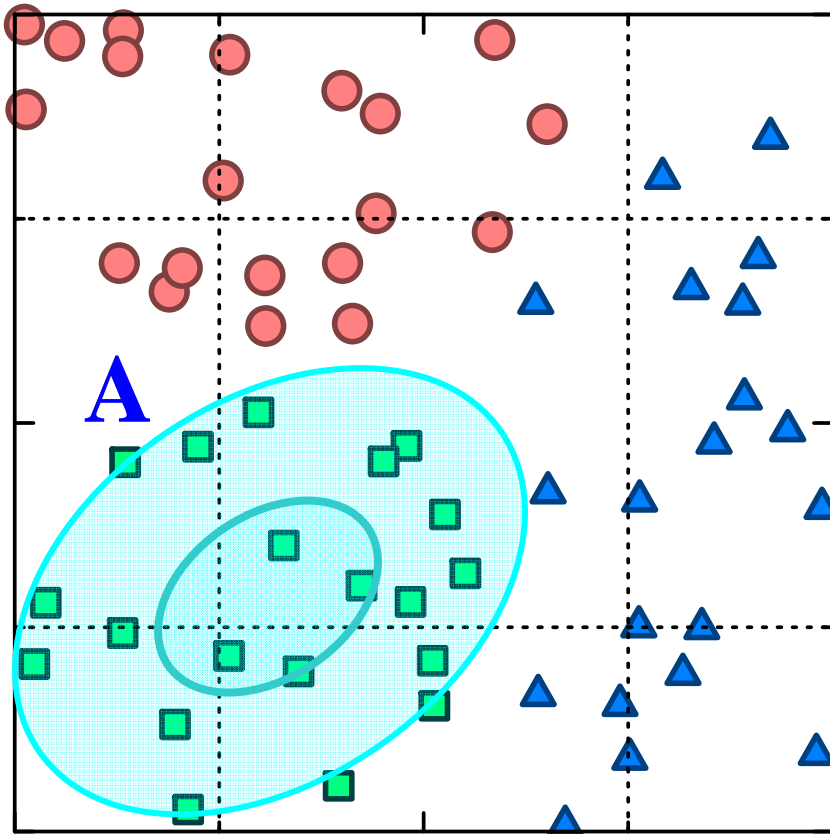


Fuzzy rules are generated in the multi-dimensional space.
=> No Exponential Increase

Scalability Improvement

Use of Multi-Dimensional Membership Functions

● Class 1 ■ Class 2 ▲ Class 3



If x is A then Class 2

A : Multi-dimensional Fuzzy Set
(Membership Function)

Fuzzy rules are generated in
the multi-dimensional space.
=> No Exponential Increase

Scalability Improvement

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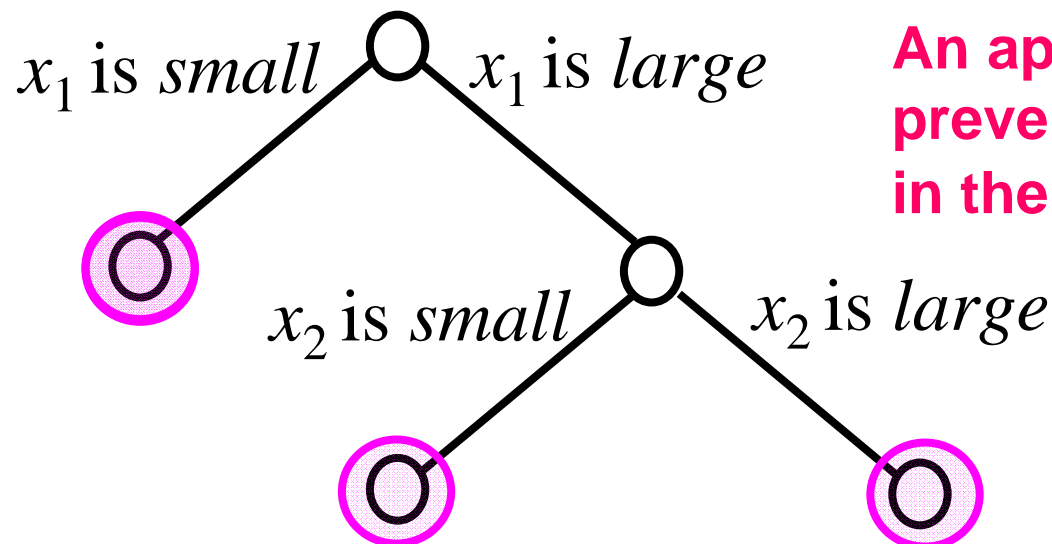
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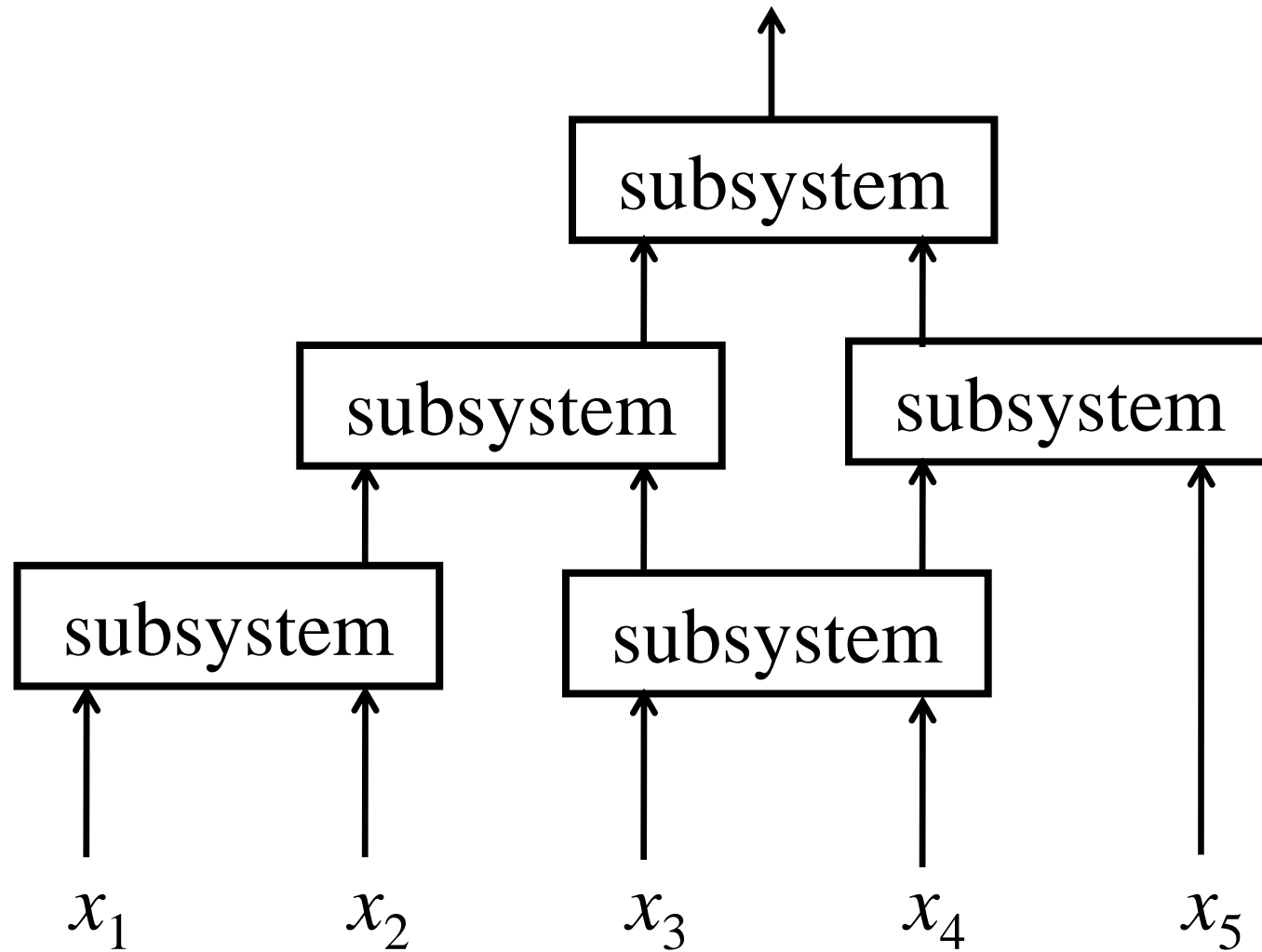
Web of Science



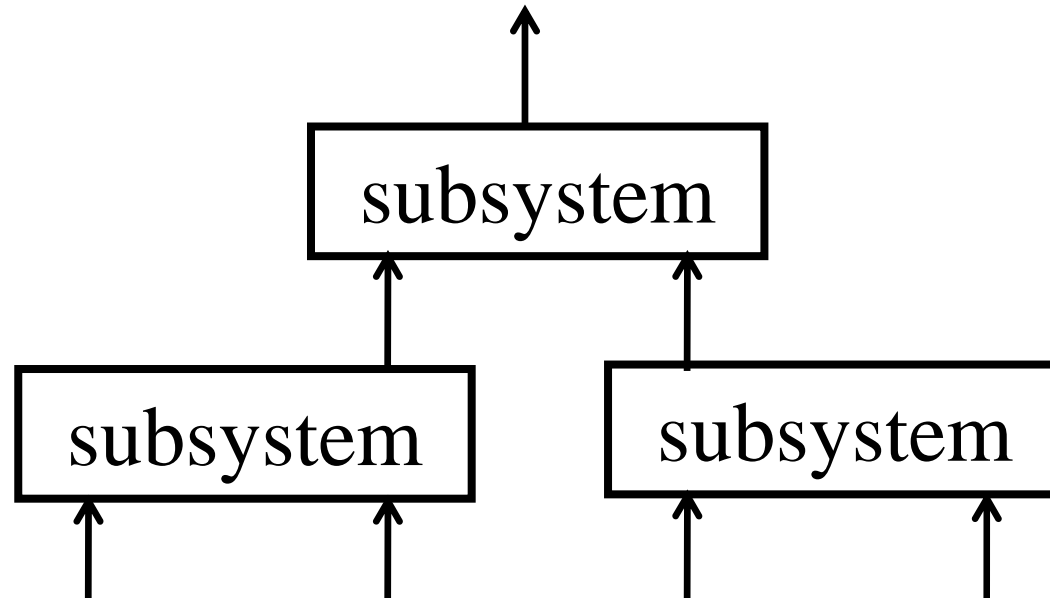
An appropriate stopping condition prevents the exponential increase in the number of fuzzy rules.

Scalability Improvement

Hierarchical Fuzzy Systems



Scalability Improvement Hierarchical Fuzzy Systems

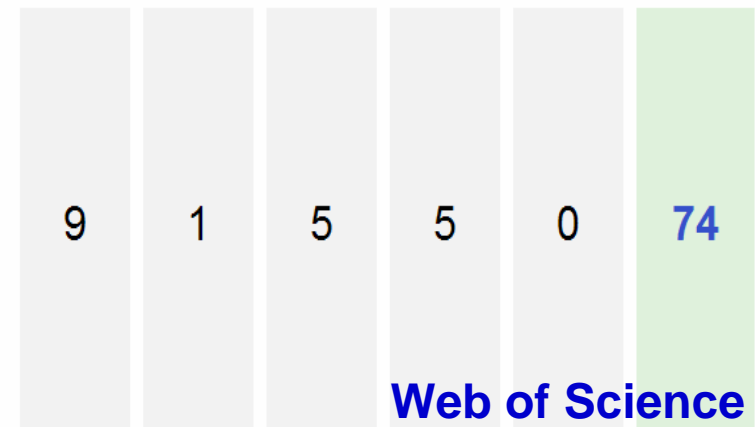


Title: SELF-TUNING FUZZY MODELING WITH
ADAPTIVE MEMBERSHIP FUNCTION, RULES, AND
HIERARCHICAL STRUCTURE-BASED ON GENETIC
ALGORITHM

Author(s): SHIMOJIMA K, FUKUDA T, HASEGAWA Y

Source: **FUZZY SETS AND SYSTEMS** Volume: 71

Issue: 3 Pages: 295-309 Published: MAY 12 1995



K. Simojima et al., *Fuzzy Sets and Systems* (1995)

Accuracy & Scalability Improvement ==> Poor Interpretability

- Use of Fine Fuzzy Partition (Accuracy)
- Use of Rule Weights (Accuracy)
- Membership Function Learning (Accuracy)
- Fuzzy Rules with Independent Membership Functions
(Accuracy and Scalability)
- Multi-Dimensional Fuzzy Systems
(Accuracy and Scalability)
- Tree-Type Fuzzy Partitions (Accuracy and Scalability)
- Hierarchical Fuzzy Systems (Accuracy and Scalability)

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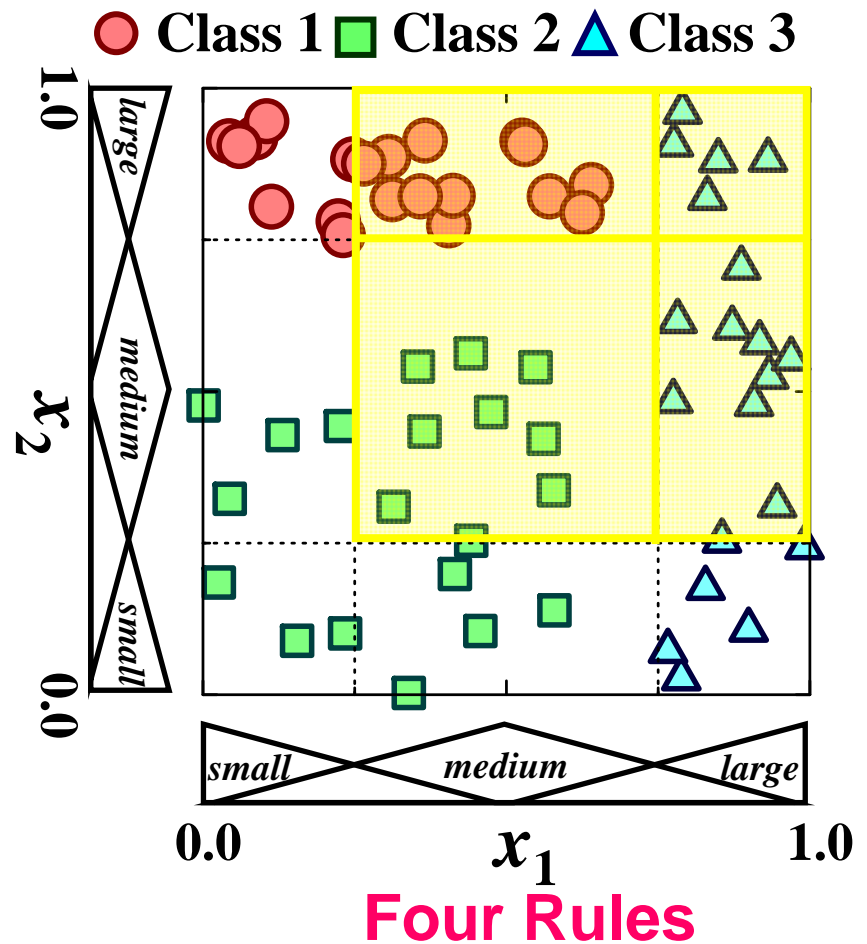
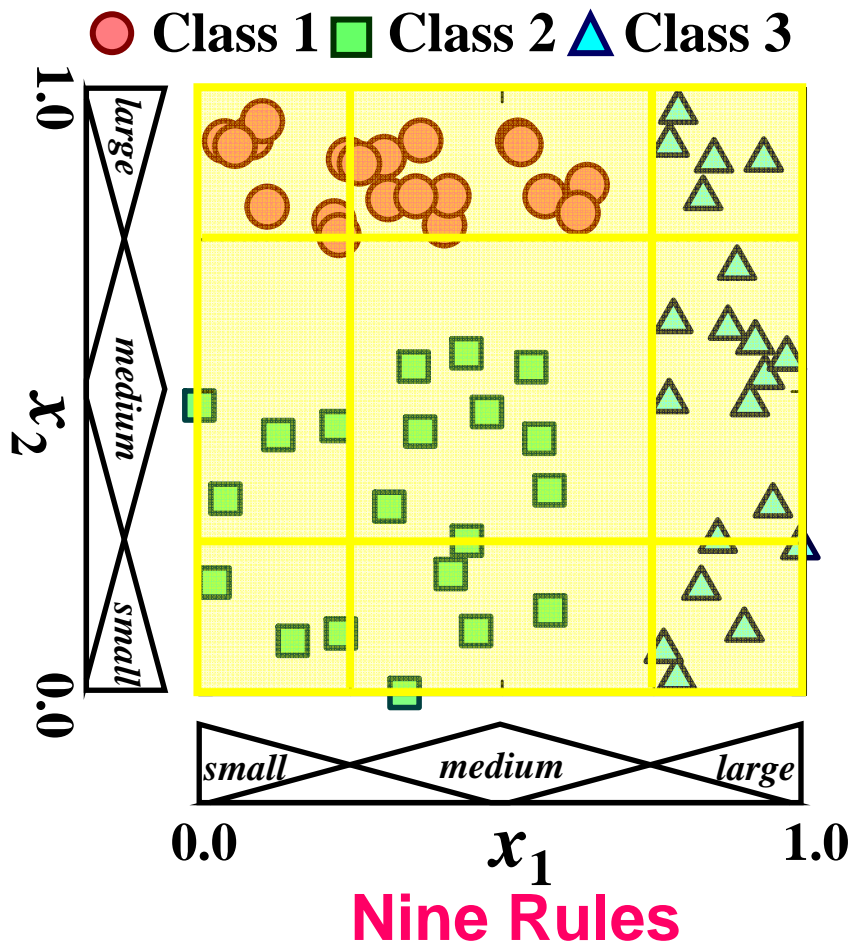
4. Current Hot Issues and Future Research Directions

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Complexity Minimization

Fuzzy Rule Selection

Rule Selection (Nine Rules ==> Four Rules)

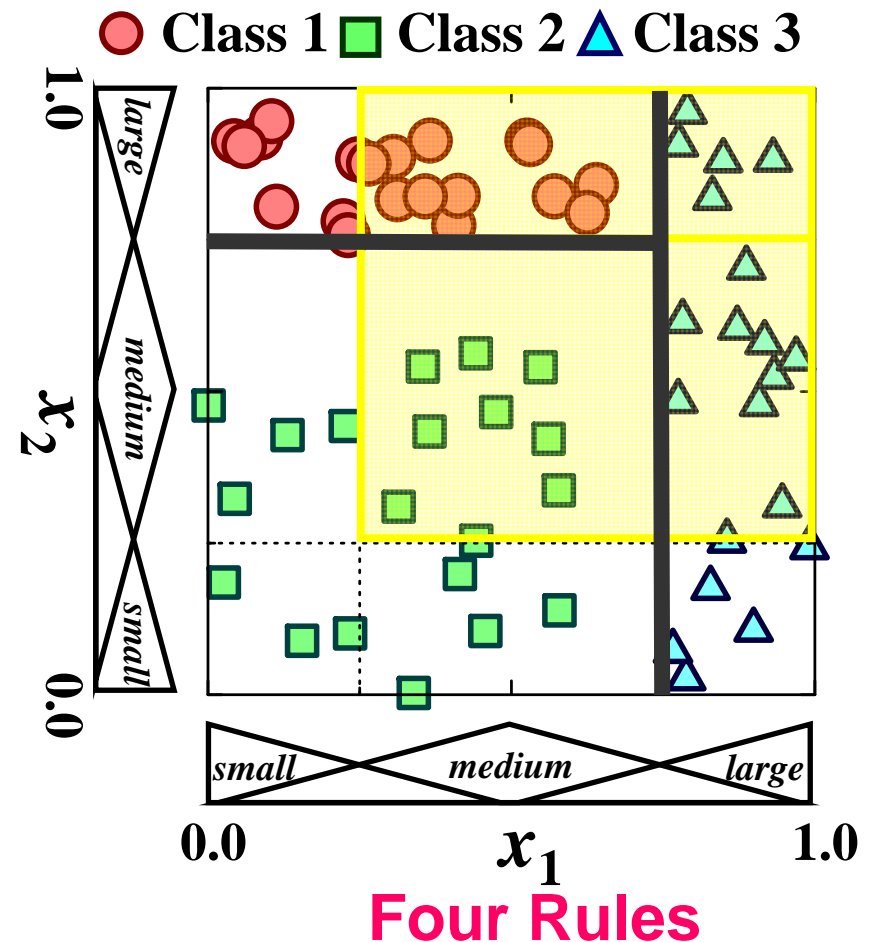
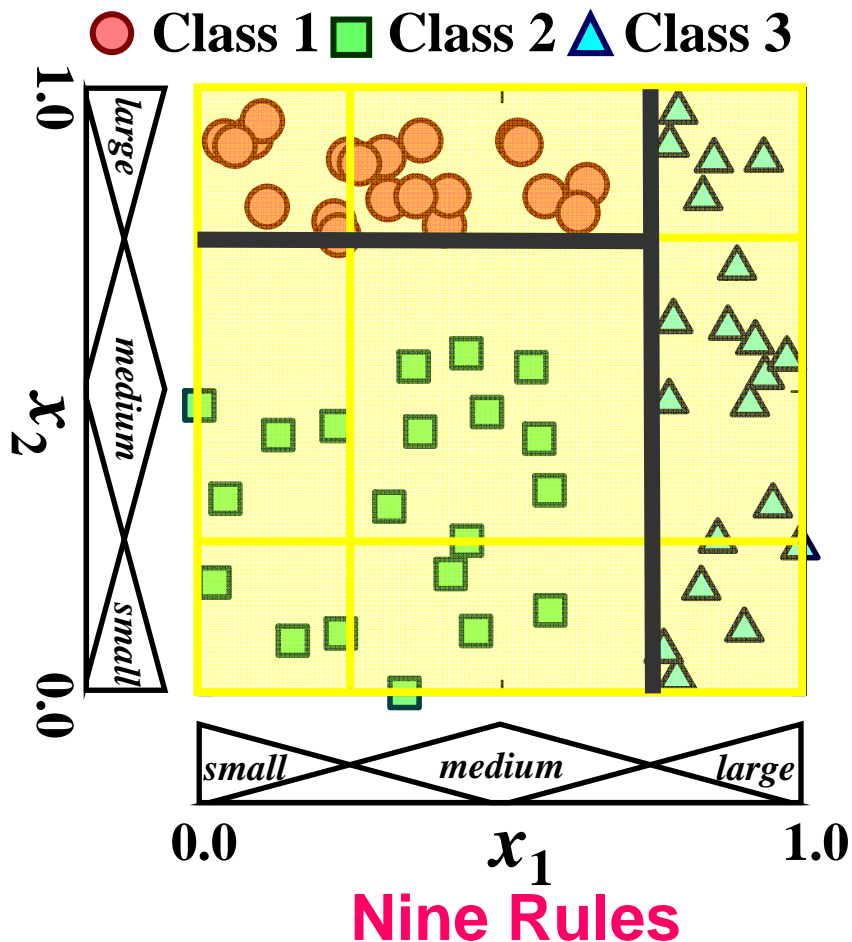


Complexity Minimization

Fuzzy Rule Selection

Rule Selection (Nine Rules ==> Four Rules)

The same classification boundaries are generated.

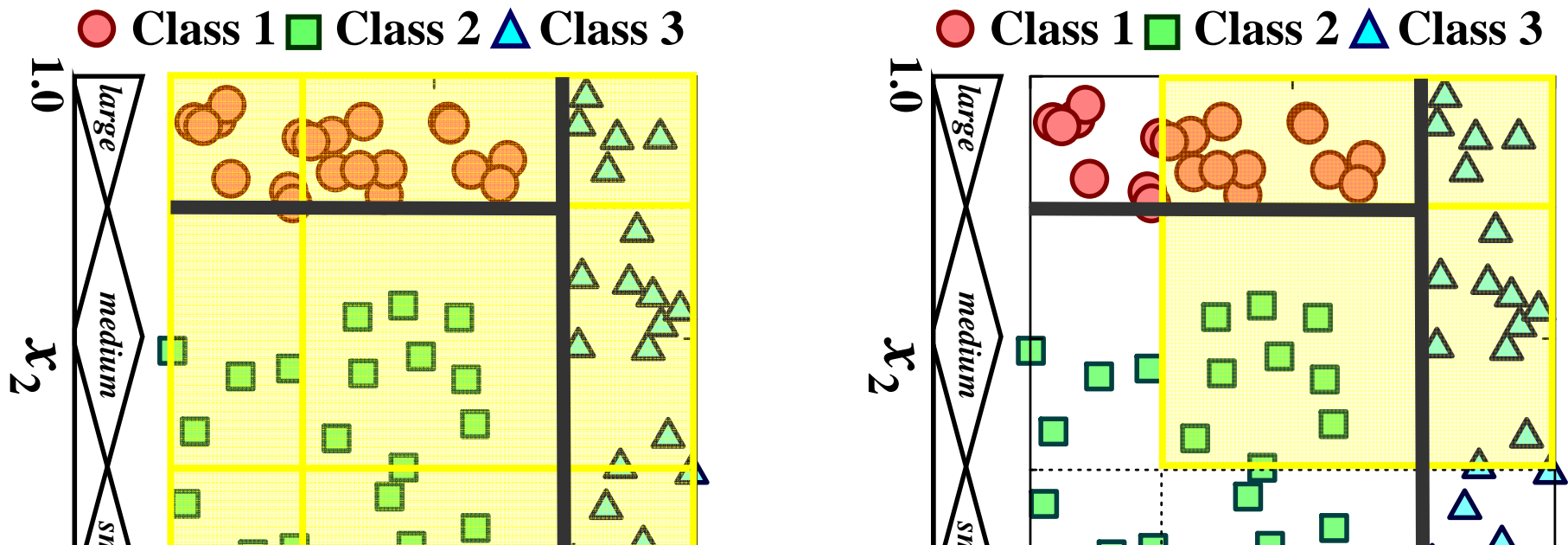


Complexity Minimization

Fuzzy Rule Selection

Rule Selection (Nine Rules ==> Four Rules)

The same classification boundaries are generated.



Title: SELECTING FUZZY IF-THEN RULES FOR CLASSIFICATION PROBLEMS USING GENETIC ALGORITHMS

Author(s): ISHIBUCHI H, NOZAKI K, YAMAMOTO N, et al.

Web of Science

Source: IEEE TRANSACTIONS ON FUZZY SYSTEMS Volume: 3 Issue: 3 Pages: 260-270

Published: AUG 1995

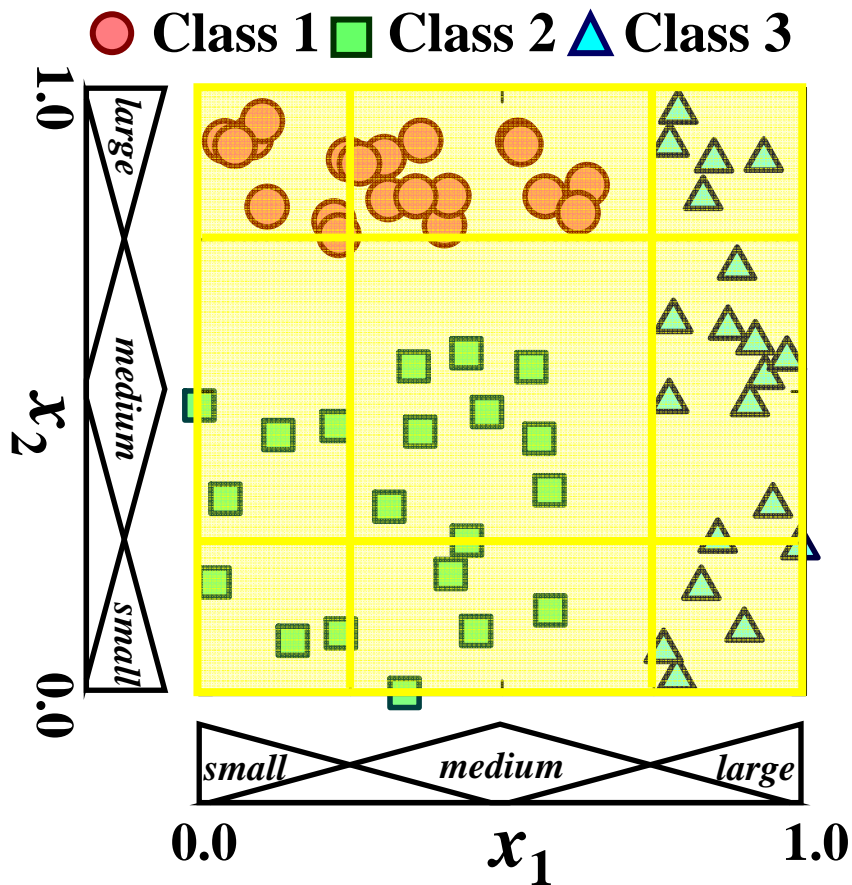
H. Ishibuchi et al., *IEEE Trans. on FS* (1995)

Complexity Minimization

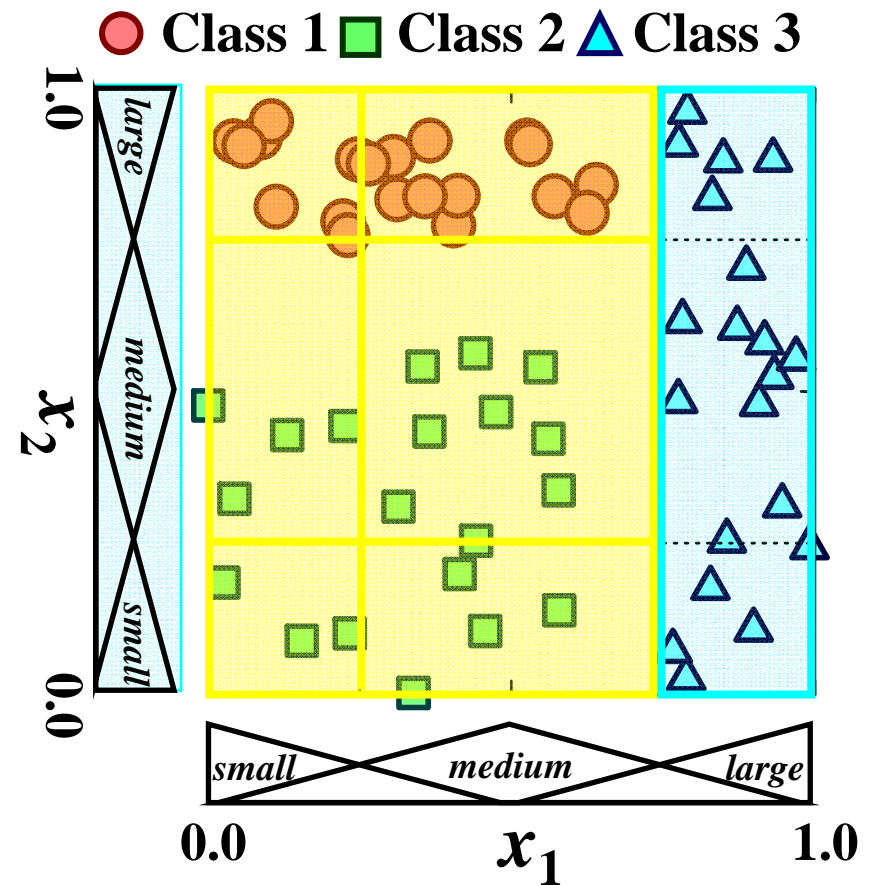
Use of “Don’t Care” Conditions

Use of “Don’t Care” Conditions

Nine Rules ==> Seven Rules (If x_1 is *large* then Class 3)



Nine Rules



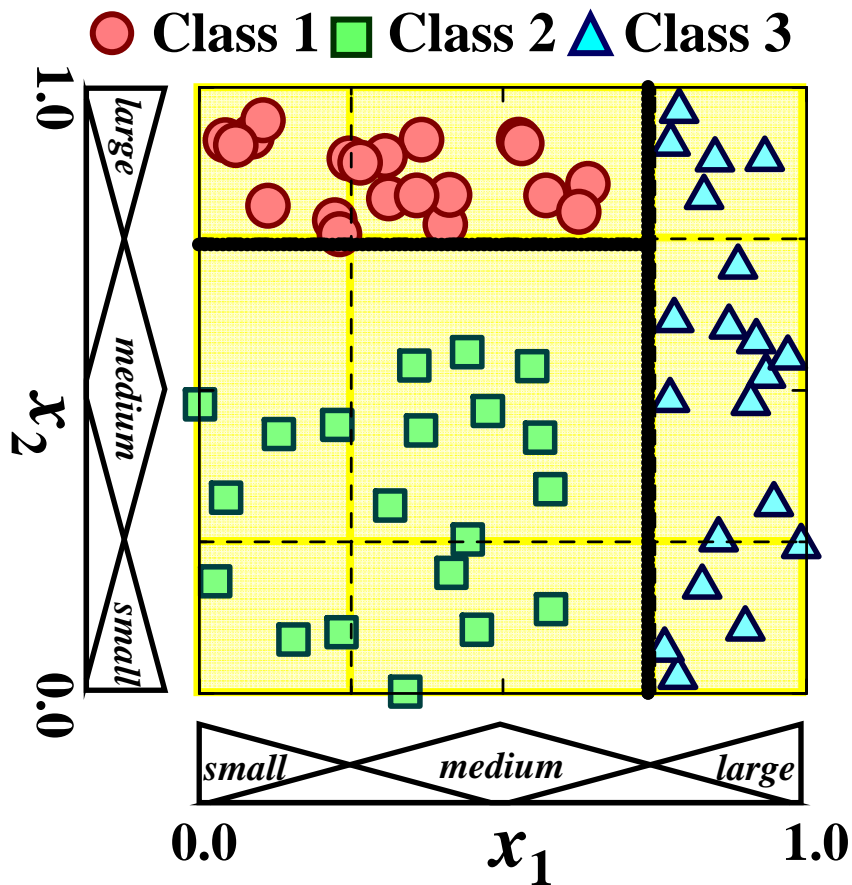
Seven Rules

Complexity Minimization

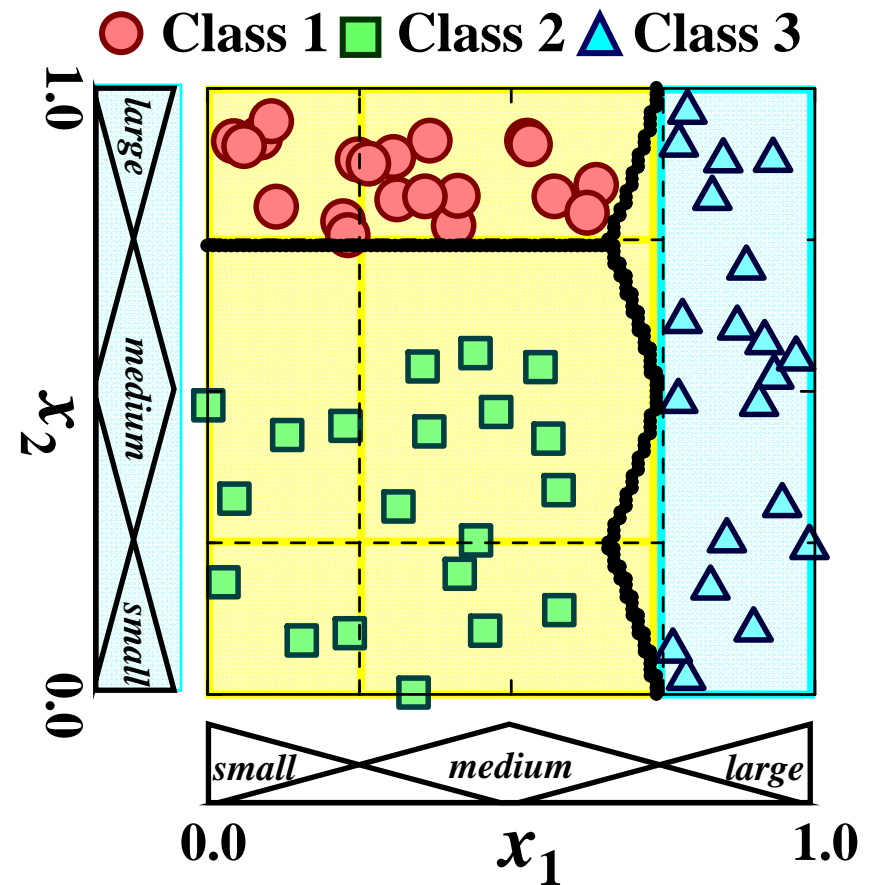
Use of “Don’t Care” Conditions

Nine Rules ==> Seven Rules (If x_1 is *large* then Class 3)

Slightly different classification boundaries are obtained.



Nine Rules



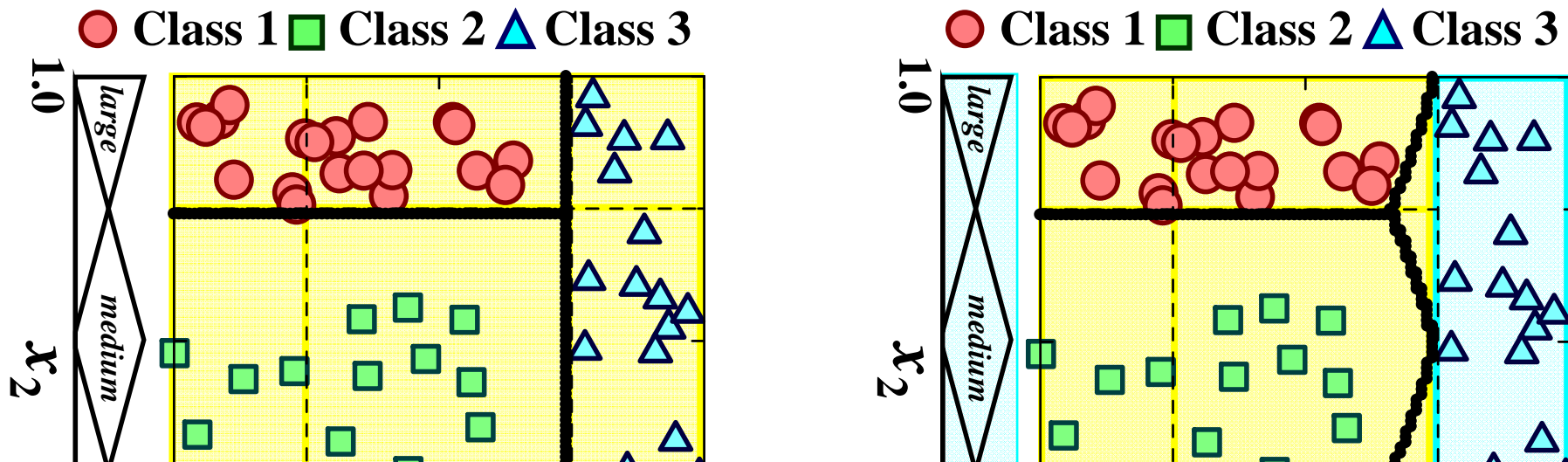
Seven Rules

Complexity Minimization

Use of “Don’t Care” Conditions

Nine Rules ==> Seven Rules (If x_1 is *large* then Class 3)

Slightly different classification boundaries are obtained.



[PDF] ► [Performance evaluation of fuzzy classifier systems for multidimensional](#)

[H Ishibuchi, T Nakashima, T Murata - IEEE Transactions on Systems, Man, and Cybernetics.](#)

Abstract— We examine the performance of a fuzzy genetics- based machine learning method for multidimensional pattern classification problems with continuous attributes. In our method, each fuzzy if-then rule is handled ...

[Google Scholar](#)

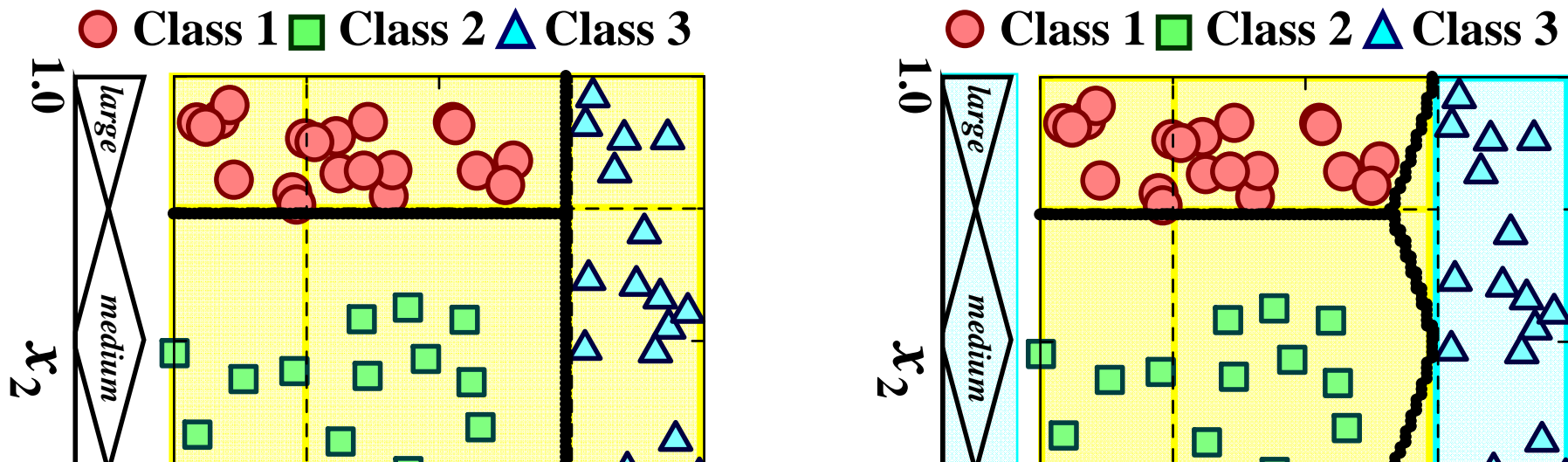
[Cited by 244](#)

[H. Ishibuchi et al., IEEE Trans. on SMC Part B \(1999\)](#)

Complexity Minimization

Use of “Don’t Care” Conditions

The use of “Don’t Care” conditions can be viewed as an scalability improvement method. If x_1 is *small* and x_{10} is ...



[PDF] ► [Performance evaluation of fuzzy classifier systems for multidimensional](#)
H Ishibuchi, T Nakashima, T Murata - IEEE Transactions on Systems, Man, and Cybernetics.

Abstract— We examine the performance of a fuzzy genetics- based machine learning method for multidimensional pattern classification problems with continuous attributes. In our method, each fuzzy if–then rule is handled ...

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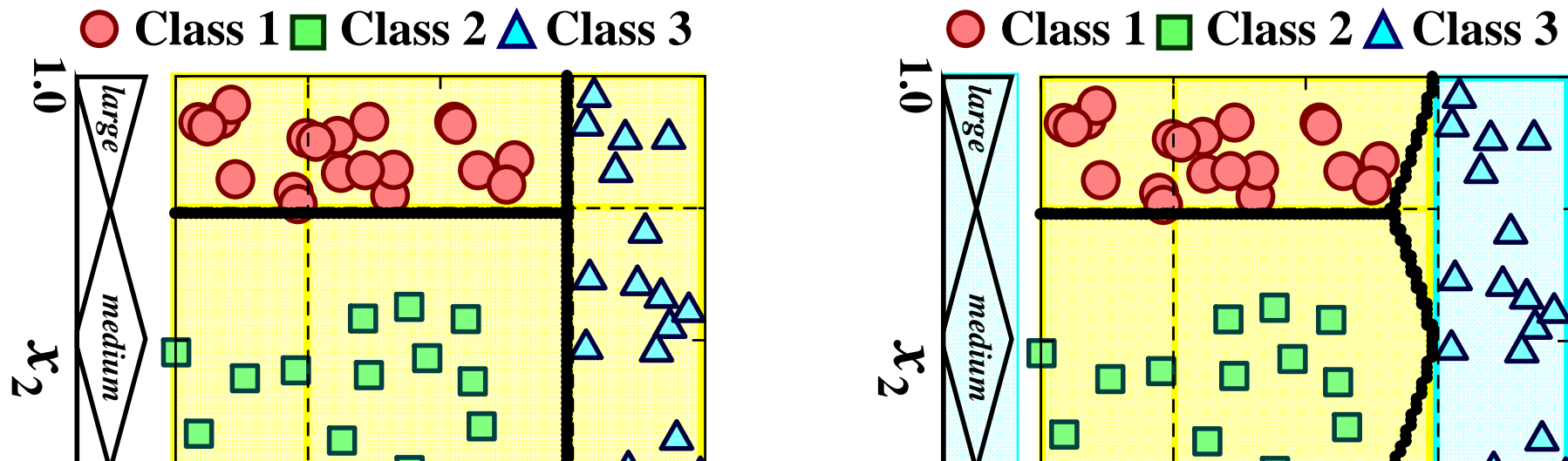
[Cited by 244](#)

H. Ishibuchi et al., *IEEE Trans. on SMC Part B* (1999)

Complexity Minimization

Use of “Don’t Care” Conditions

The use of “Don’t Care” conditions can be also viewed as input selection for each rule (rule-wise input selection).



[PDF] ► [Performance evaluation of fuzzy classifier systems for multidimensional](#)
H Ishibuchi, T Nakashima, T Murata - IEEE Transactions on Systems, Man, and Cybernetics.

Abstract— We examine the performance of a fuzzy genetics- based machine learning method for multidimensional pattern classification problems with continuous attributes. In our method, each fuzzy if-then rule is handled ...

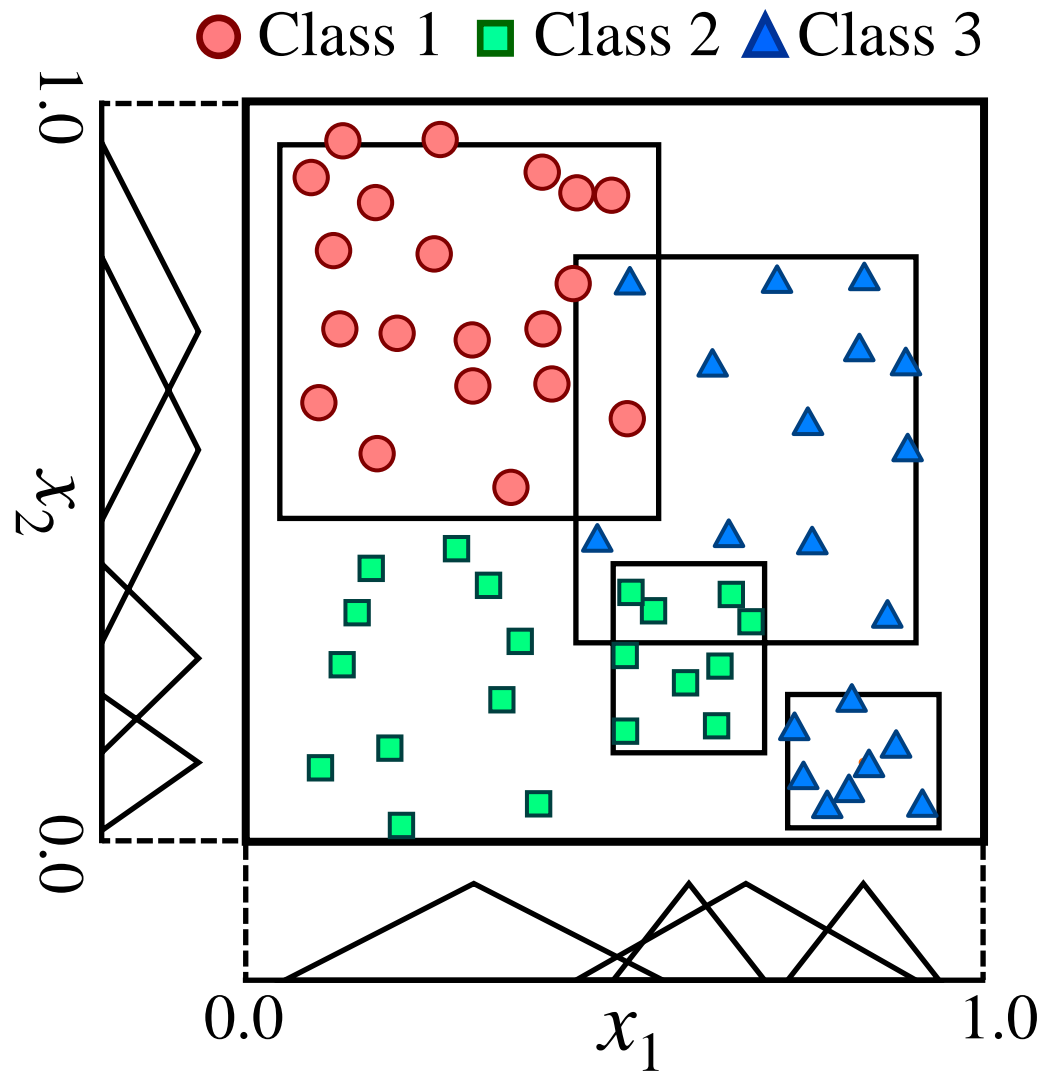
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H. Ishibuchi et al., *IEEE Trans. on SMC Part B* (1999)

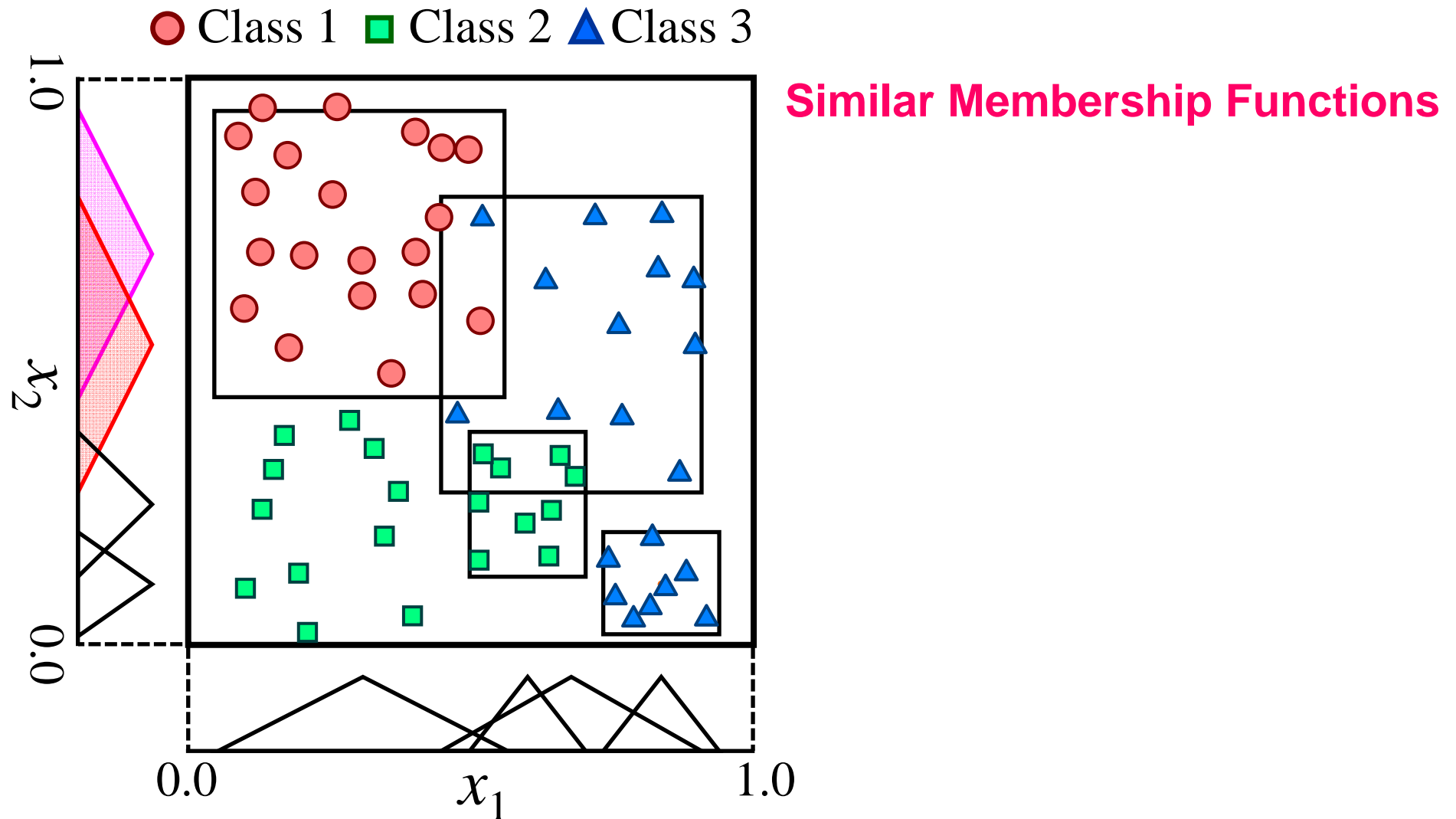
Complexity Minimization

Merging Similar Membership Functions



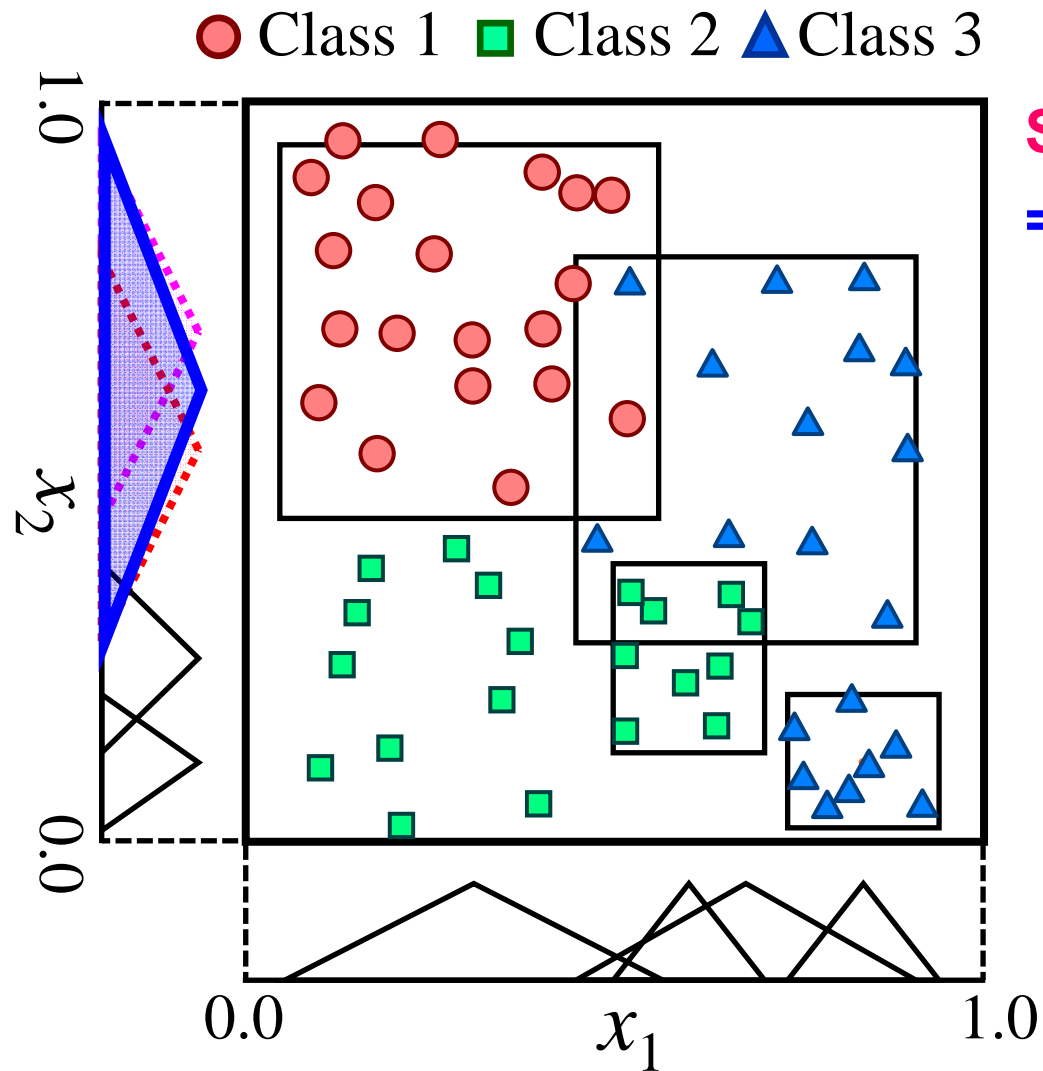
Complexity Minimization

Merging Similar Membership Functions



Complexity Minimization

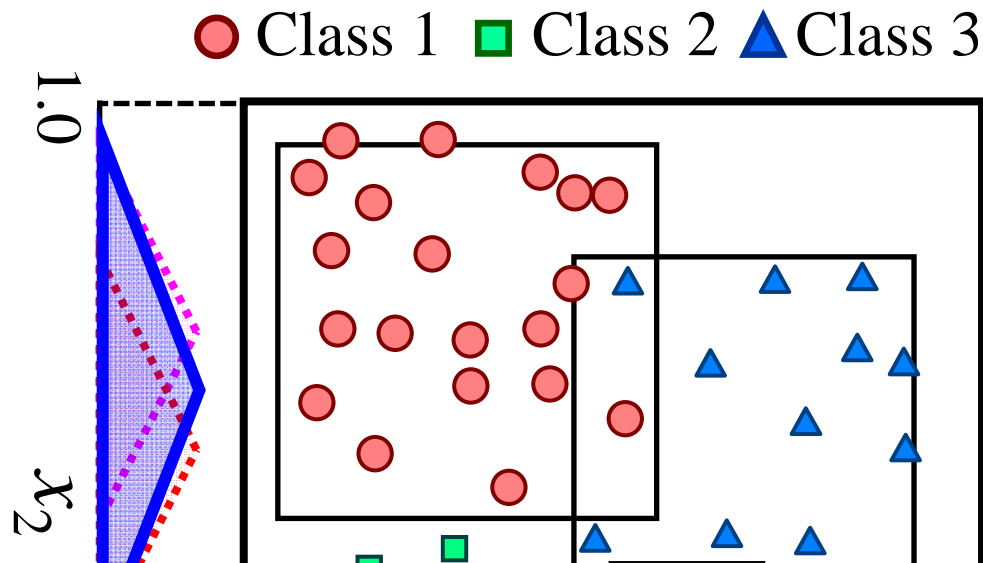
Merging Similar Membership Functions



Similar Membership Functions
==> One Membership Function

Complexity Minimization

Merging Similar Membership Functions



Similar Membership Functions
==> One Membership Function

[PDF] ► [Similarity measures in fuzzy rule base simplification](#)

M Setnes, R Babuska, U Kaymak, HR van ... - IEEE Transactions on Systems, Man, repository.tudelft.nl [Google Scholar](#)

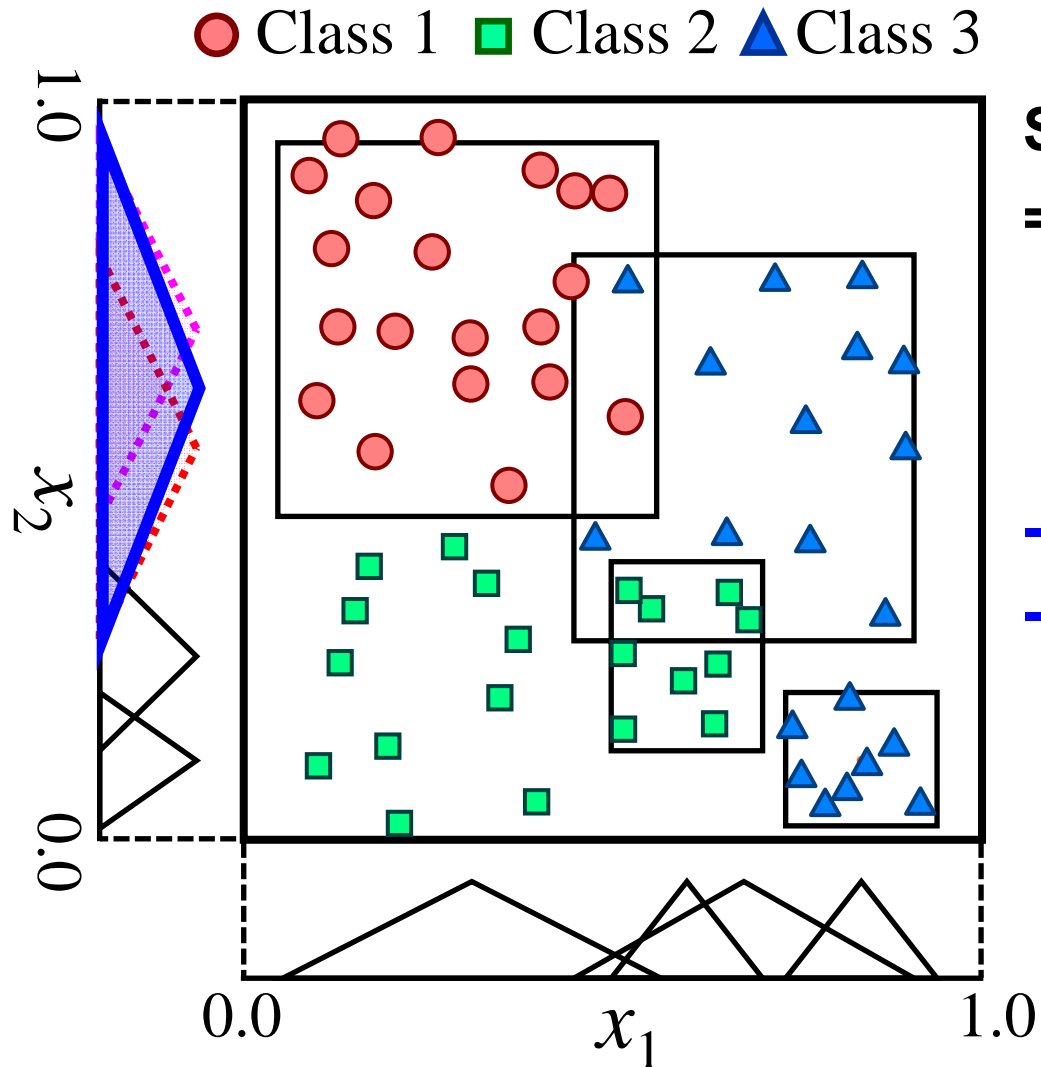
Abstract—In **fuzzy rule-based** models acquired from numerical data, redundancy may be present in the form of similar **fuzzy** sets that represent compatible concepts. This results in an un- necessarily complex and less transparent ...

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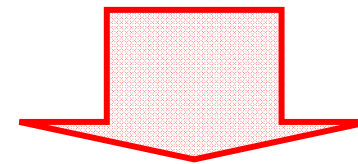
M. Setnes et al., *IEEE Trans. on SMC Part B* (1998)

Complexity Minimization

Merging Similar Membership Functions



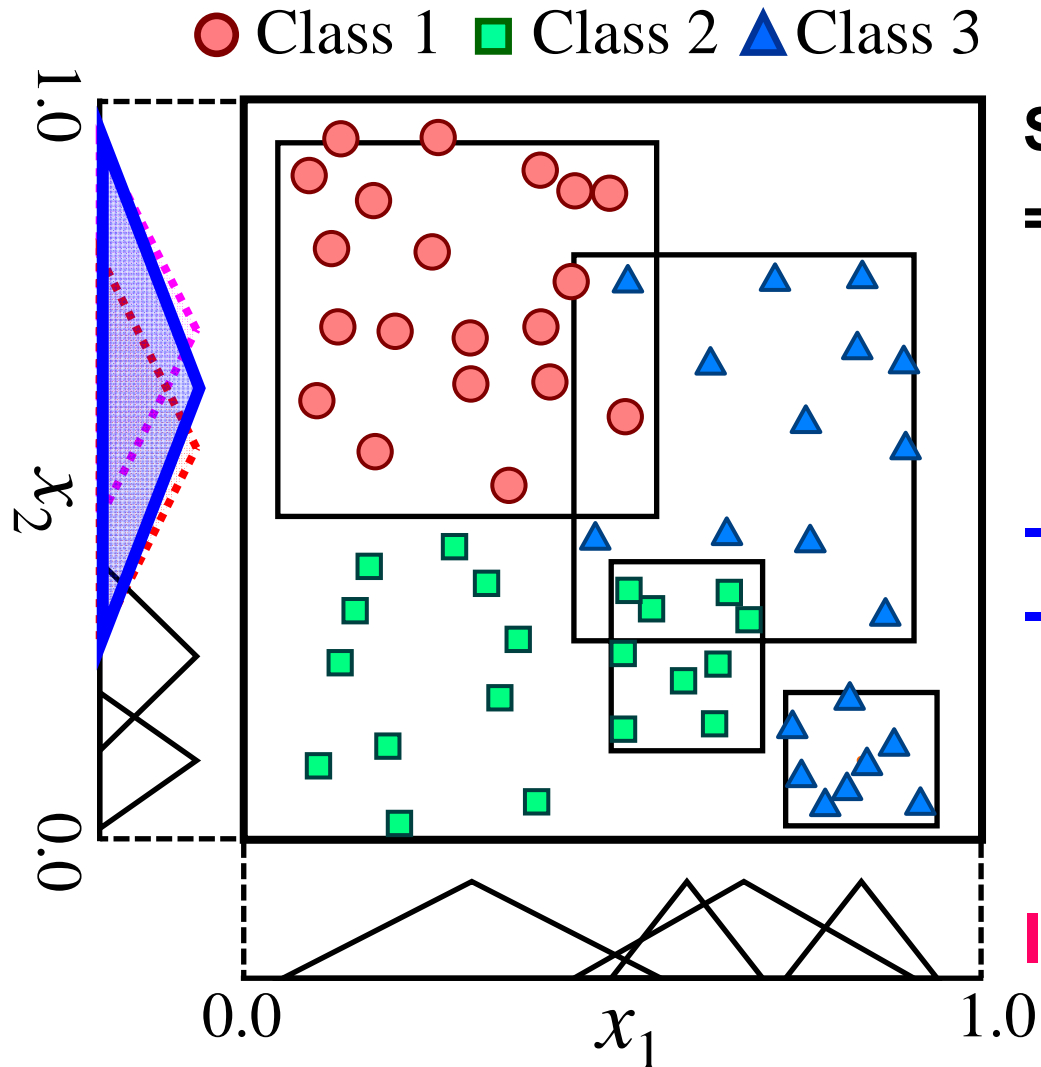
Similar Membership Functions
==> One Membership Function



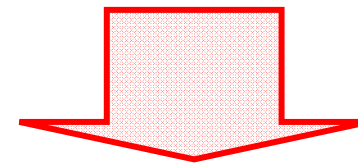
- Interpretability is improved.
- Accuracy is degraded.

Complexity Minimization

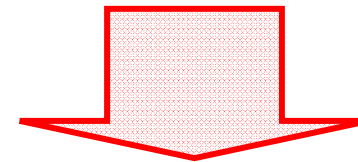
Merging Similar Membership Functions



Similar Membership Functions
==> One Membership Function



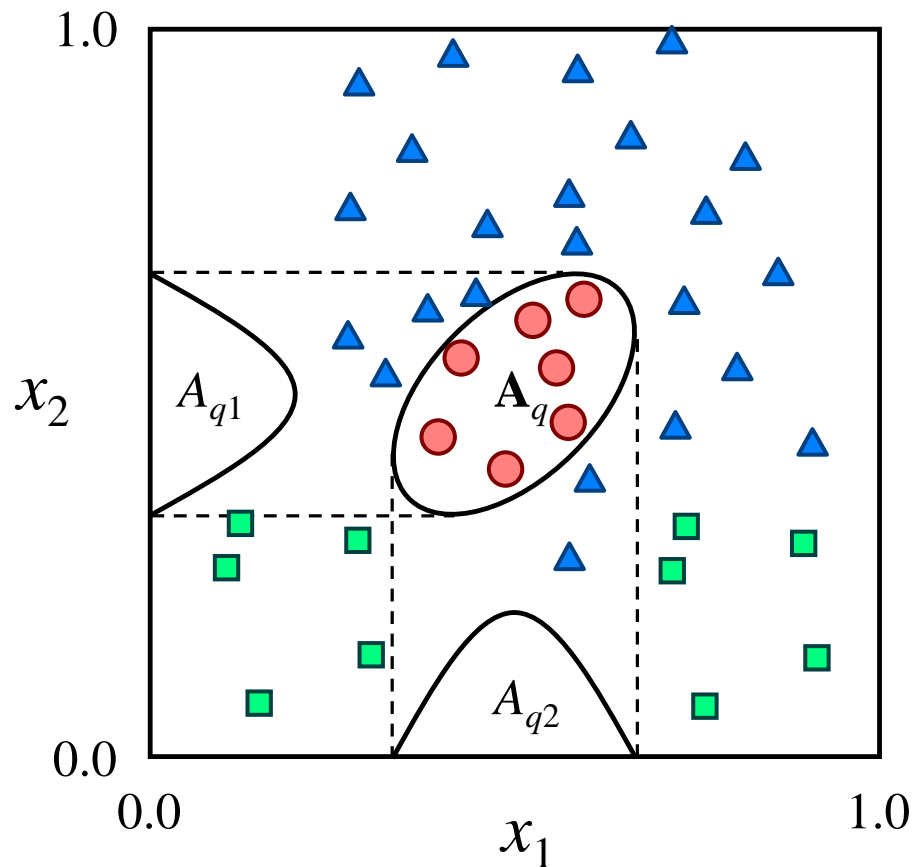
- Interpretability is improved.
- Accuracy is degraded.



Interpretability-Accuracy
Tradeoff

Complexity Minimization

Projection of Multi-Dimensional Fuzzy Sets



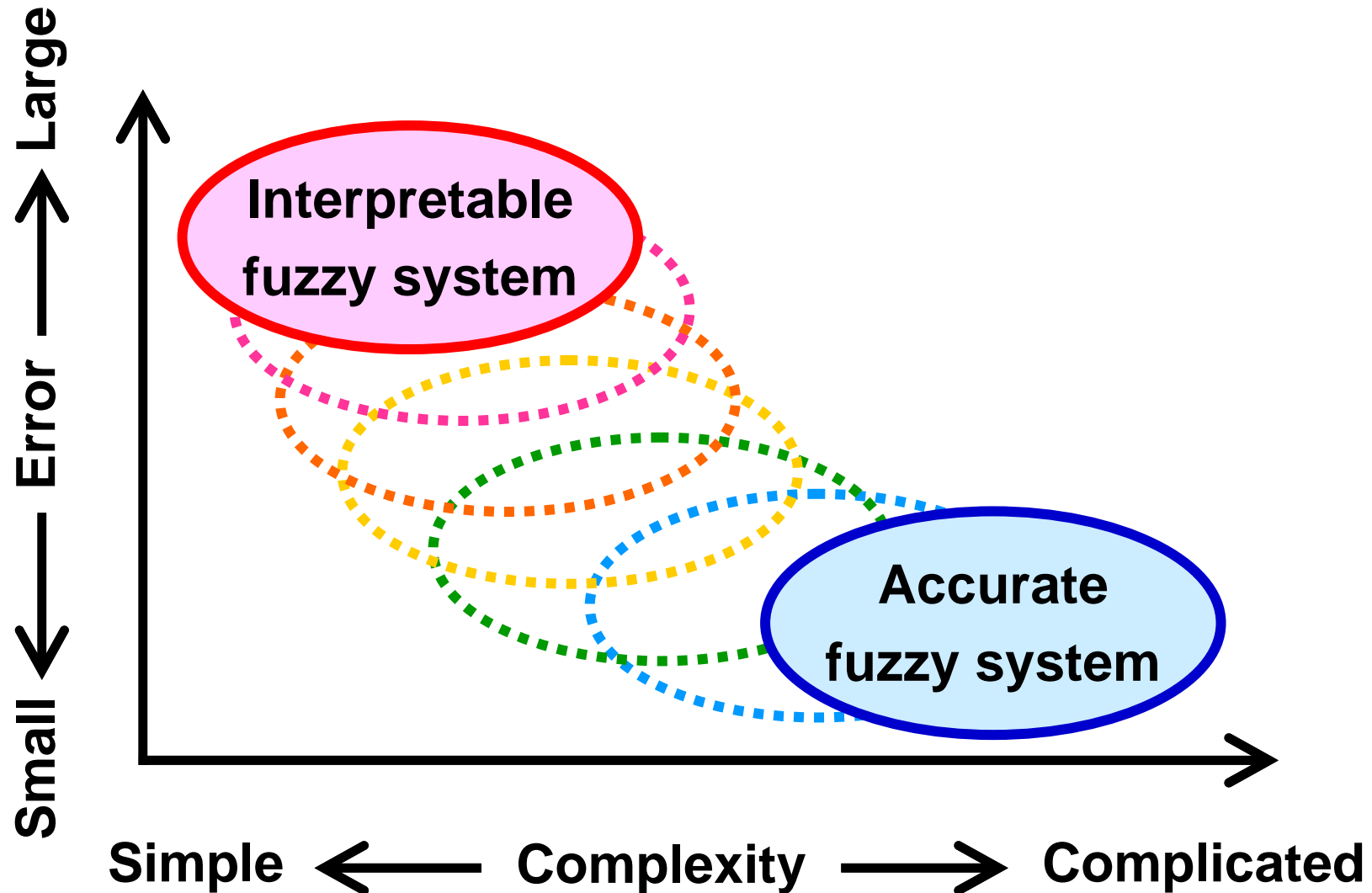
Change of Fuzzy Rule Form

If \mathbf{x} is A_q then ...
If x_1 is A_{q1} and x_2 is A_{q2} then

- Interpretability is improved.
- Accuracy is degraded.

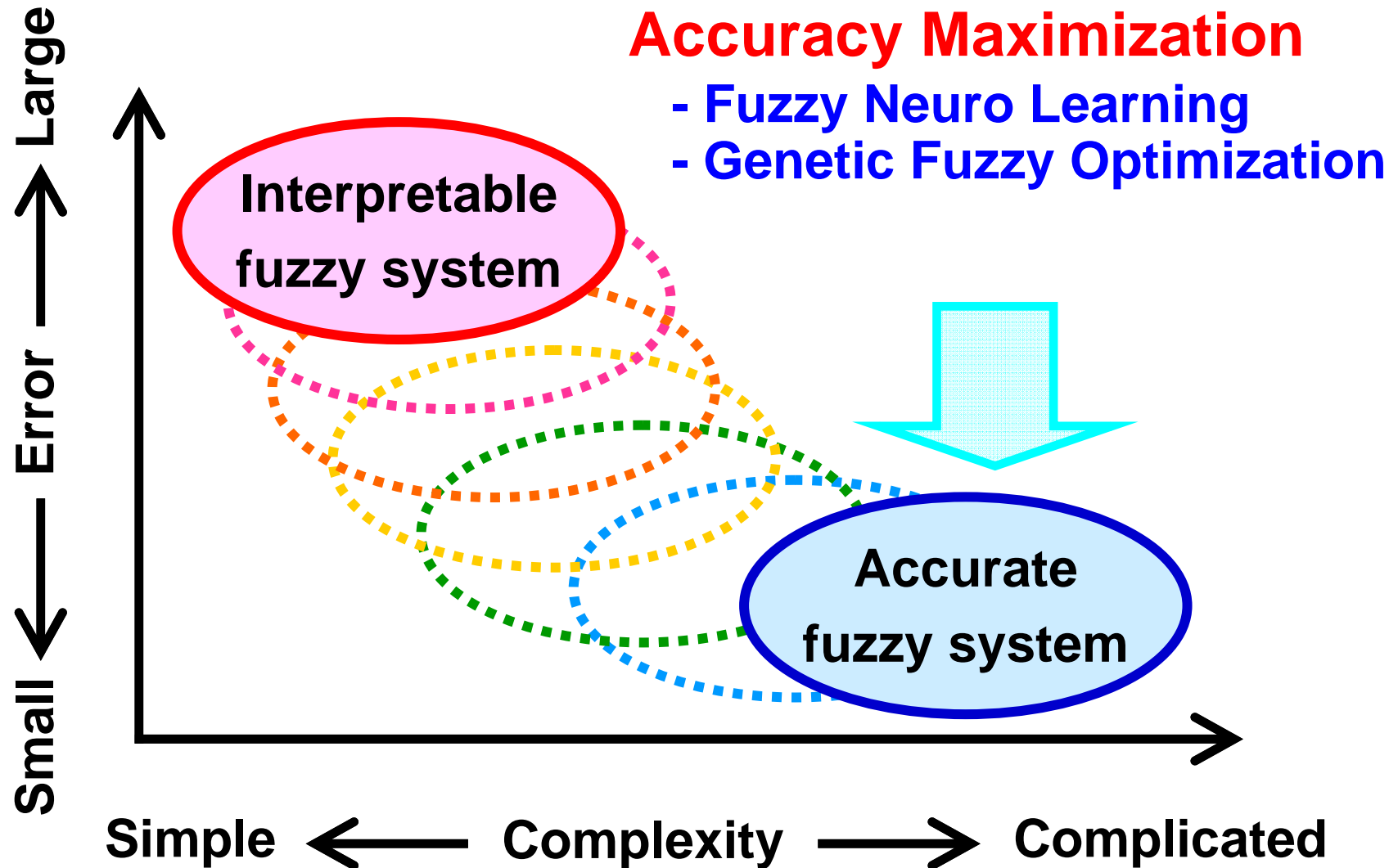
Interpretability-Accuracy
Tradeoff

Interpretability-Accuracy Tradeoff (Accuracy-Simplicity Tradeoff)



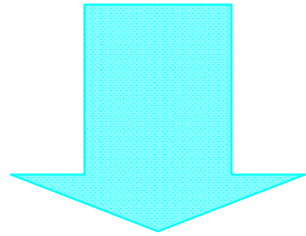
Accuracy Maximization

Main Research Direction Since the Early 1990s



Possible Difficulties

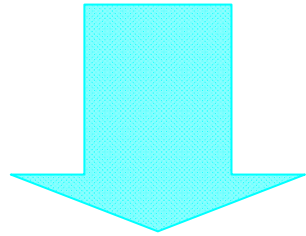
Accuracy Maximization



- **Poor Interpretability**
- **Overfitting to Training Data**

Possible Difficulties

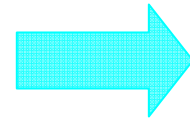
Accuracy Maximization



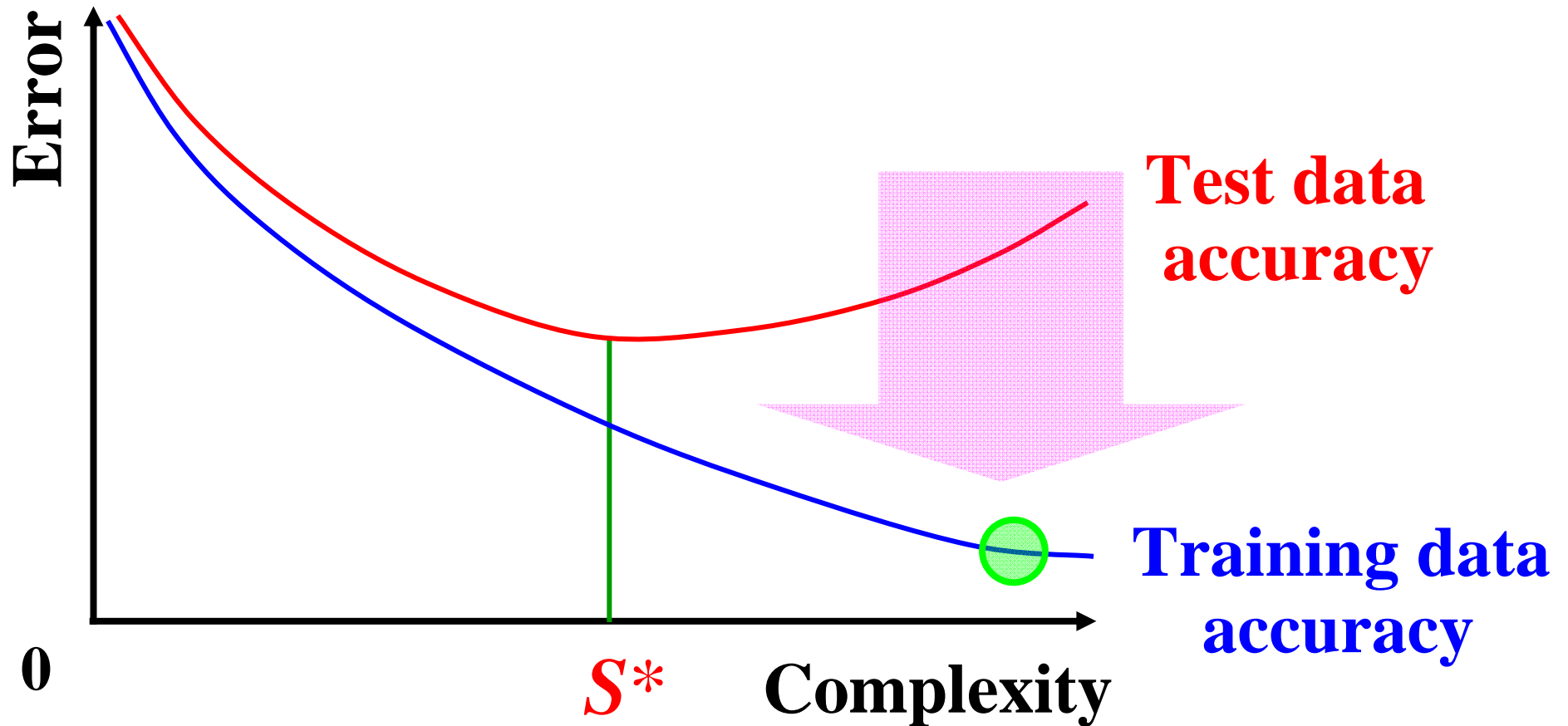
- *Poor Interpretability*
- **Overfitting to Training Data**

Difficulty in Accuracy Maximization

Accuracy maximization

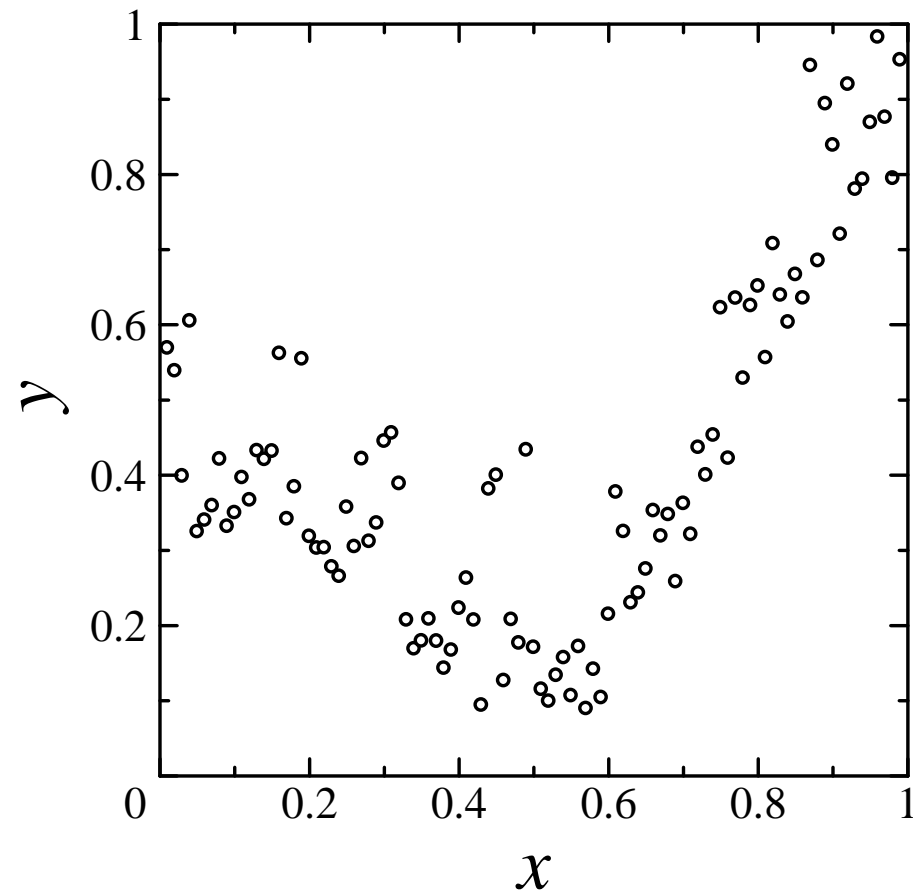
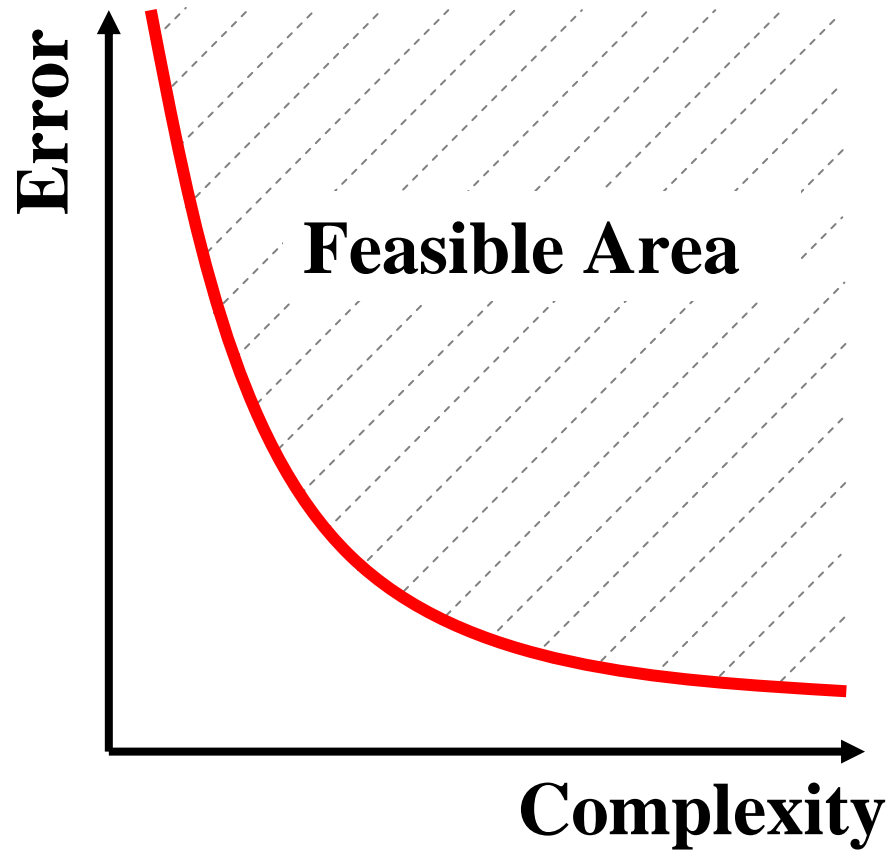


Overfitting



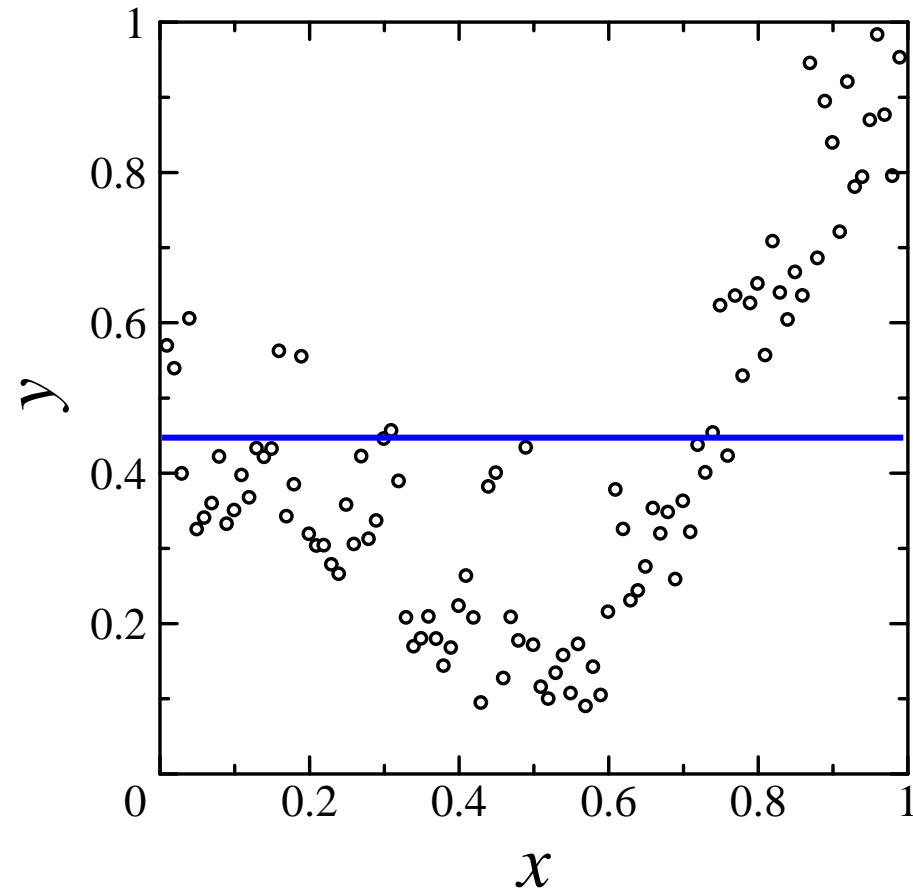
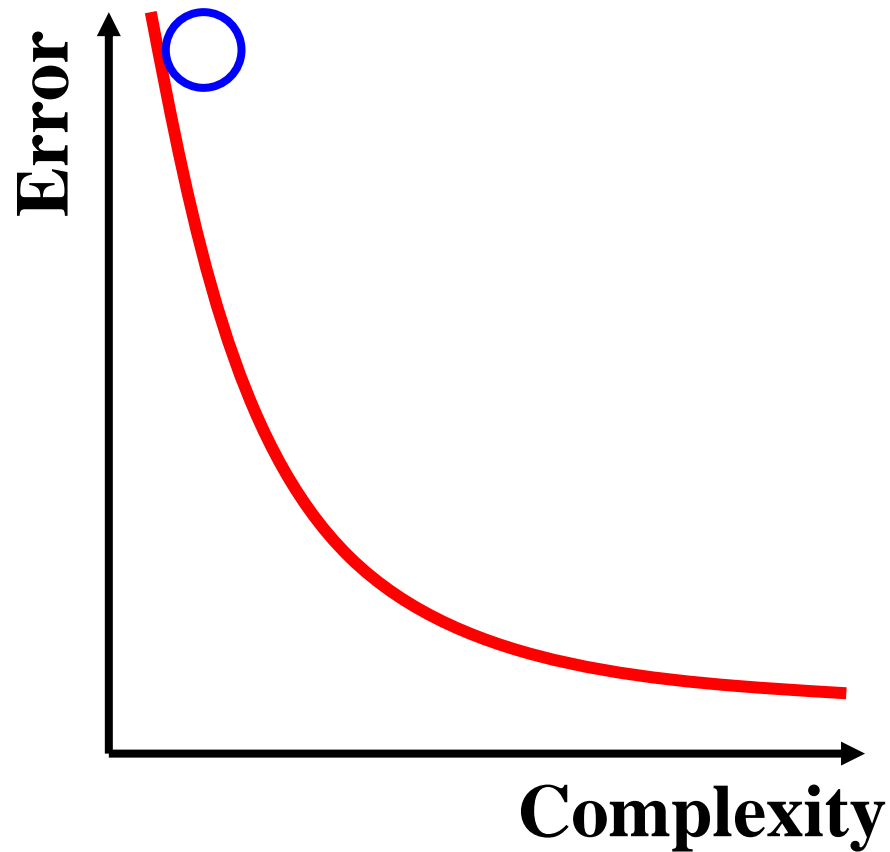
Accuracy-Complexity Tradeoff

Curve Fitting



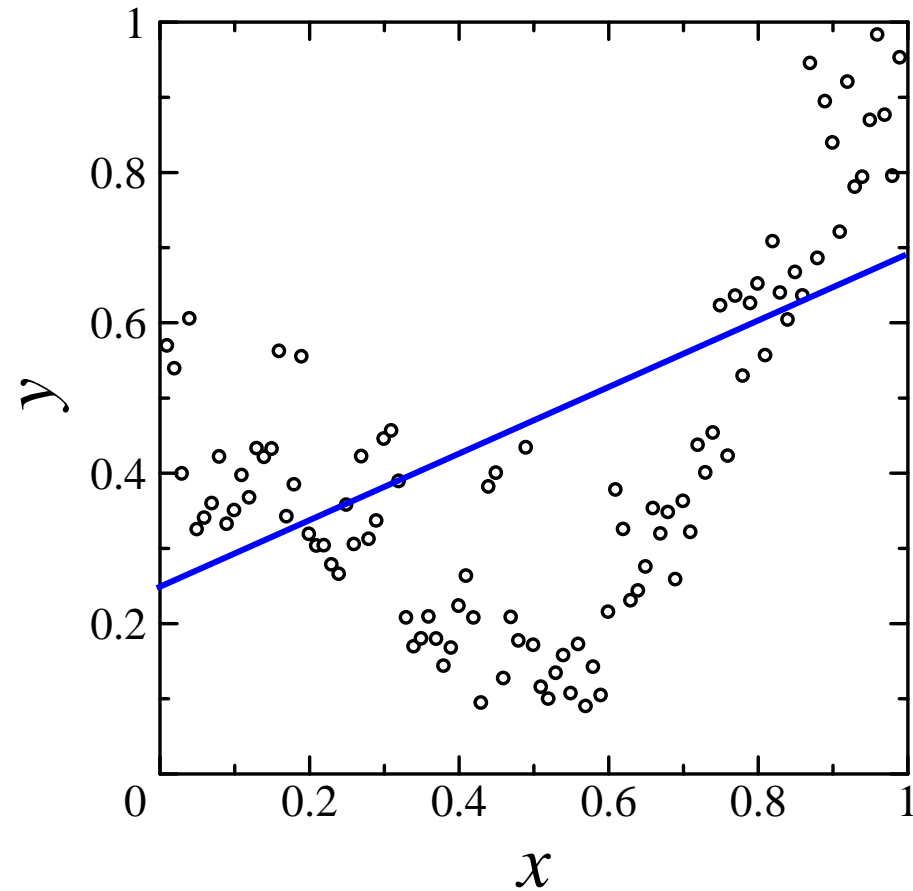
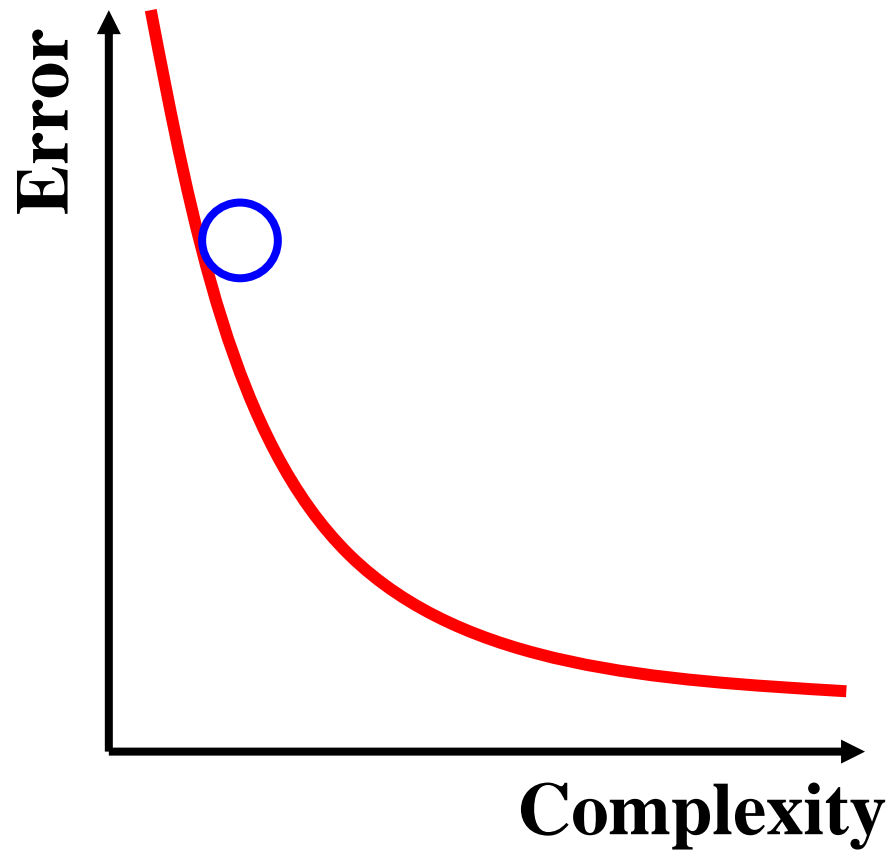
Accuracy-Complexity Tradeoff

Curve Fitting



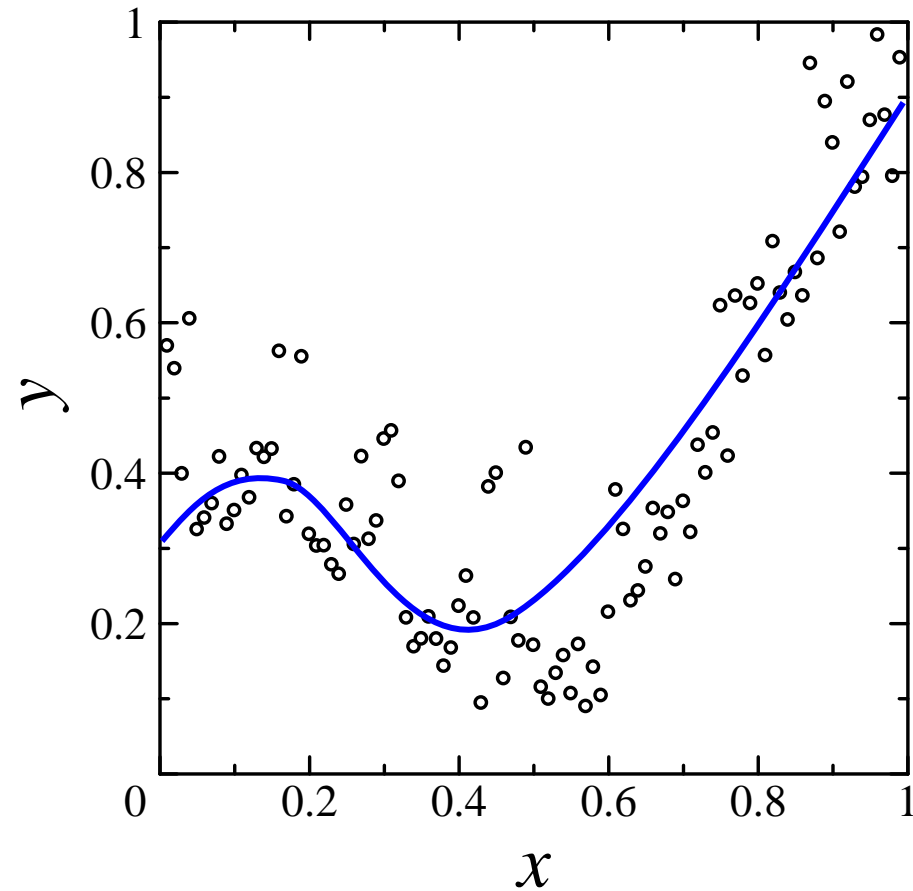
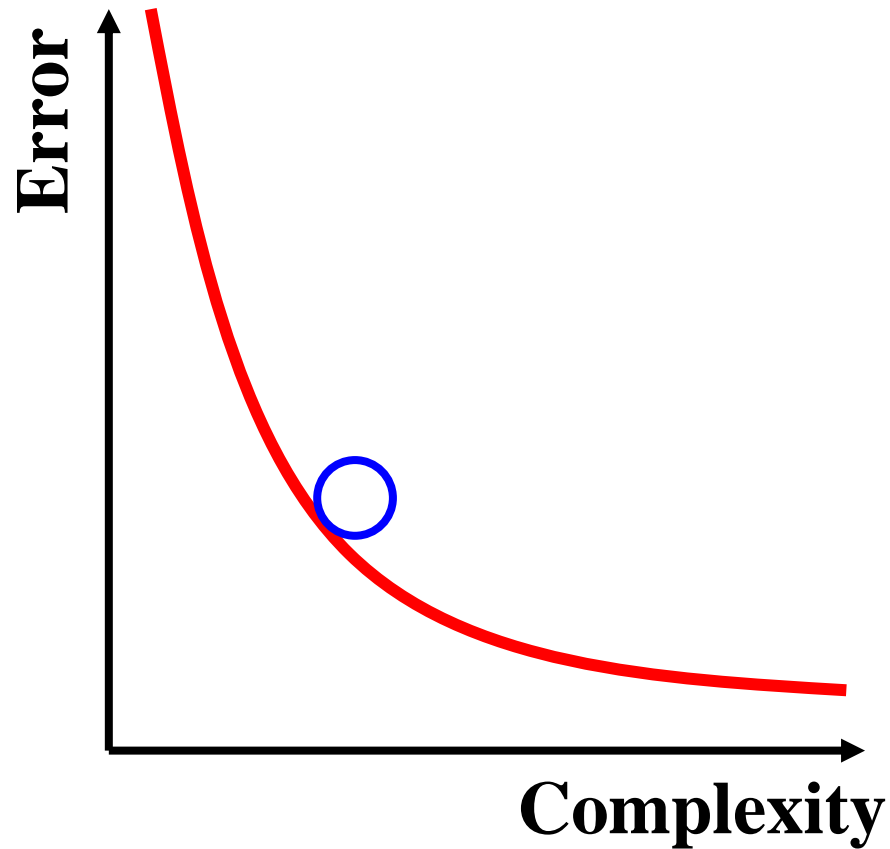
Accuracy-Complexity Tradeoff

Curve Fitting



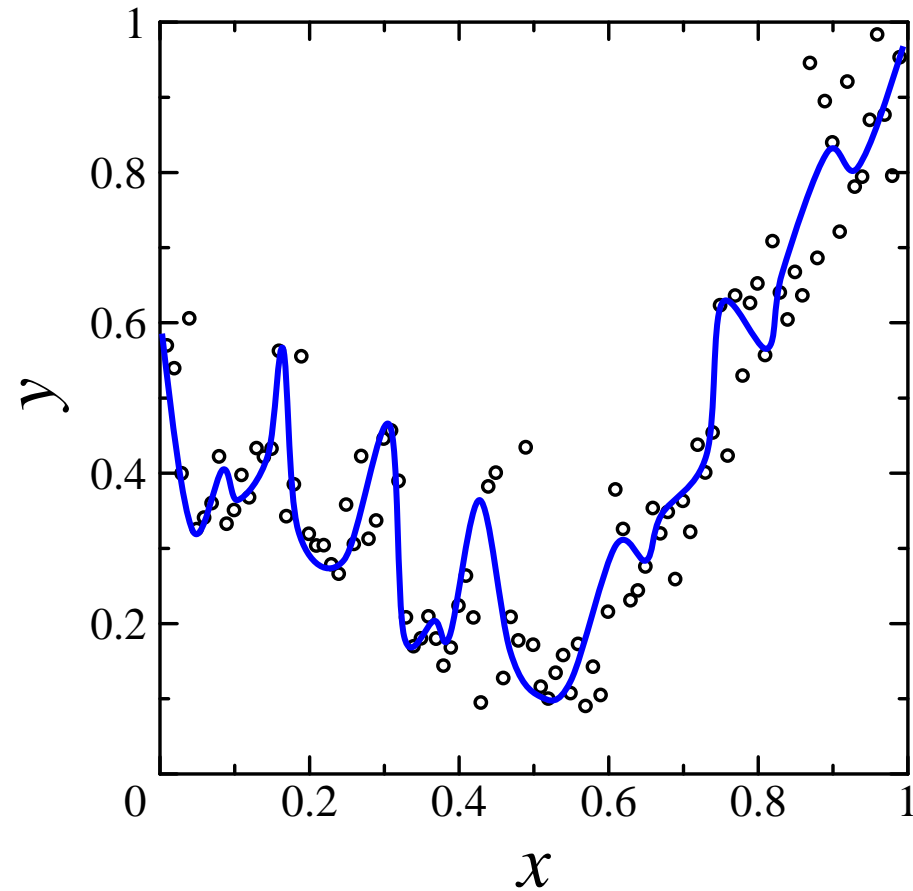
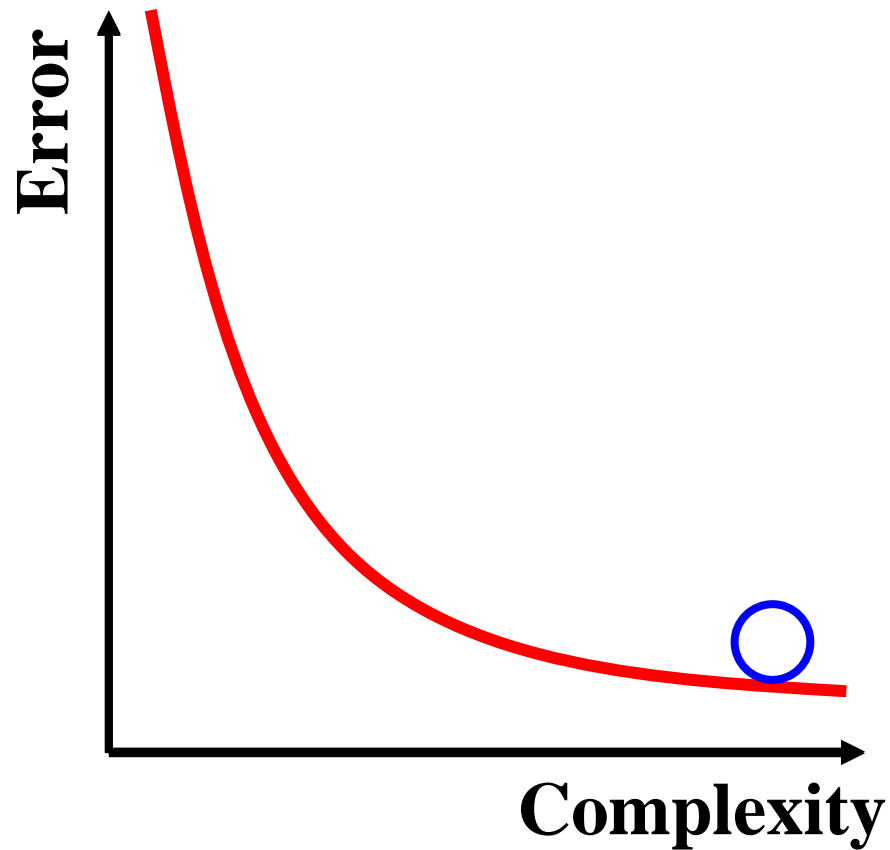
Accuracy-Complexity Tradeoff

Curve Fitting



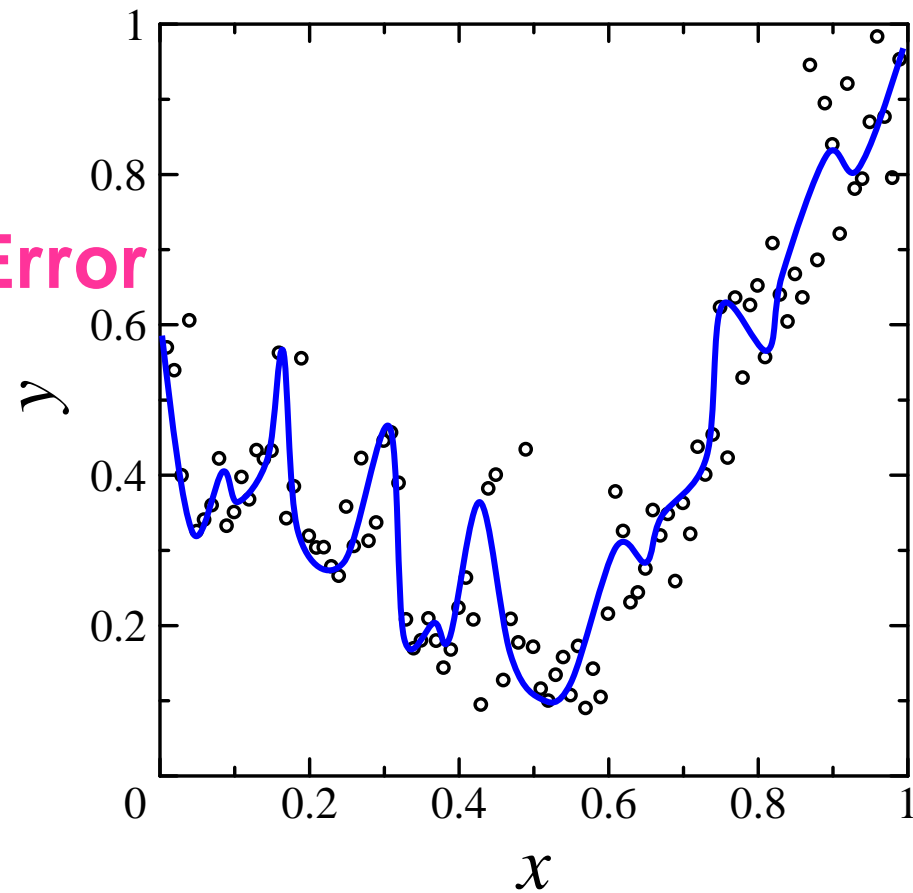
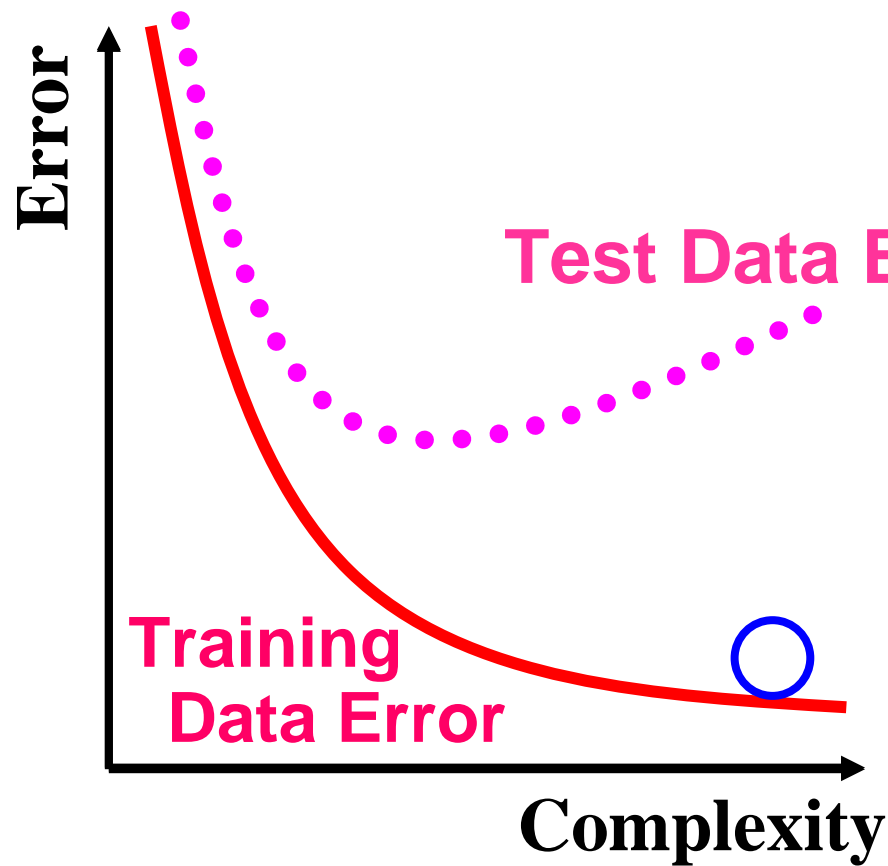
Accuracy-Complexity Tradeoff

Curve Fitting



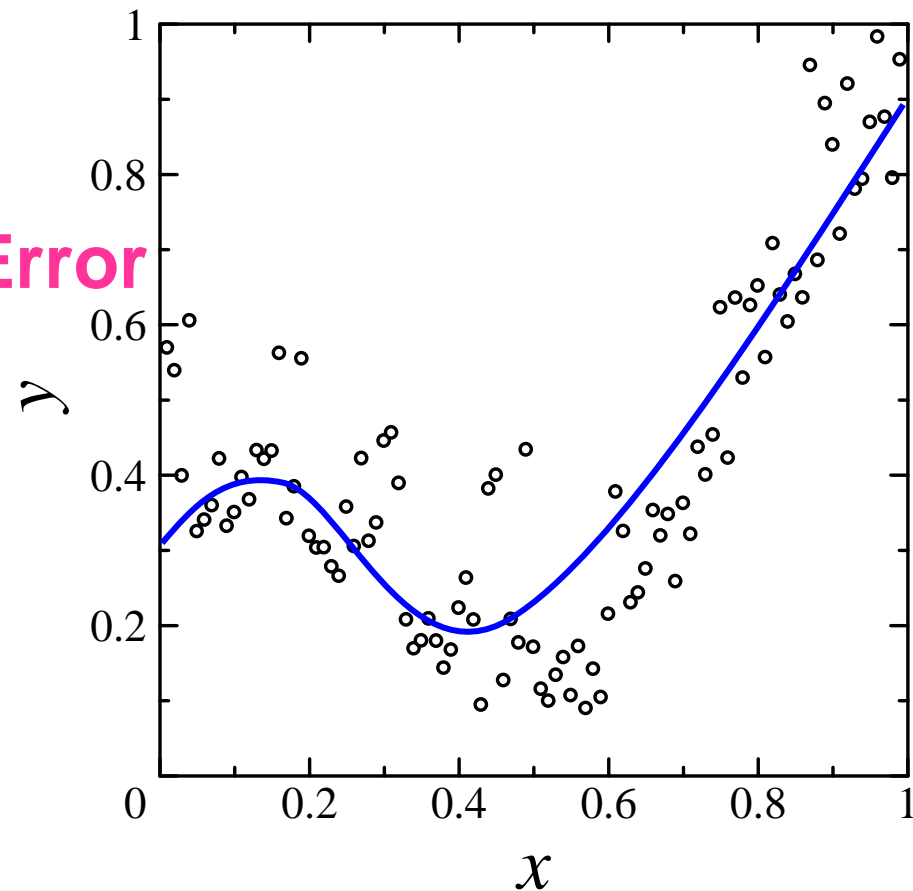
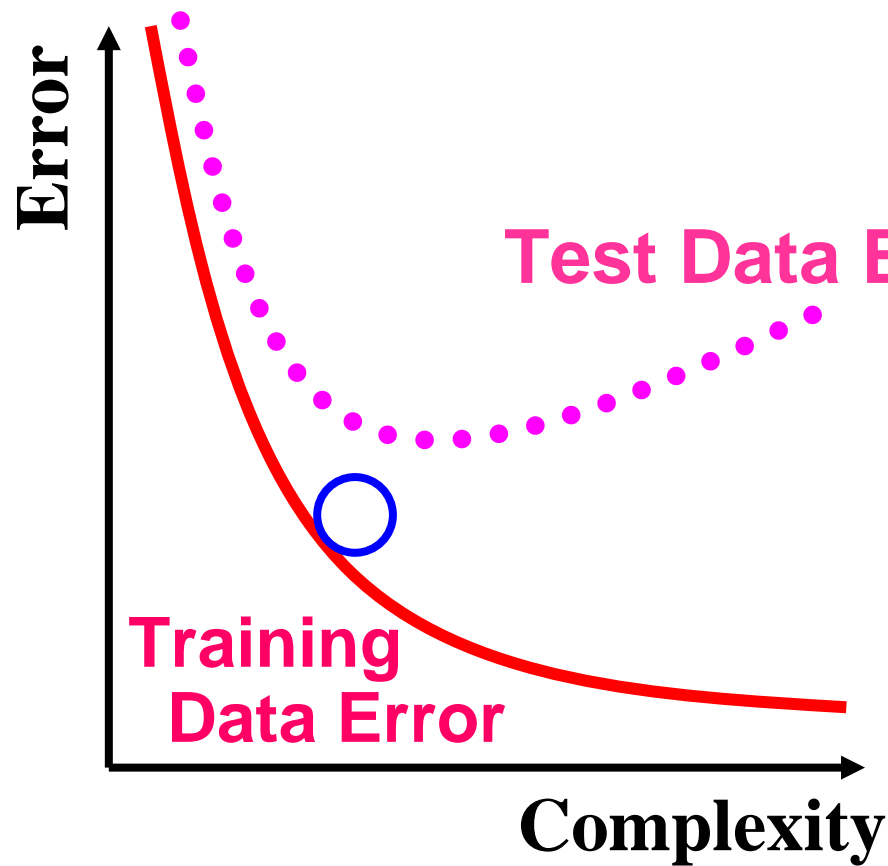
Accuracy-Complexity Tradeoff

Curve Fitting



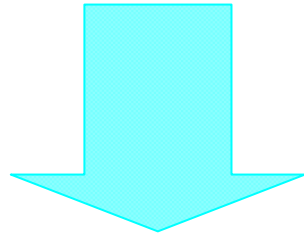
Accuracy-Complexity Tradeoff

Curve Fitting



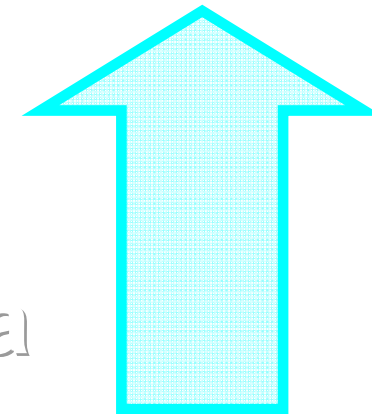
Possible Difficulties

Accuracy Maximization



Interpretability maintenance while maximizing the accuracy.

- **Poor Interpretability**
- *Overfitting to Training Data*

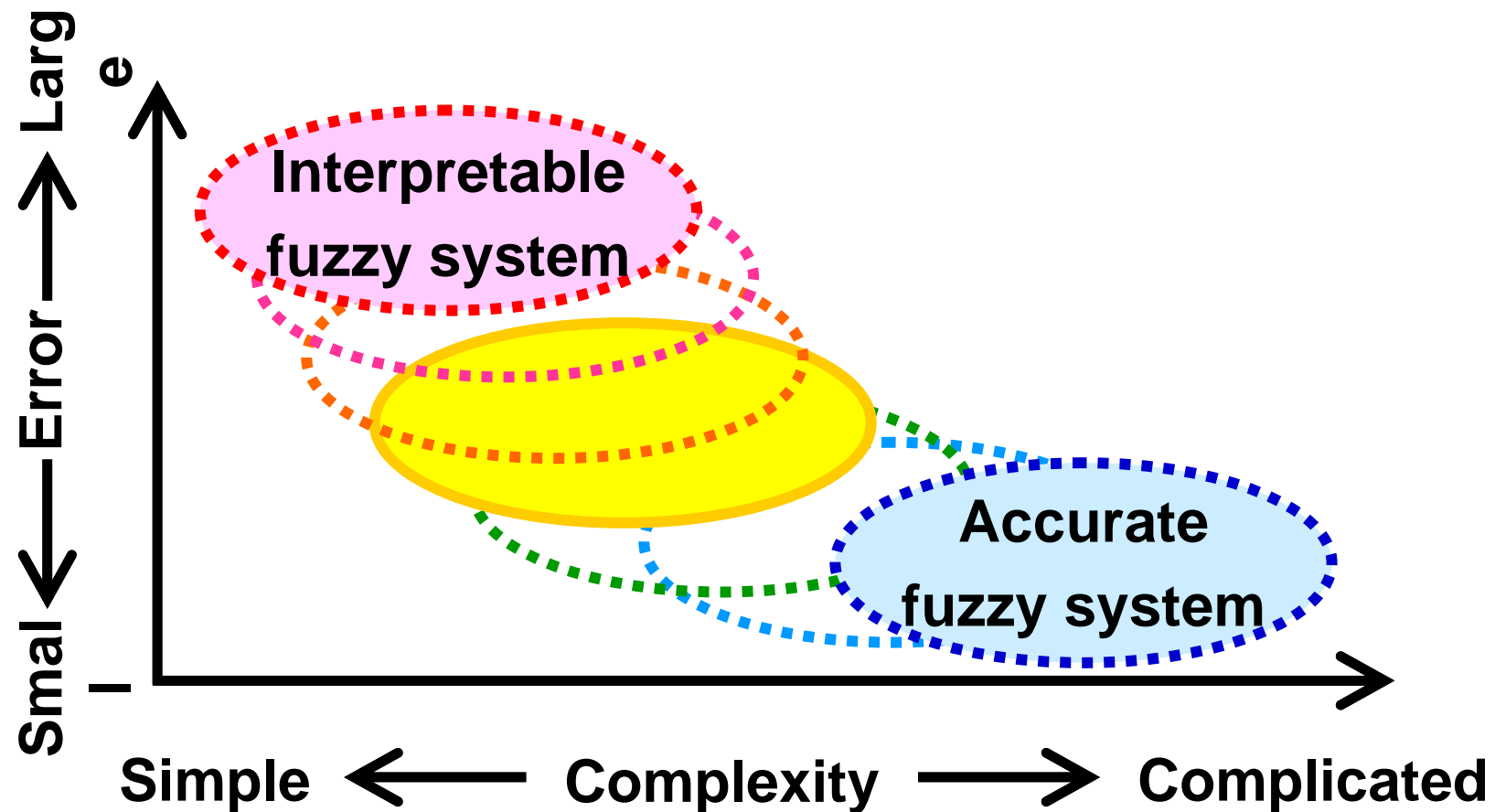


In the design of fuzzy systems, emphasis should be placed on their linguistic interpretability.

Accuracy and Interpretability Maximization

Active Research Direction Since the Late 1990s

Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)



Accuracy and Interpretability Maximization

Active Research Direction Since the Late 1990s

Compromise between Accuracy and Complexity
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Some Ideas

- **Aggregated Objective Function:** To combine the error minimization and the complexity minimization into a single scalar fitness function

Accuracy and Interpretability Maximization

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Accuracy and Interpretability Maximization

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- **Aggregated Objective Function:** To combine the error minimization and the complexity minimization into a single scalar fitness function
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- **Two-Step Fuzzy System Design:** 1st Step: Search for accurate and complicated fuzzy rule-based systems. 2nd Step: Simplification of obtained fuzzy rule-based systems.

Accuracy and Interpretability Maximization

Active Research Direction Since the Late 1990s

Accuracy Maximization and Complexity Minimization

[PDF] ► [GA-fuzzy modeling and classification: complexity and performance](#)

M Setnes, H Roubos - IEEE Transactions on Fuzzy Systems, 2000 - Citeseer

Manuscript received (...); revised (...). This work was supported in part by the Research Council of Norway. The authors are with the Delft University of Technology, Faculty of Information Technology and Systems, Control ...

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[PDF] ► [Compact and transparent fuzzy models and classifiers](#)

H Roubos, M Setnes - IEEE Transactions on Fuzzy Systems, 2001 - repec

Abstract—In our previous work we showed that genetic algorithms (GAs) provide a powerful tool to increase the accuracy of fuzzy models for both systems modeling and classification. In addition to these results, we ...

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[BOOK] [Interpretability issues in fuzzy modeling](#)

J Casillas, O Cordon, F Herrera, L Magdalena, 2003 - books.google.com

Dr. Jorge Casillas Dr. Luis Magdalena E-mail: casillas@decsai.ugr.es E-mail: llyos@mat.upm.es Dr. Oscar Cordón Dpto. Matematicas Aplicadas E-mail: ocordon@decsai.ugr.es a las Tecnologias de la Information Dr. Francisco ...

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Accuracy and Interpretability Maximization

Active Research Direction Since the Late 1990s

Compromise between Accuracy and Complexity (Search for a good accuracy-complexity tradeoff)

Basic Idea

- **Aggregated Objective Function:** To combine the error minimization and the complexity minimization into a single scalar fitness function
- **Constraint Condition:** To use constraint conditions on the position and the shape of membership functions
- **Two-Step Fuzzy System Design:** 1st Step: Search for accurate and complicated fuzzy rule-based systems. 2nd Step: Simplification of obtained fuzzy rule-based systems.

Accuracy and Interpretability Maximization

Active Research Direction Since the Late 1990s

Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)

Aggregated Objective Function

- To combine the error minimization and the complexity minimization into a single scalar objective function

Accuracy and Interpretability Maximization

Active Research Direction Since the Late 1990s

Compromise between Accuracy and Complexity
(Search for a good accuracy-complexity tradeoff)

Aggregated Objective Function

- To combine the error minimization and the complexity minimization into a single scalar objective function

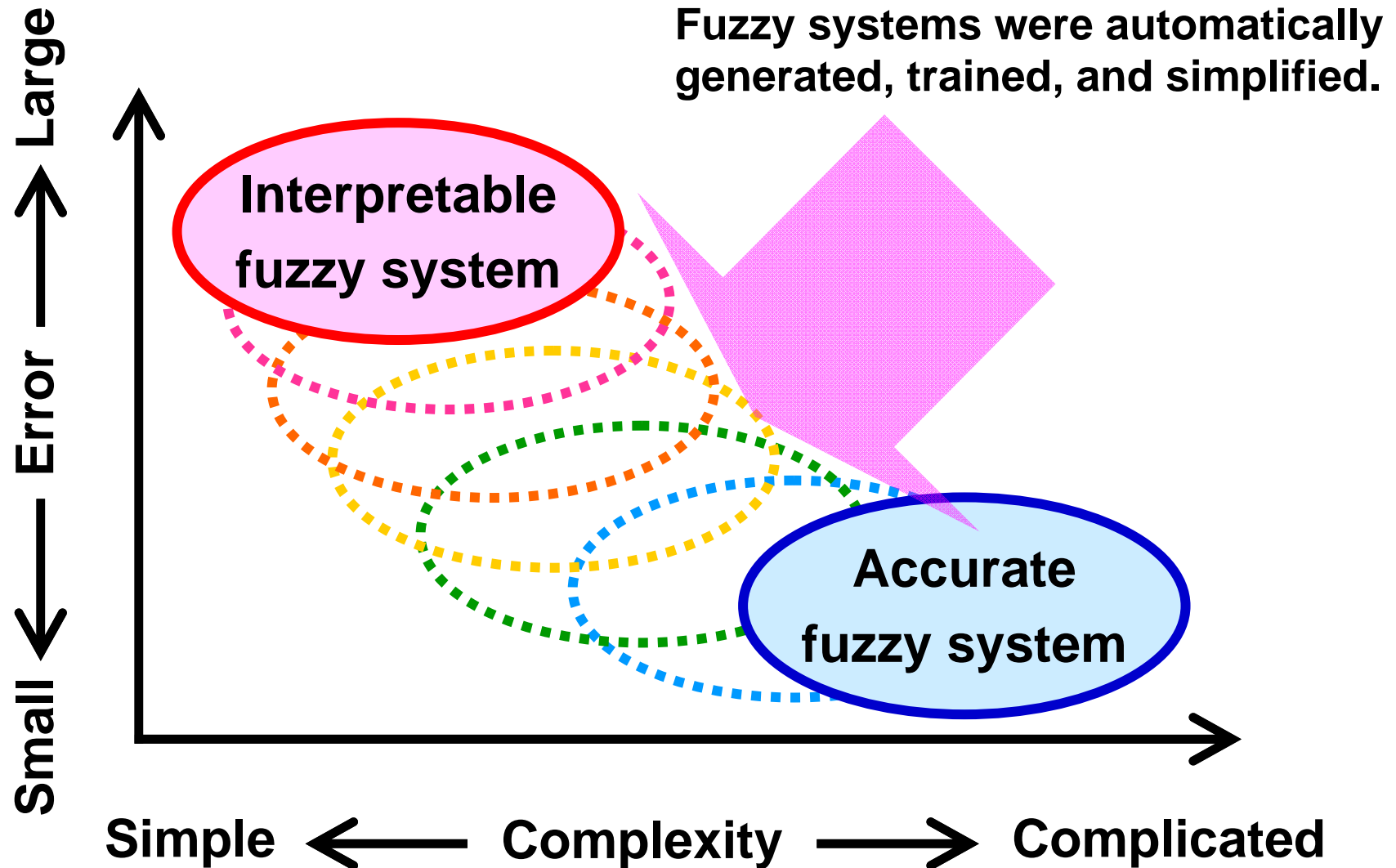
Example: Combination of the average error rate and the number of fuzzy rules

Example of a scalar objective function: Weighted sum

$$f(S) = w_1 \cdot f_{\text{Error}}(S) + w_2 \cdot f_{\text{Complexity}}(S)$$

Accuracy and Interpretability Maximization

Active Research Direction Since the Late 1990s



Difficulty in Weighted Sum Approach

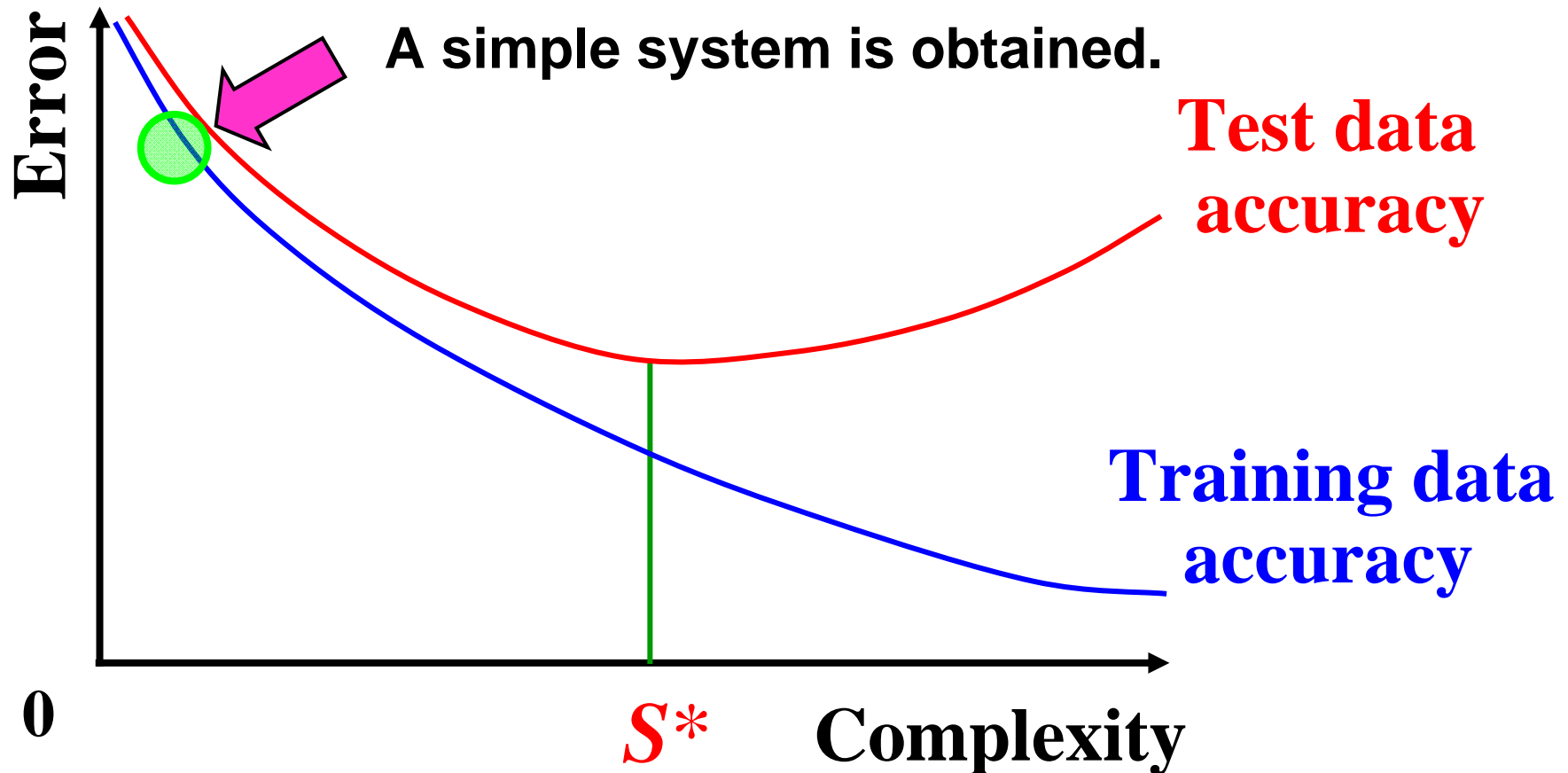
Sensitivity to the weight vector:

The obtained system strongly depends on the specification of the weight vector.

Difficulty in Weighted Sum Approach

Minimize $w_1 \cdot \text{Error} + w_2 \cdot \text{Complexity}$

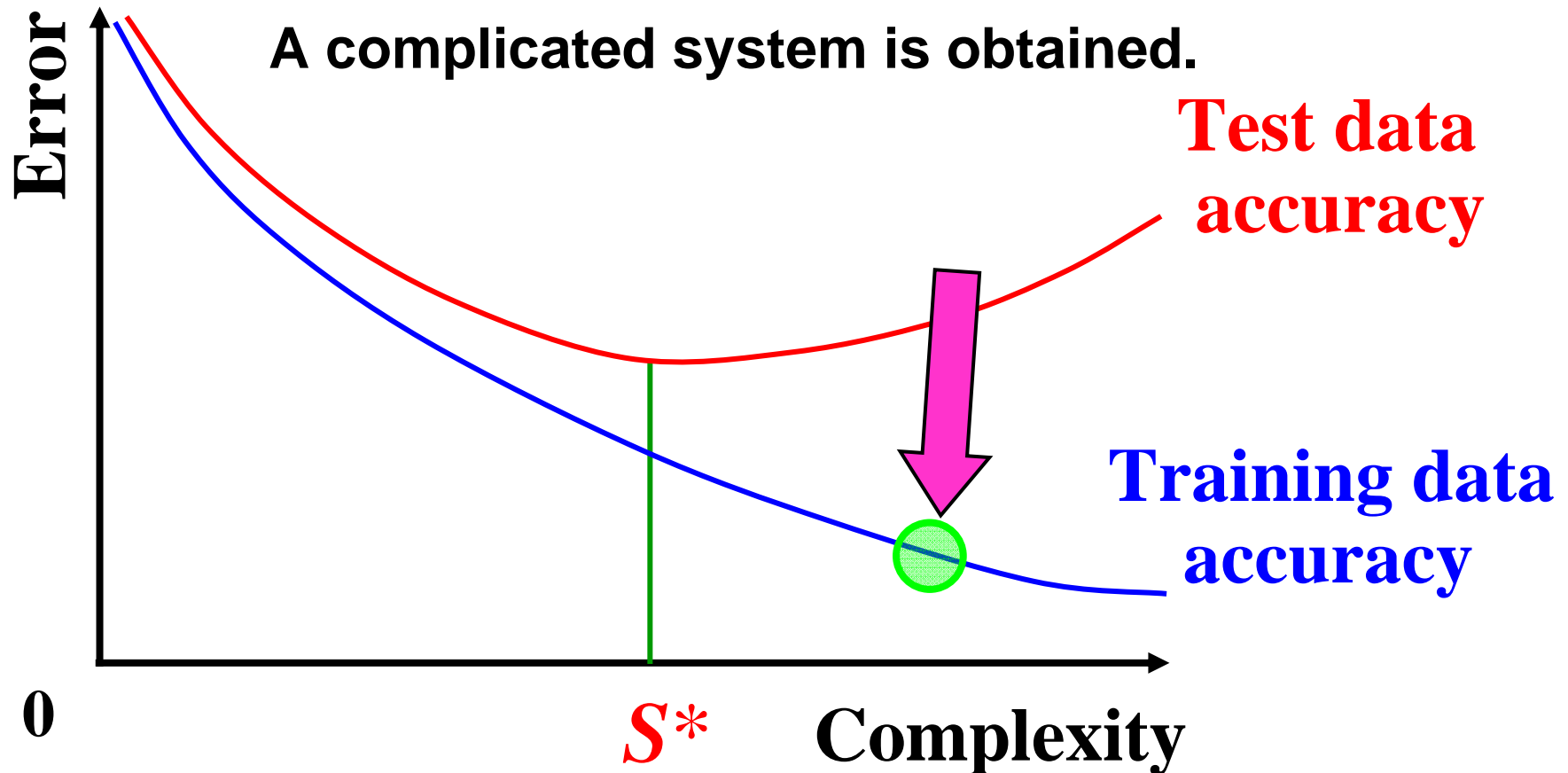
When the weight for the complexity minimization is large:



Difficulty in Weighted Sum Approach

Minimize $w_1 \cdot \text{Error} + w_2 \cdot \text{Complexity}$

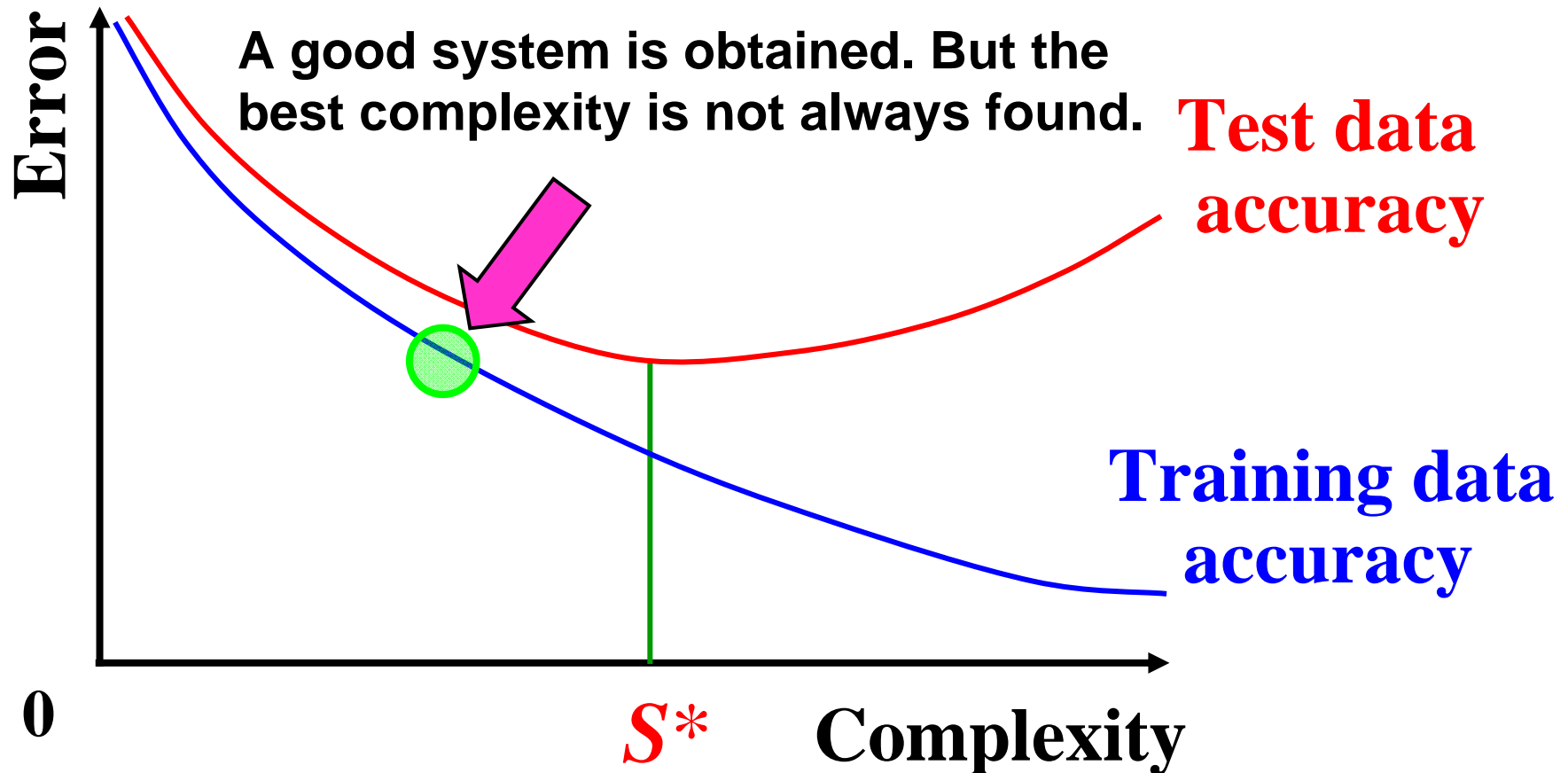
When the weight for the error minimization is large:



Difficulty in Weighted Sum Approach

Minimize $w_1 \cdot \text{Error} + w_2 \cdot \text{Complexity}$

When the two weights are appropriately specified:



Multiobjective Fuzzy System Design

Currently An Active Research Issue

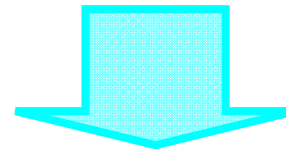
Basic Idea

To search for a number of non-dominated fuzzy systems with respect to the accuracy maximization and the interpretability maximization (instead of searching for a single fuzzy system).

Aggregation Approach

$$f(S) = w_1 \cdot f_{\text{Error}}(S) + w_2 \cdot f_{\text{Complexity}}(S)$$

Multiobjective Approach



Minimize $\{f_{\text{Error}}(S), f_{\text{Complexity}}(S)\}$

Multiobjective Fuzzy System Design

Currently An Active Research Issue

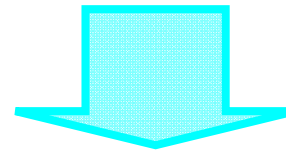
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Multiobjective Approach

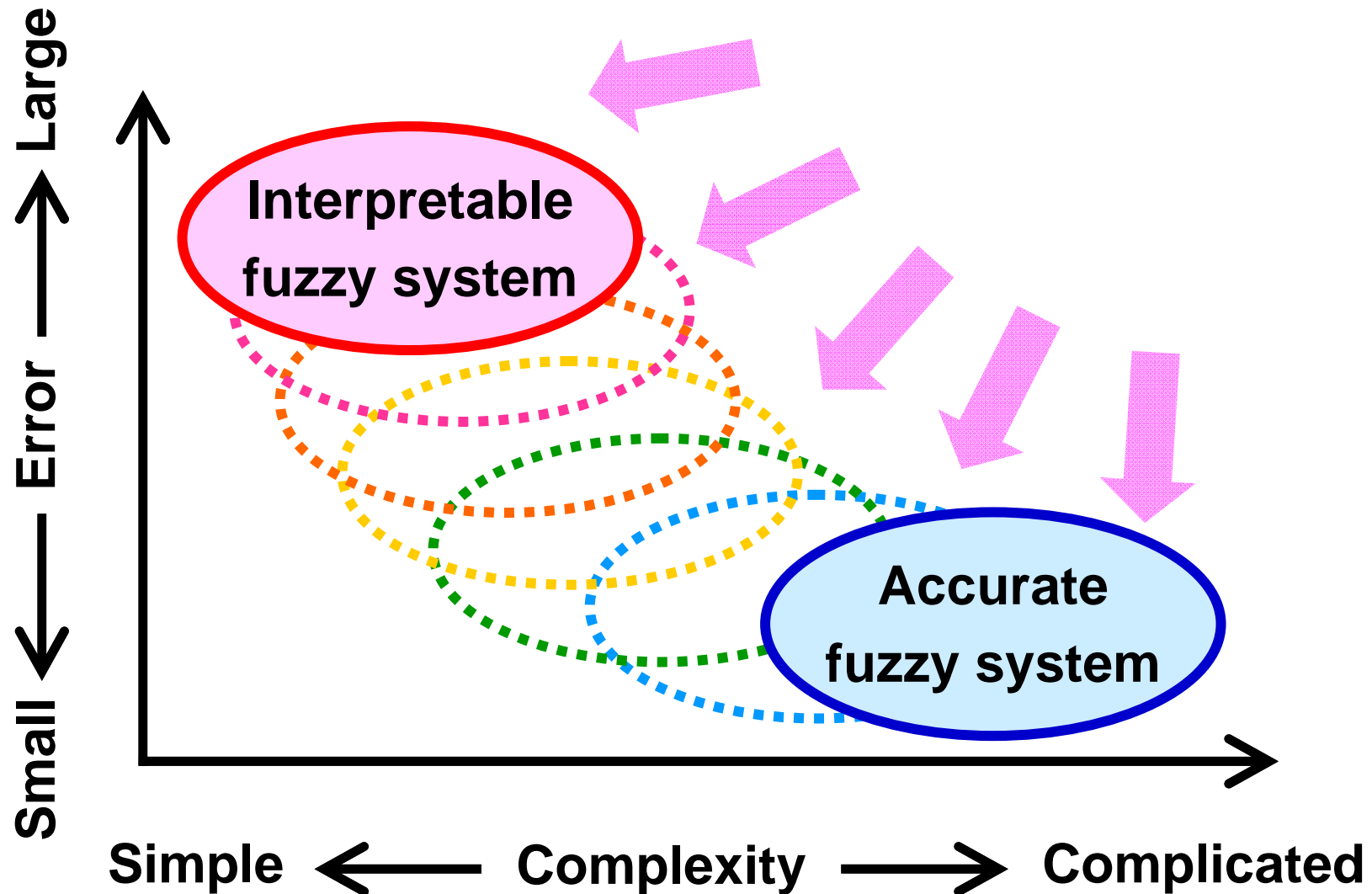


Minimize $\{f_{\text{Error}}(S), f_{\text{Complexity}}(S)\}$

Search for Pareto Optimal Fuzzy Rule-Based Systems

Multiobjective Fuzzy System Design

Currently An Active Research Issue



Contents of This Presentation

Accurate and Interpretable Fuzzy Rule-Based Classifier Design

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- Accuracy Improvement
- Scalability to High-Dimensional Problems
- Complexity Minimization

3. Multiobjective Fuzzy Rule-Based Classifier Design

- Formulation of Multi-objective Problems
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- Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions

- Search Ability of EMO for Fuzzy System Design
- Definition of Interpretability of Fuzzy Systems
- Explanation Ability of Fuzzy Rule-Based Systems
- Various Classification Problems: Imbalanced, Online, ...

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The Evolutionary Multiobjective Optimization of Fuzzy Rule-Based Systems Bibliography Page

Abstract

Since pioneering works by [Prof. Hisao Ishibuchi](#) in middle nineties, Pareto-based Evolutionary Multiobjective Optimization (EMO) of Fuzzy Rule-Based Systems (FRBSs) is nowadays a well-established research area. It is a branch of the more general Evolutionary/Genetic Fuzzy Systems (see [F. Herrera](#), "Genetic Fuzzy systems: Taxonomy, current research trends and

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[Evolutionary/Genetic Fuzzy Systems](#) (see [F. Herrera](#), "Genetic

Fuzzy systems: Taxonomy, current research trends and prospects", *Evo. Intel.* (2008), 1:27-46 and [this](#) bibliography page on recent publications on the topic, maintained by [R. Alcalá](#) and [M. J. Gacto](#)). In Pareto-based evolutionary optimization the set of objectives used are not aggregated in order to reconduct the problem to a single objective optimization problem. This page is intended to collect as many references as possible to papers dealing with Pareto-based EMO of FRBSs. (Pareto-based) EMOs of FRBSs are special cases of Multiobjective Evolutionary Fuzzy Systems (MEFSs), which include the class of Multiobjective Genetic Fuzzy Systems (MGFSs). For a review on the last topic,

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6). For a more general overview of multiobjective optimization in machine learning please refer to [Y. Jin](#) and B. Sendhoff, "Pareto-Based Multiobjective Machine Learning: An Overview and Case Studies", IEEE Trans. on Syst., Man and Cyb., part C, (2008), 38(3):397- 415. For a more general bibliography on EMO, please refer to the [EMOO](#) bibliography page, mantained by [Prof. Carlos A. Coello Coello](#).

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Ishibuchi, H., Murata, T., Turksen, I.B.	Selecting linguistic classification rules by two-objective genetic algorithms	1995	in: Proc. of the 1995 IEEE International Conference on Systems, Man and Cybernetics	inproceedings
Ishibuchi, H., Murata, T.	Minimizing the fuzzy rule base and maximizing its performance by a multi-objective	1997	in: Proc. of the 1997 IEEE International Conference on Fuzzy Systems	inproceedings

Contents of This Presentation

1. Introduction to Fuzzy Rule-Based Classification

- Is Fuzzy Rule-Based Classification a Popular Research Area?

2. Fuzzy Rule-Based Classifier Design

- Accuracy Improvement
- Scalability to High-Dimensional Problems
- Complexity Minimization

3. Multiobjective Fuzzy Rule-Based Classifier Design

- **Formulation of Multi-objective Problems**
- Accuracy-Complexity Tradeoff Analysis
- Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions

- Search Ability of EMO for Fuzzy System Design
- Definition of Interpretability of Fuzzy Systems
- Explanation Ability of Fuzzy Rule-Based Systems
- Various Classification Problems: Imbalanced, Online, ...

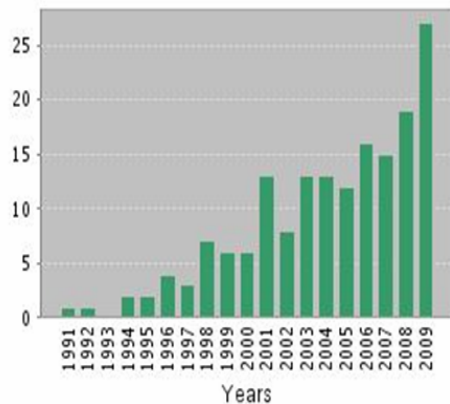
Multi-Objective Fuzzy Rule-Based Systems

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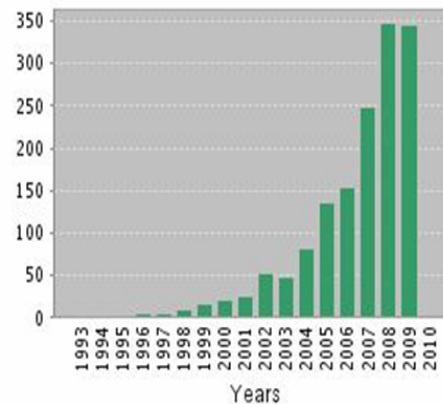
Citation Report Topic=((Fuzzy Rule*) OR (Fuzzy Rule-Based System*)) AND Topic=((Multi-Objective) OR (Multiobjective) OR (Two-Objective) OR (Three-Objective) OR (Multiple Criteria))
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Multi-Objective Fuzzy Rule Selection for Fuzzy Rule-Based Classifier Design

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Two Objectives:

- The number of correctly classified training patterns
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<input type="checkbox"/>	2.	Title: Optimal operation of multireservoir systems: State-of-the-art review Author(s): Labadie JW Source: JOURNAL OF WATER RESOURCES PLANNING AND MANAGEMENT-ASCE Volume: 130 Issue: 2 Pages: 93-111 Published: MAR-APR 2004	14	24	23	30	0	99
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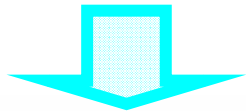
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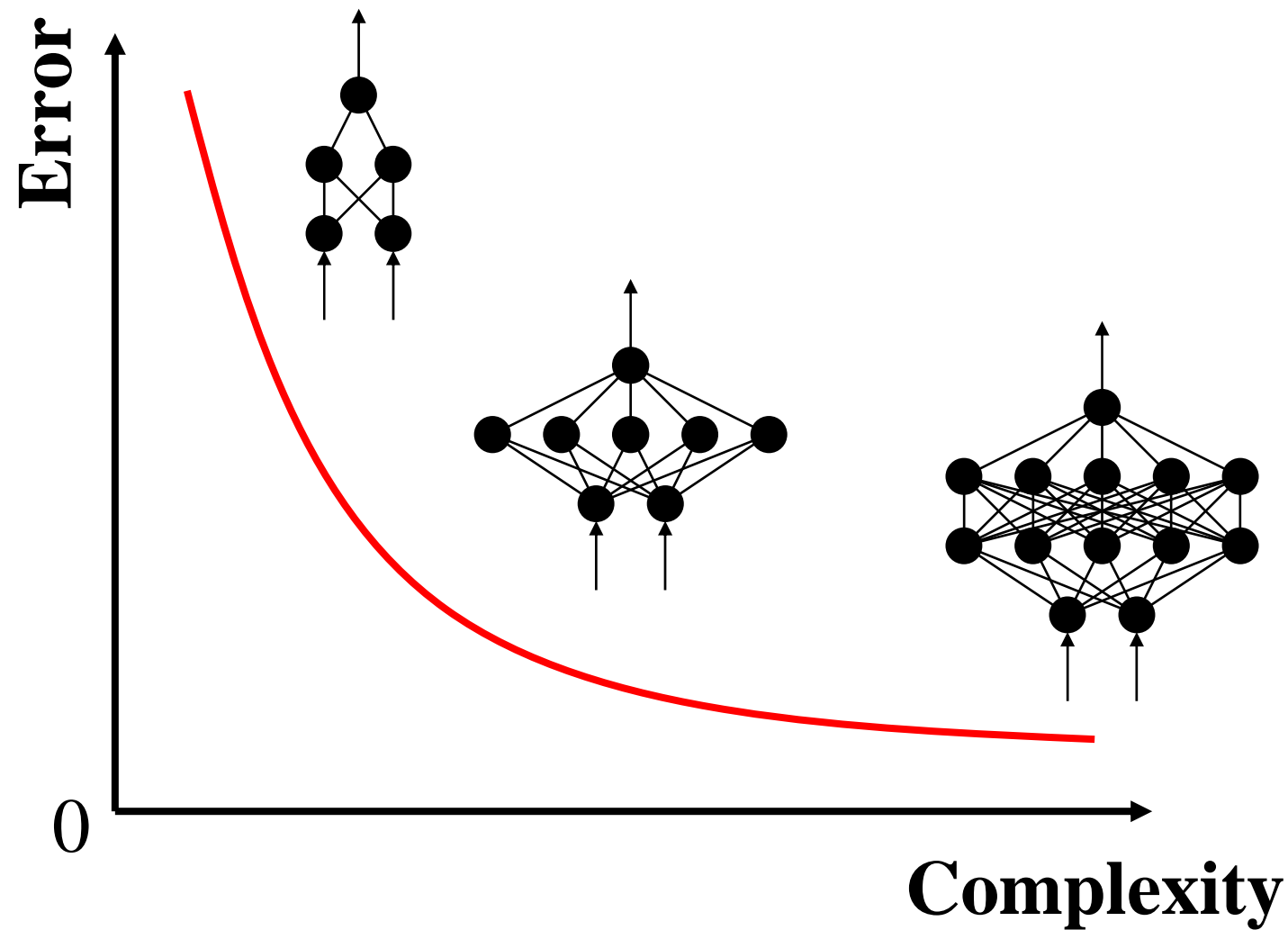
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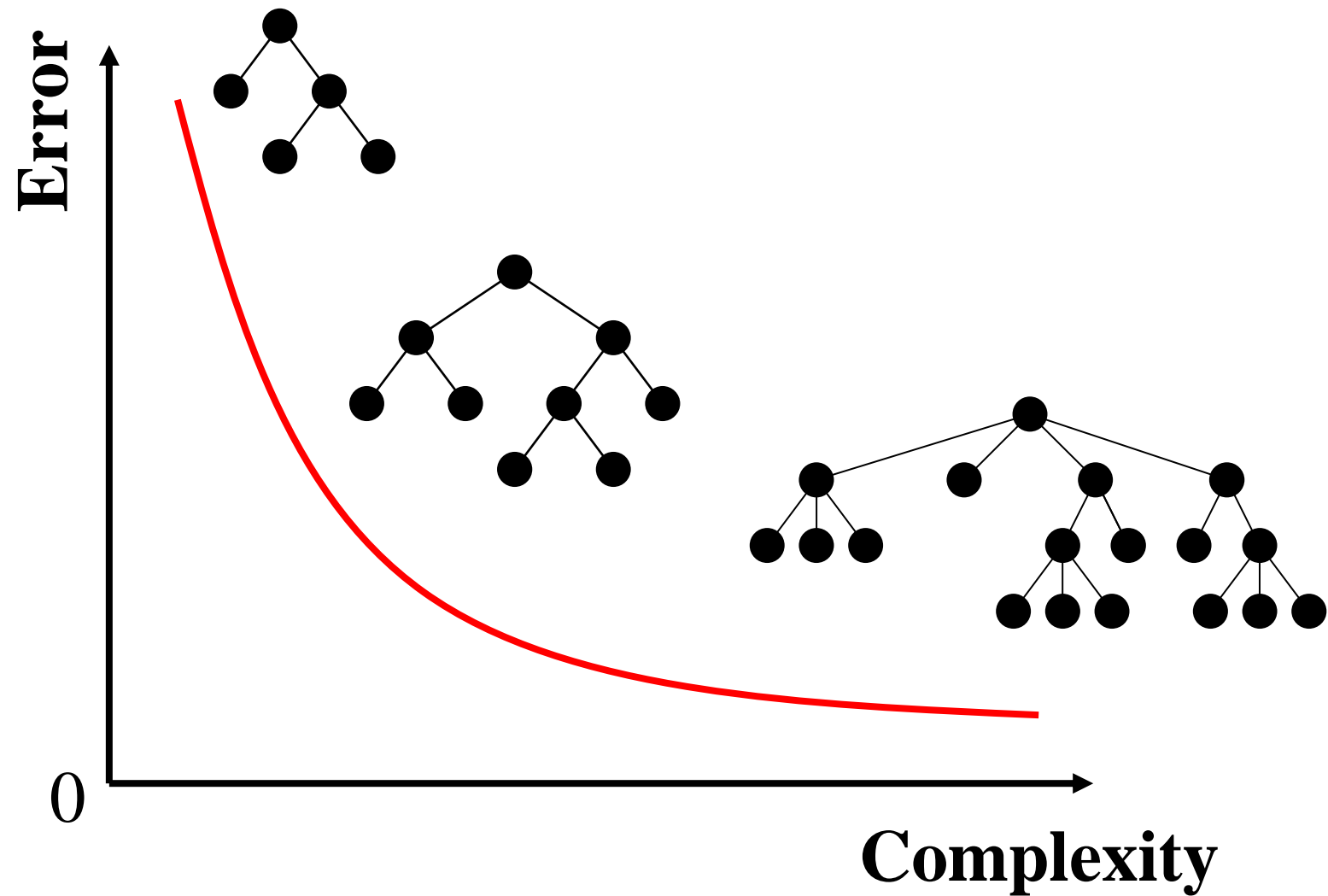
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Multi-Objective Neural Network Design and Learning

Multiobjective Neural Networks



Multiobjective Decision Trees (GP)



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Author(s): Alcala R, Ducange P, Herrera F, et al.
Source: **IEEE TRANSACTIONS ON FUZZY SYSTEMS** Volume: **17** Issue: **5** Pages: **1106-1122**
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- 2. Title: [Optimum energy management of a photovoltaic water pumping system](#)
Author(s): Sallem S, Chaabene M, Kamoun MBA
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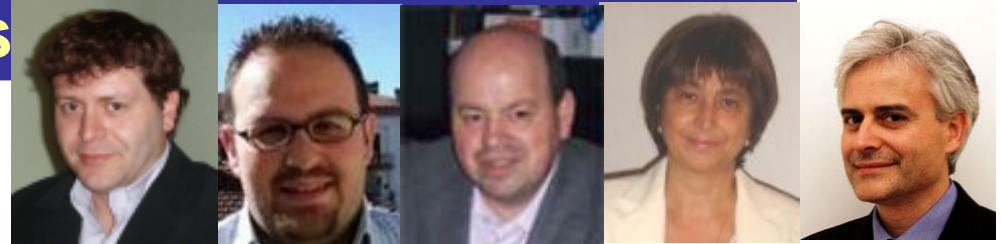
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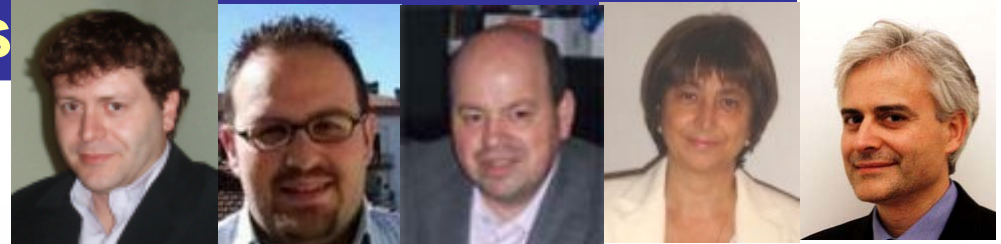
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Paper Title: Learning concurrently partition granularities and rule bases of Mamdani fuzzy systems in a multi-objective evolutionary framework



Author(s): Antonelli M (Antonelli, Michela)¹, Ducange P (Ducange, Pietro)¹, Lazzerini B (Lazzerini, Beatrice)¹, Marcelloni F (Marcelloni, Francesco)¹

Source: INTERNATIONAL JOURNAL OF APPROXIMATE REASONING **Volume:** 50 **Issue:** 7 **Pages:** 1066-1080 **Published:** JUL 2009

[1. Univ Pisa](#)

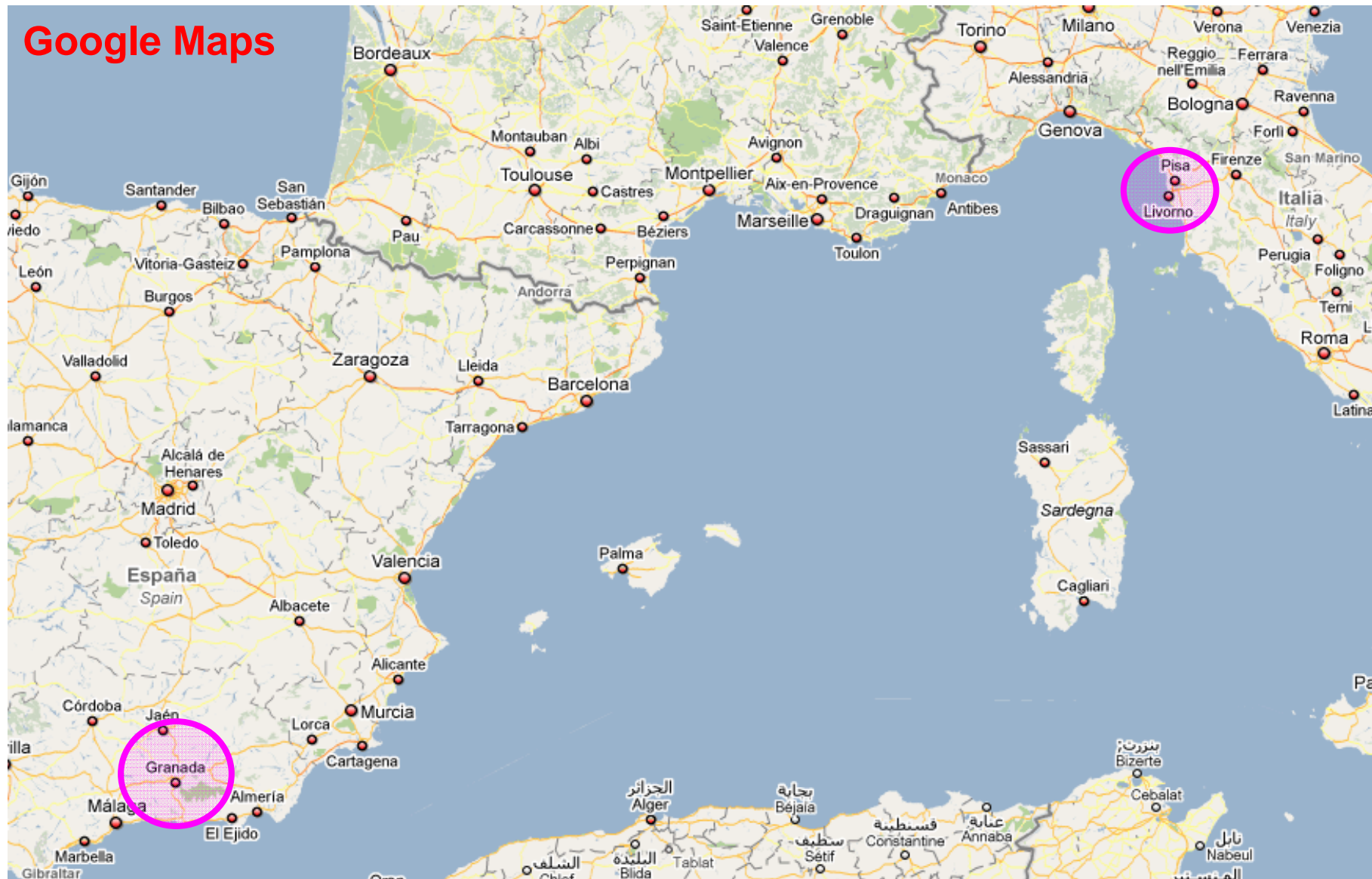
Times Cited: 0 **References:** 43 [Citation Map](#)

[Web of Science](#)

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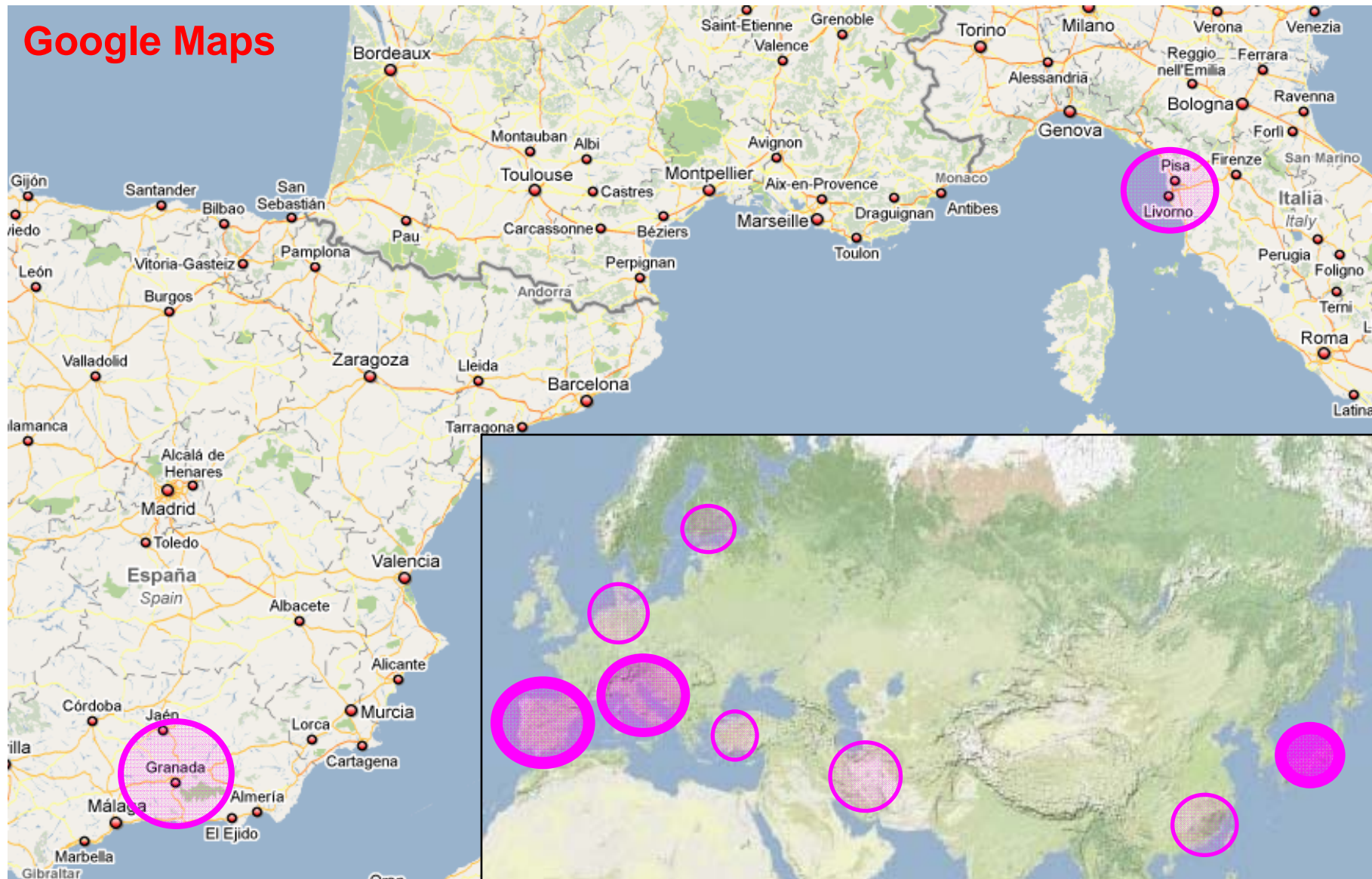
Multi-Objective Fuzzy System Design Research

Active Geographical Regions



Multi-Objective Fuzzy System Design Research

Active Geographical Regions



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4. Current Hot Issues and Future Research Directions

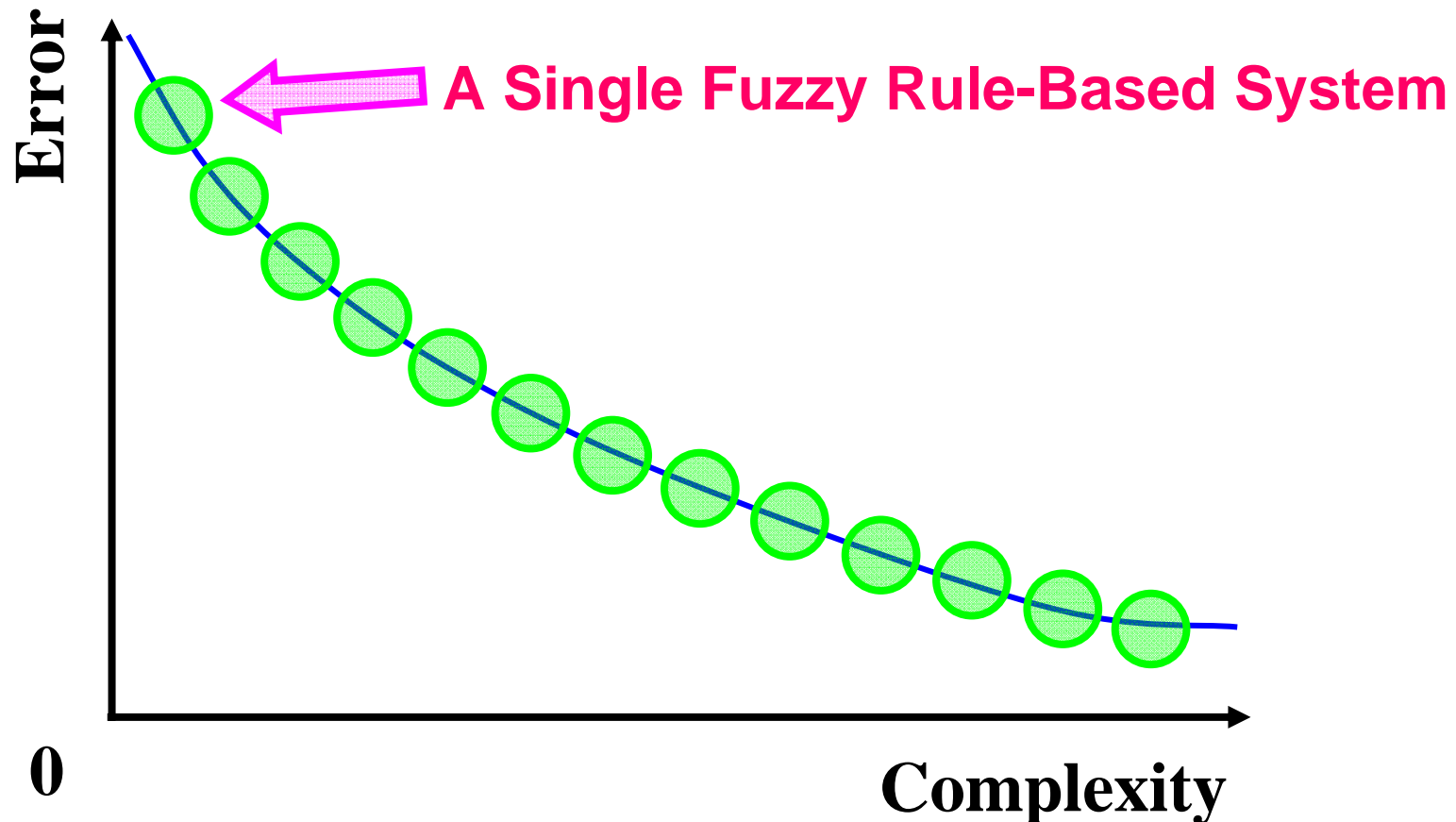
- Search Ability of EMO for Fuzzy System Design
- Definition of Interpretability of Fuzzy Systems
- Explanation Ability of Fuzzy Rule-Based Systems
- Various Classification Problems: Imbalanced, Online, ...

Results of Multi-Objective Search

Non-Dominated Fuzzy Rule-Based Systems

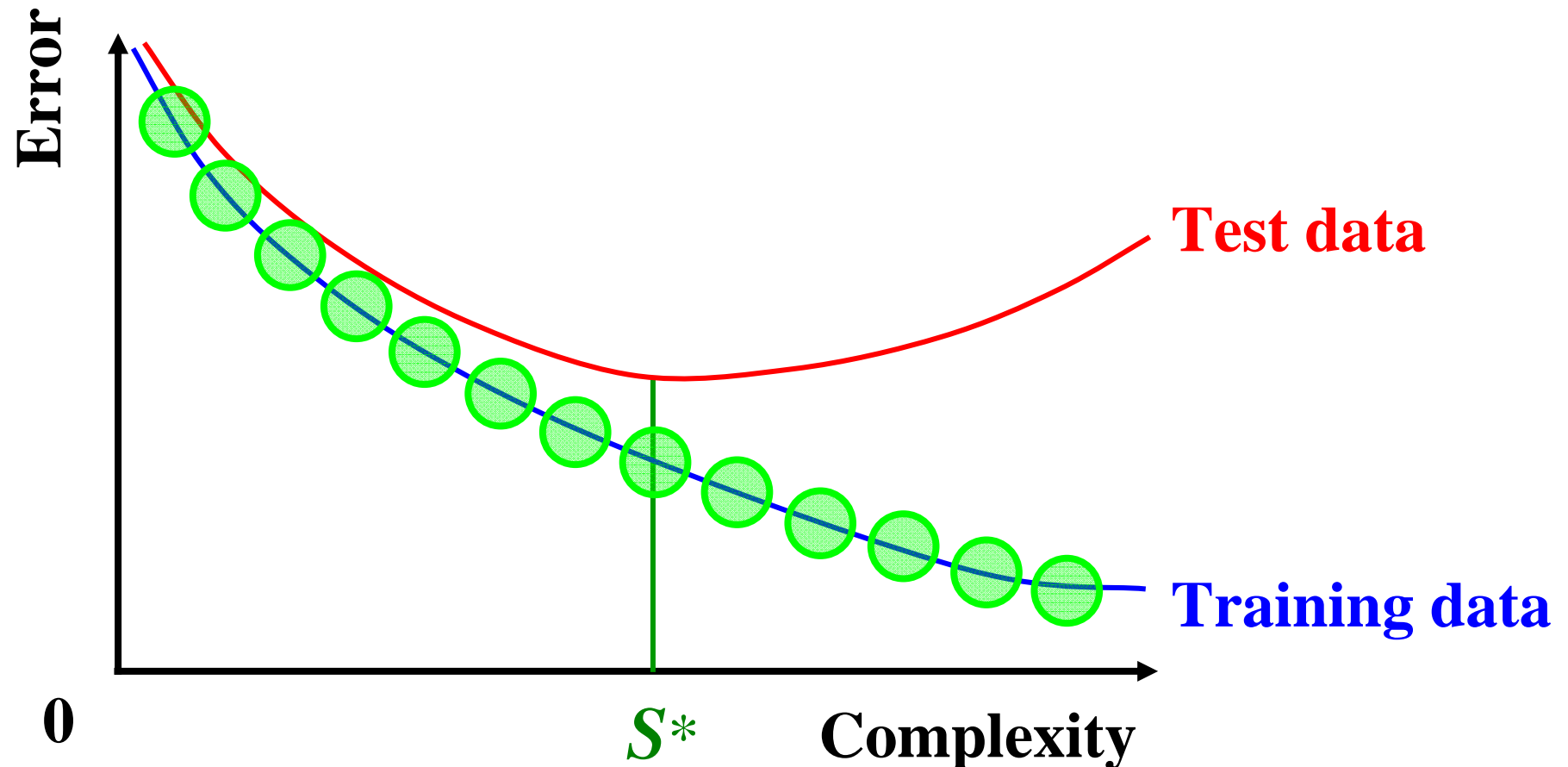
Many non-dominated fuzzy systems can be obtained along the tradeoff surface by a single run of an EMO algorithm.

EMO: Evolutionary Multi-Objective Optimization



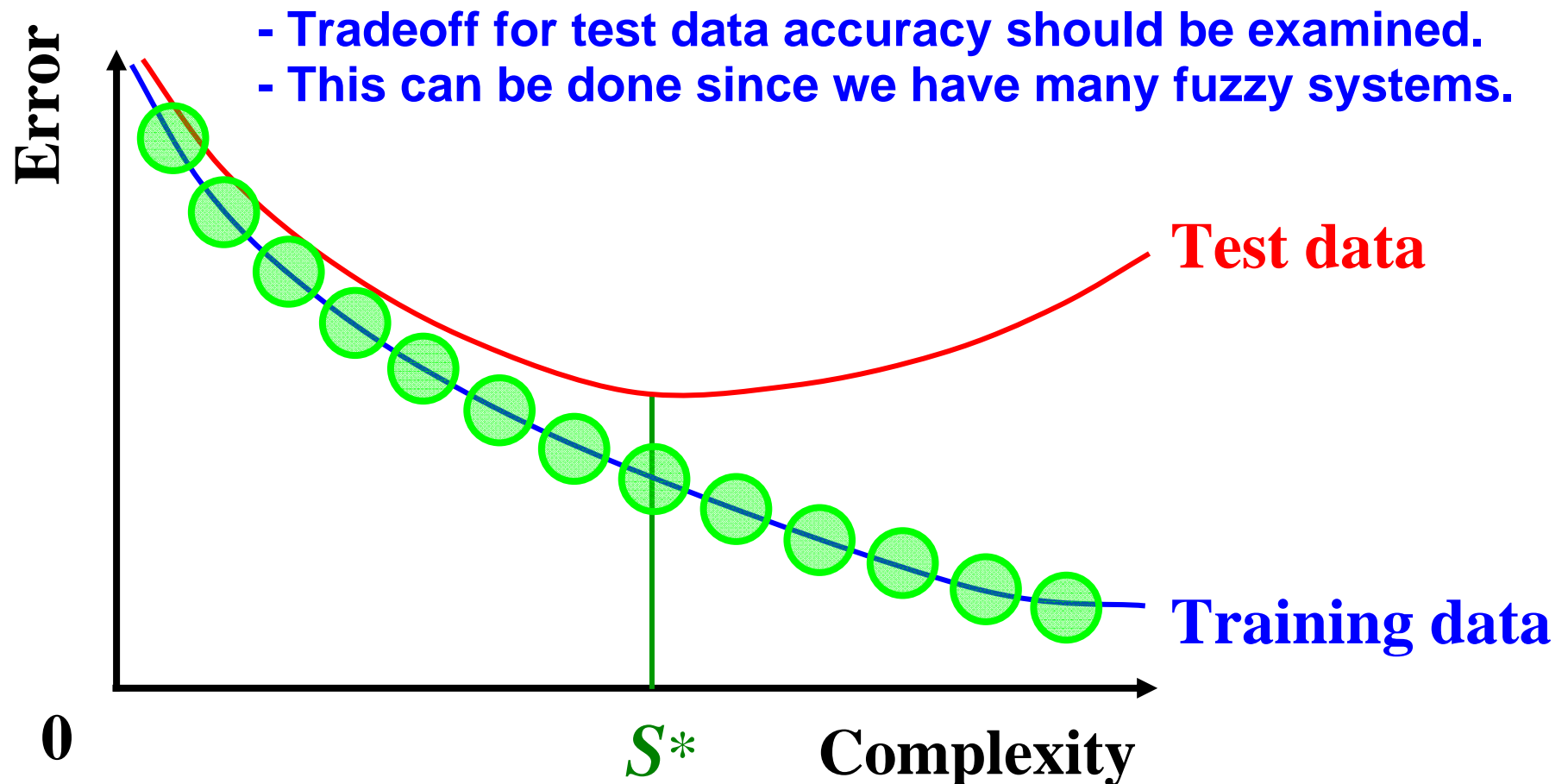
Accuracy-Complexity Tradeoff for Training Data and Test Data

The obtained non-dominated fuzzy systems show the tradeoff between the complexity and the training data accuracy (not the tradeoff between the complexity and the test data accuracy).

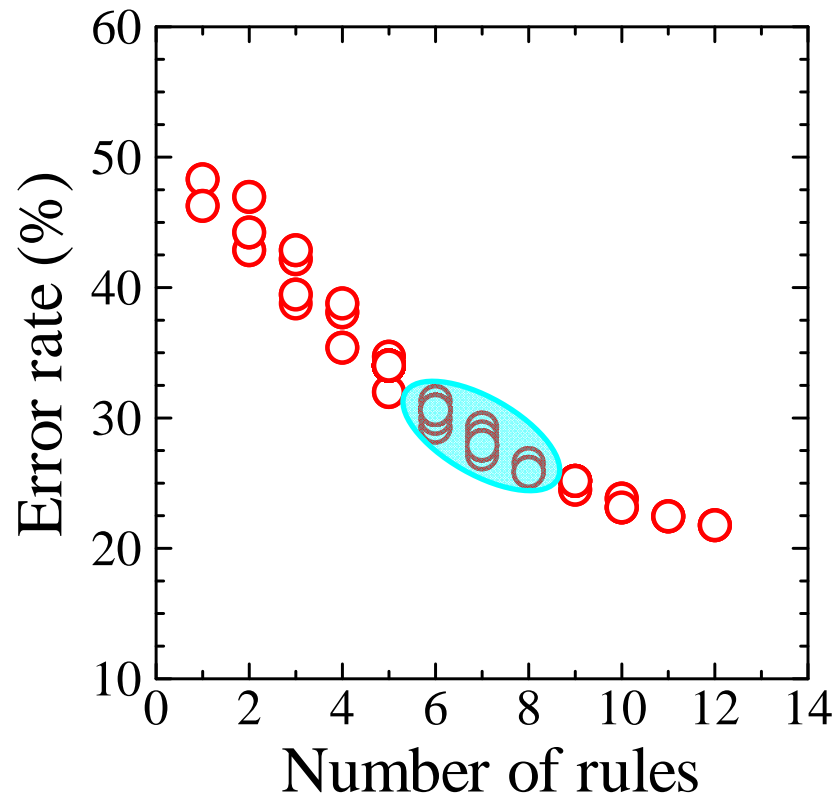


Accuracy-Complexity Tradeoff for Training Data and Test Data

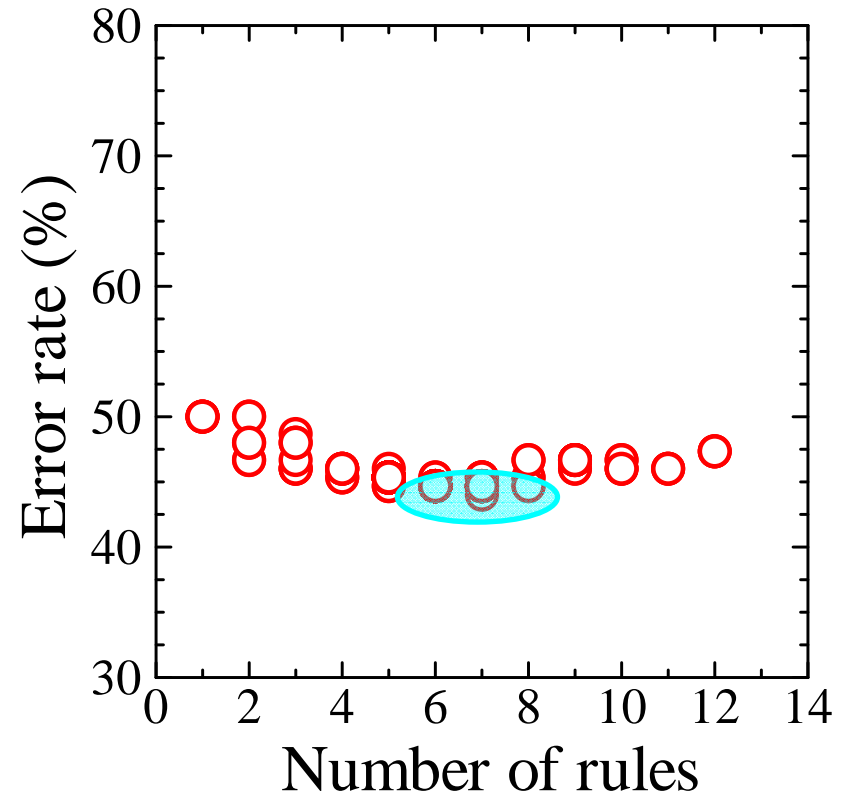
The obtained non-dominated fuzzy systems show the tradeoff between the complexity and the training data accuracy (not the tradeoff between the complexity and the test data accuracy).



Example: Obtained Rule Sets (Heart C)



Training data accuracy

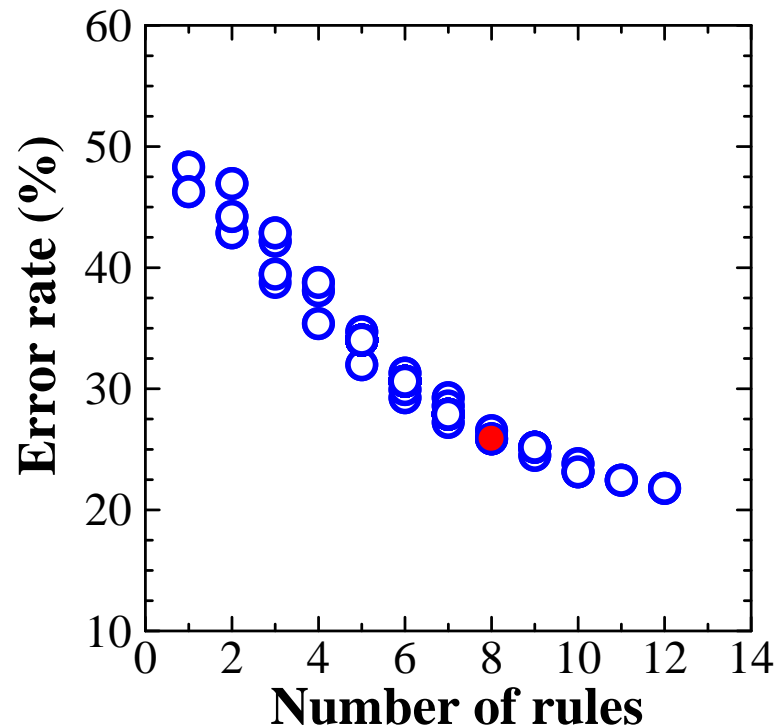


Testing data accuracy

Obtained rule sets help us to find the optimal complexity of fuzzy systems. (Rule sets with six, seven and eight rules may be good)

A Rule Set with High-Generalization Ability

A rule set with eight fuzzy rules

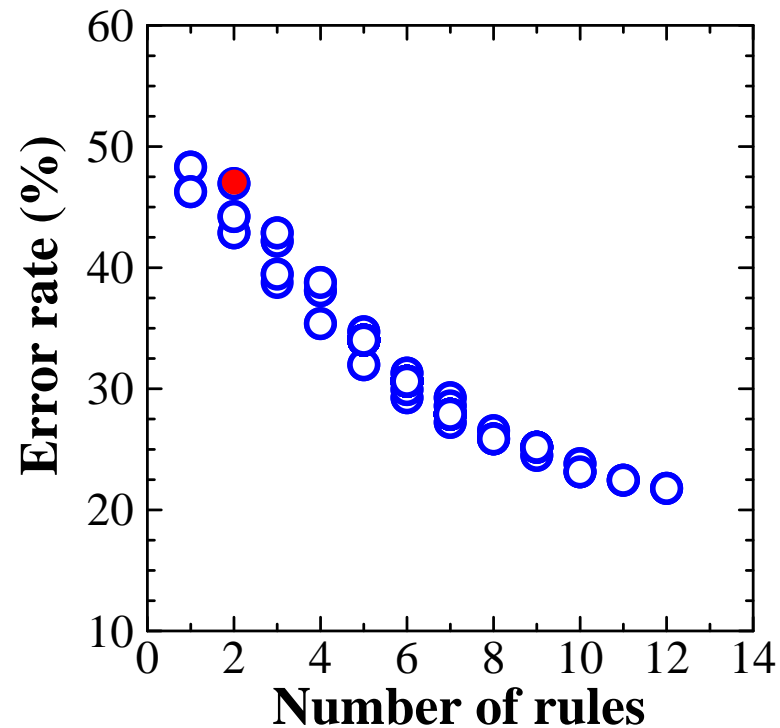


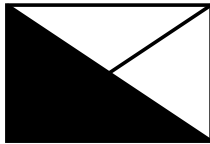
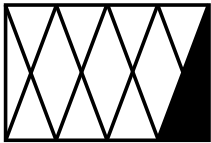
	x_1	x_3	x_4	x_6	x_7	x_8	x_{10}	x_{11}	x_{12}	Consequent
R_1	DC	DC	▲	DC	DC	DC	DC	DC	▲	Class 1 (0.46)
R_2	DC	▲	DC	DC	DC	DC	DC	▲	▲	Class 1 (0.23)
R_3	DC	DC	DC	DC	DC	▲	DC	DC	DC	Class 1 (0.81)
R_4	▲	DC	▲	DC	DC	DC	DC	▲	DC	Class 2 (0.63)
R_5	DC	DC	DC	▲	DC	DC	DC	▲	▲	Class 2 (0.20)
R_6	DC	DC	DC	DC	DC	DC	▲	DC	DC	Class 2 (1.00)
R_7	▲	DC	DC	DC	DC	DC	▲	DC	▲	Class 3 (0.35)
R_8	DC	DC	DC	DC	▲	DC	DC	DC	DC	Class 3 (0.24)

Some human users may prefer simpler rule sets.

A Rule Set with High Interpretability

A very simple rule set with only two fuzzy rules



	x_{10}	x_{11}	Consequent
R_1	DC		Class 1 (0.26)
R_2		DC	Class 2 (1.00)

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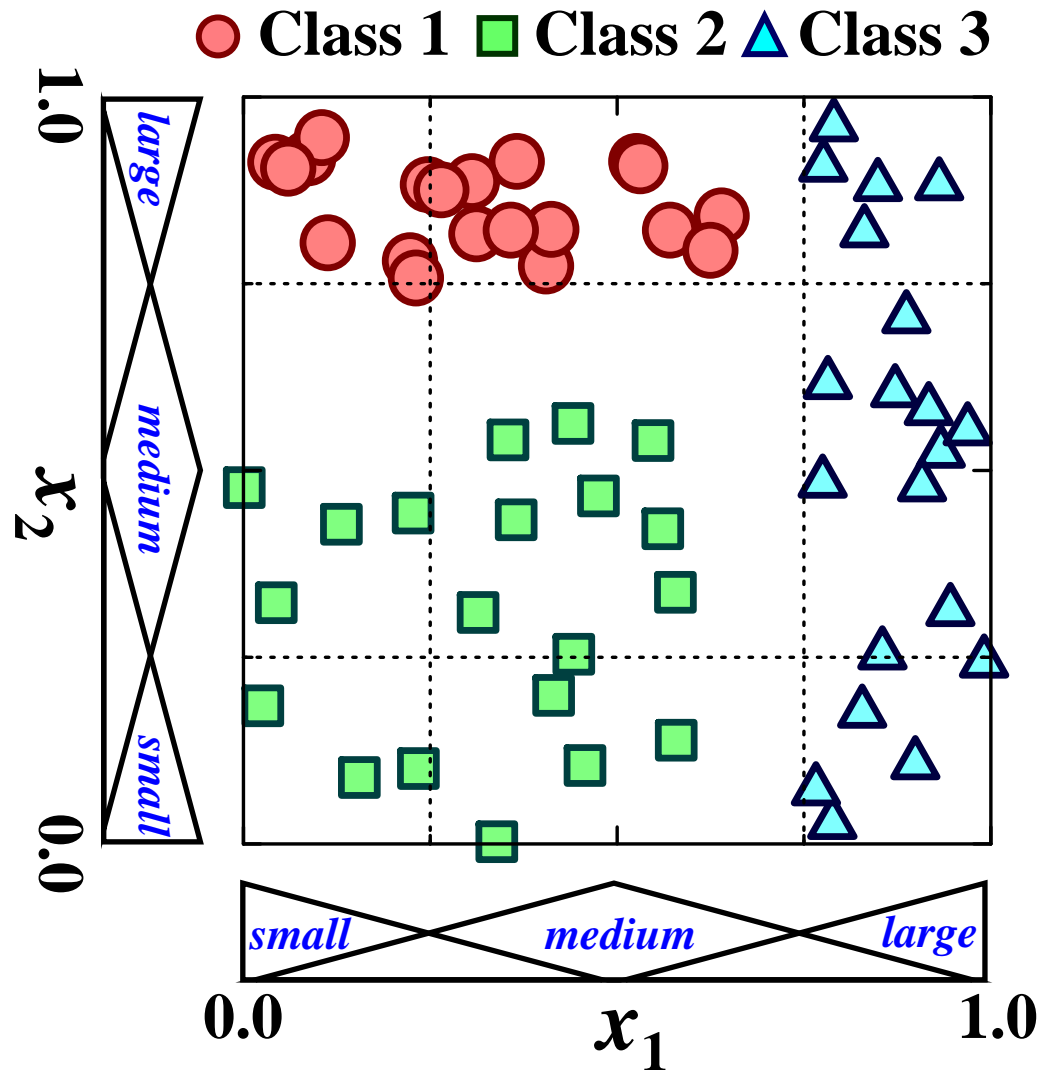
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Why is the fuzzy system design difficult?

1. Large Search Space: Difficulty in Search

The search space exponentially increases with the number of attributes (i.e., with the dimensionality of the pattern space).

Number of Fuzzy Rules



Basic Form

If x_1 is *small* and x_2 is *small*
then Class 2

If x_1 is *small* and x_2 is *medium*
then Class 2

...

If x_1 is *large* and x_2 is *large*
then Class 3

Number of Fuzzy Rules:

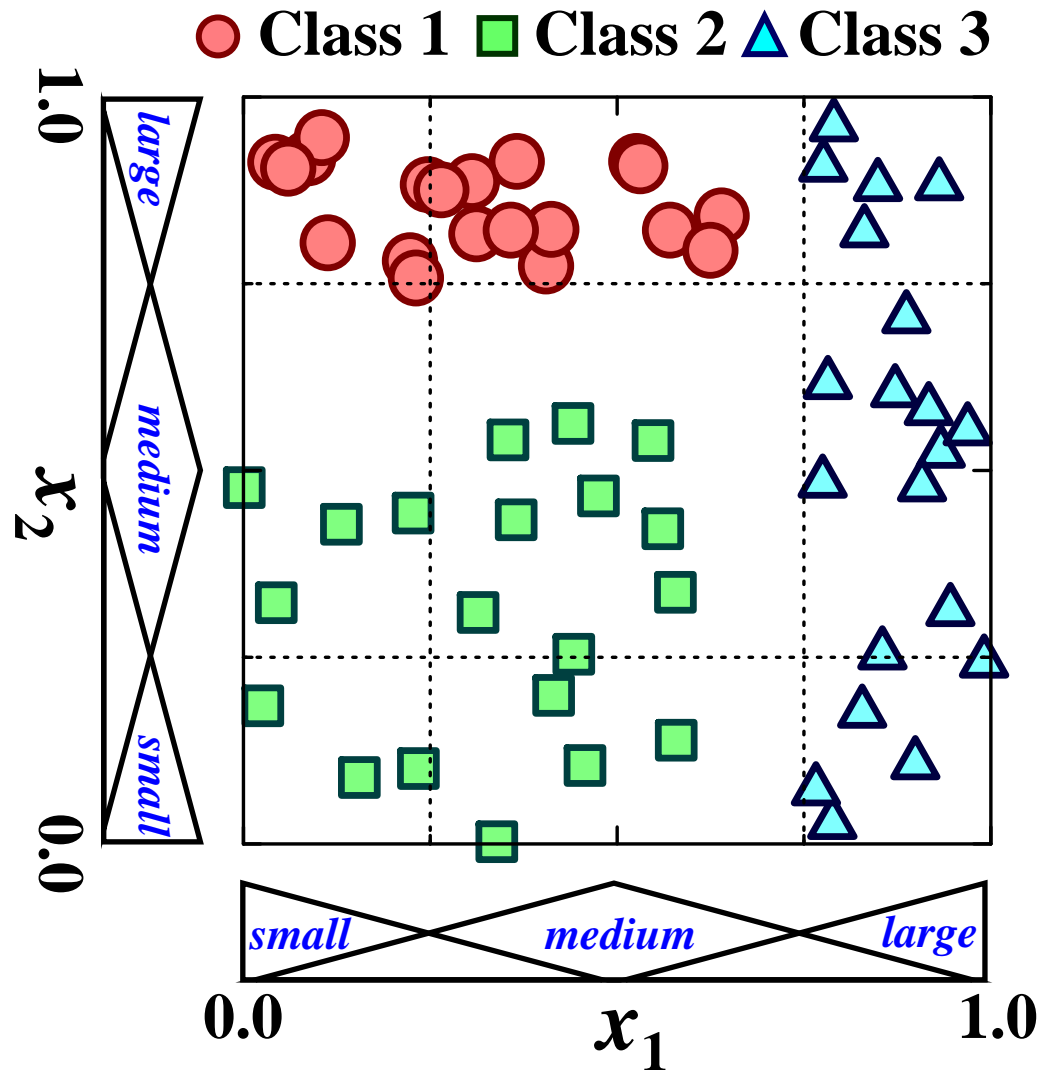
2-D Problem: 3×3

3-D Problem: $3 \times 3 \times 3$

4-D Problem: $3 \times 3 \times 3 \times 3$

5-D Problem: $3 \times 3 \times 3 \times 3 \times 3$

Number of Fuzzy Rules



Use of Don't Care

If x_1 is *small* and x_2 is *small*
then Class 2

If x_1 is *small* and x_2 is *medium*
then Class 2

...

If x_1 is *large* and
 x_2 is *don't care* then Class 3

Number of Fuzzy Rules:

2-D Problem: $(3+1) \times (3+1)$

3-D Problem: $(3+1)^3$

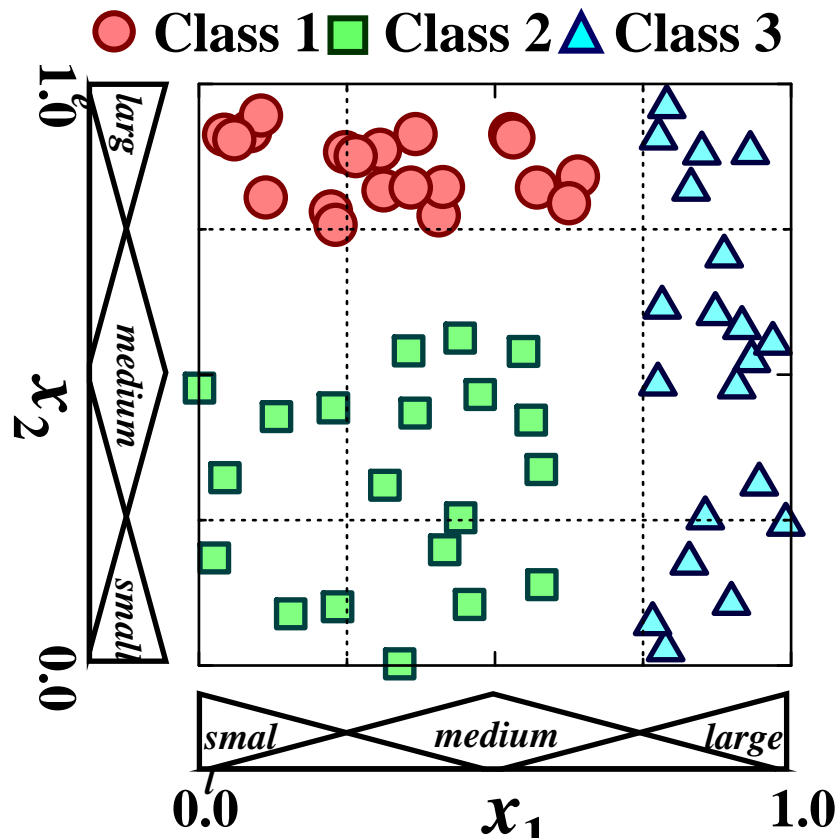
4-D Problem: $(3+1)^4$

5-D Problem: $(3+1)^5$

Number of Fuzzy Rules and Number of Rule Sets

Search Space Size: Large

Example: Classification problem with **50 attributes**
and **3 linguistic values** for each attribute



The total number of fuzzy rules (i.e., antecedent condition combinations):

$$(3+1) \times \dots \times (3+1) = 4^{50} = 2^{100}$$

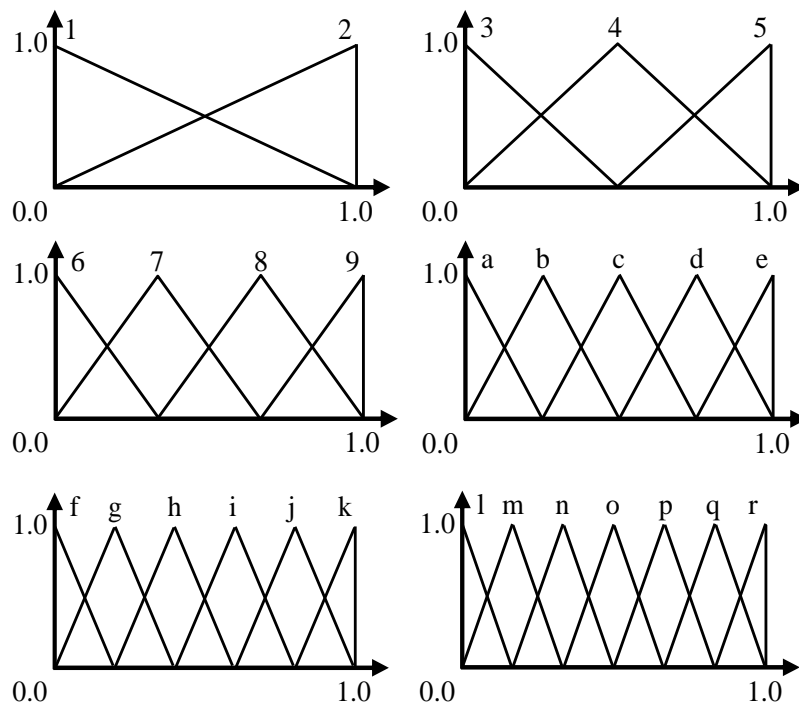
The total number of fuzzy rule sets with 20 rules (i.e., combinations of 20 fuzzy rules):

$$N C_{20} \sim 2^{2000} \text{ where } N = 2^{100}$$

Number of Fuzzy Rules and Number of Rule Sets

Search Space Size: Large

Example: Classification problem with **50 attributes**
and **1-7 fuzzy partition** for each attribute



The total number of fuzzy rules (i.e., antecedent condition combinations):

$$(1+2+ \dots 7) \times \dots = 28^{50} > 2^{400}$$

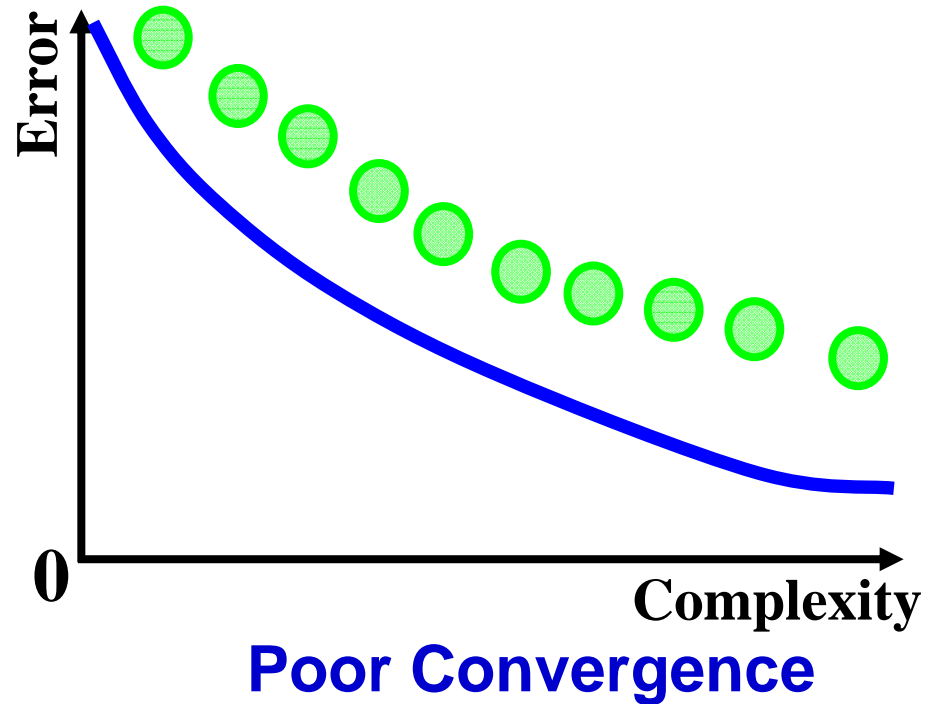
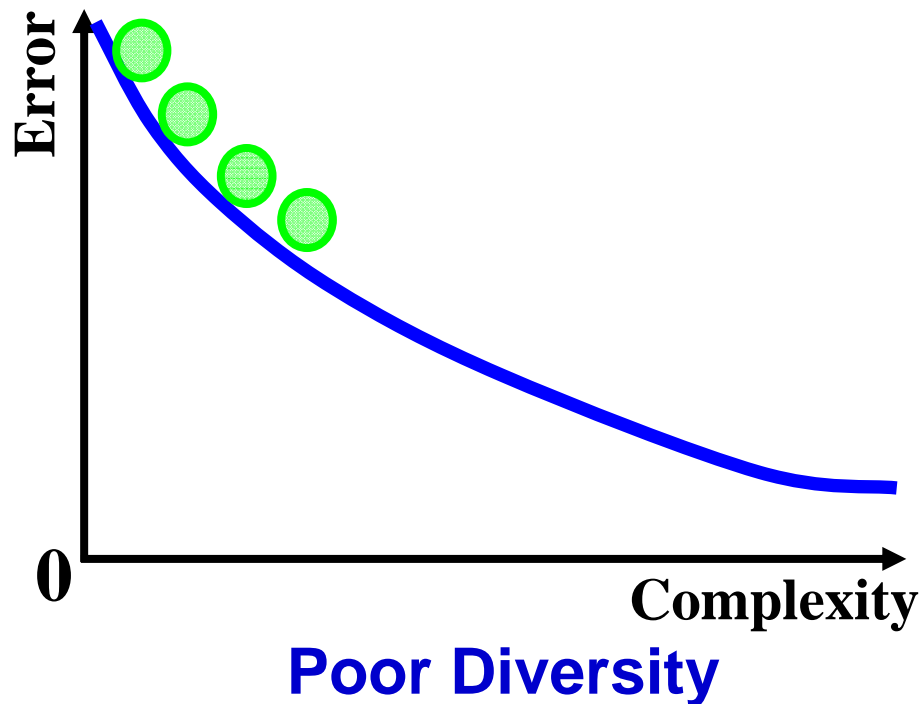
The total number of fuzzy rule sets with 20 rules (i.e., combinations of 20 fuzzy rules):

$$N C_{20} \sim 2^{8000} \text{ where } N > 2^{400}$$

Why is the fuzzy system design difficult?

1. Large Search Space: Difficulty in Search

The search space exponentially increases with the number of attributes (i.e., with the dimensionality of the pattern space). **It is likely that the entire tradeoff curve can not be covered well by the obtained non-dominated fuzzy rule-based systems.**



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The improvement in the training data accuracy does not always mean the improvement in the test data accuracy.

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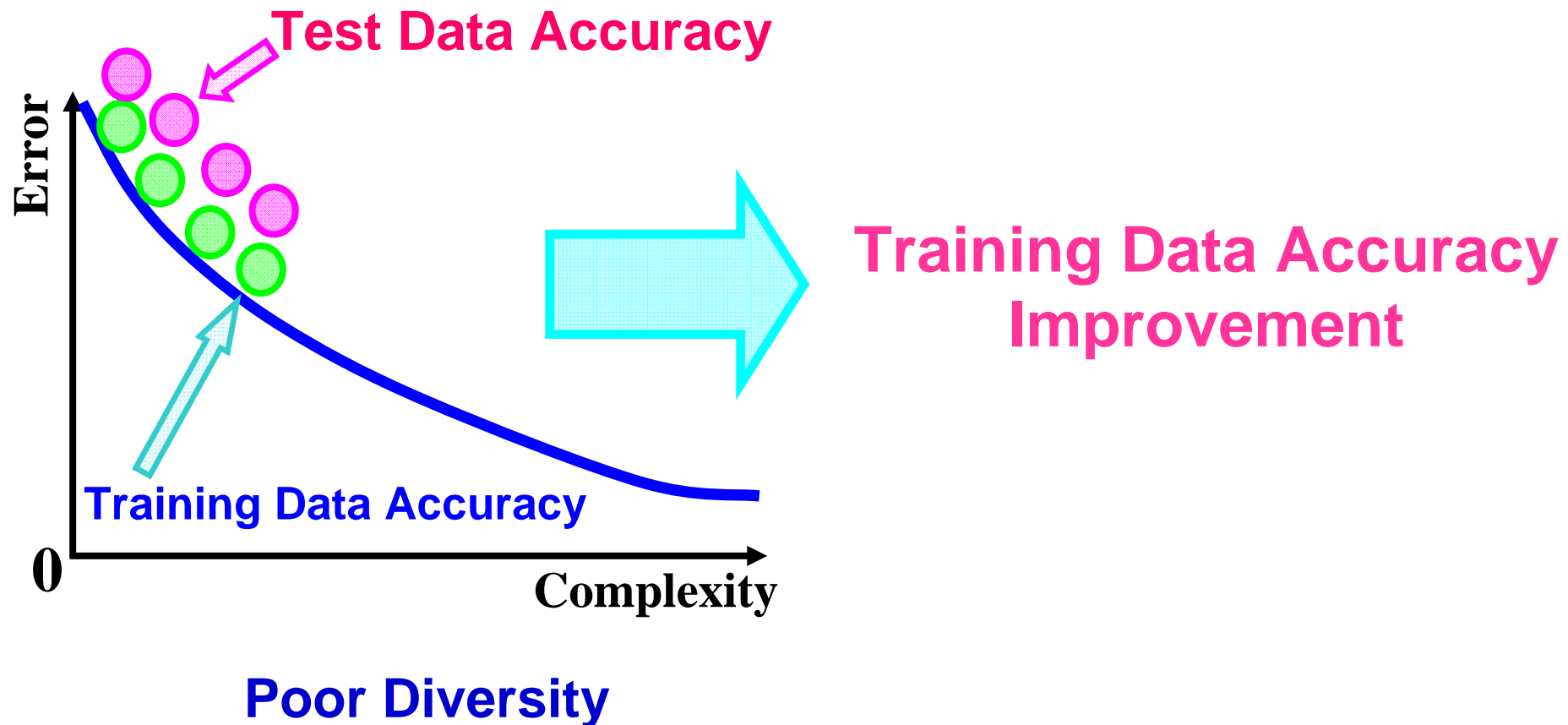
2. Possibility of Over-Fitting: Difficulty in Learning

The improvement in the training data accuracy does not always mean the improvement in the test data accuracy. **This means that the fitness function improvement does not always lead to better fuzzy rule-based classifiers (when the training data accuracy is used in the fitness function) .**

Why is the fuzzy system design difficult?

2. Possibility of Over-Fitting: Difficulty in Learning

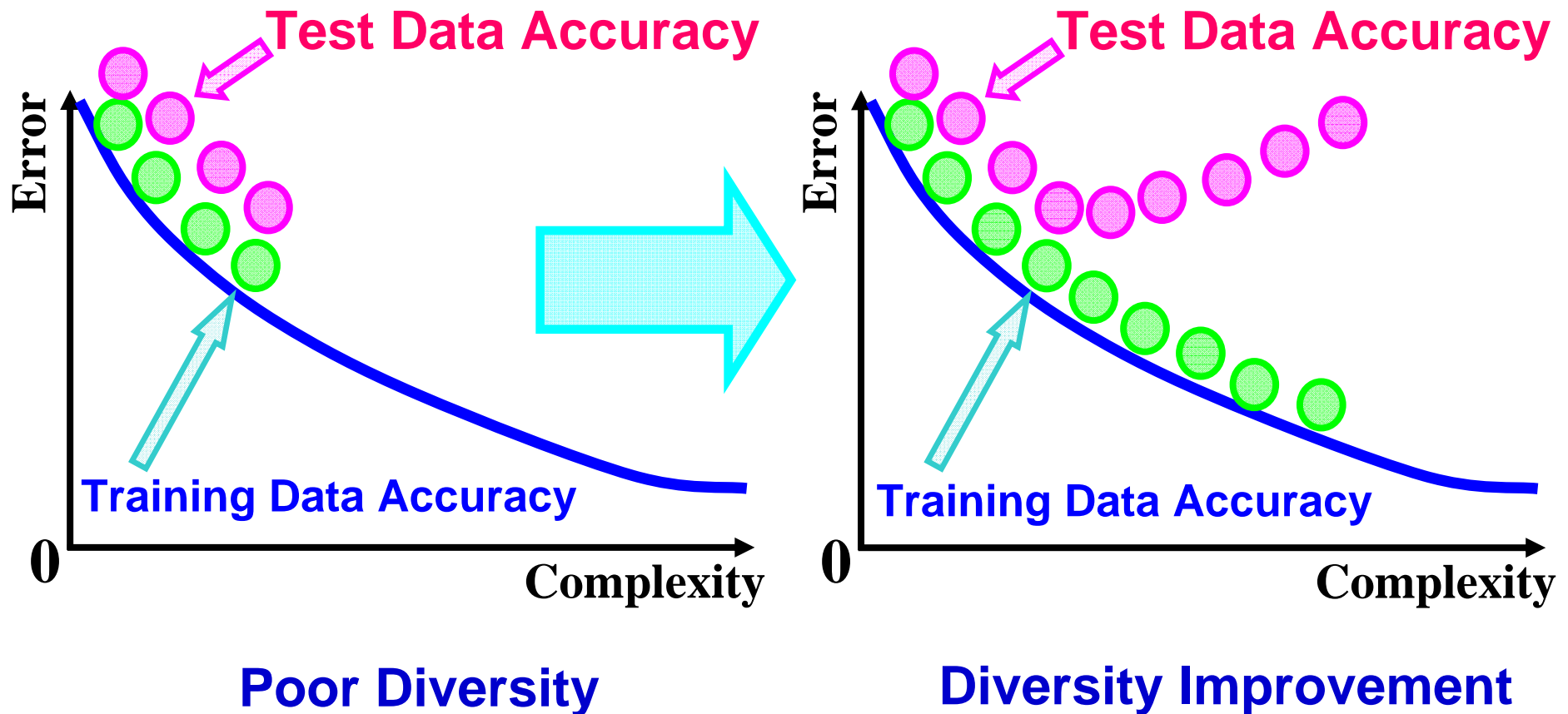
The improvement in the training data accuracy does not always mean the improvement in the test data accuracy.



Why is the fuzzy system design difficult?

2. Possibility of Over-Fitting: Difficulty in Learning

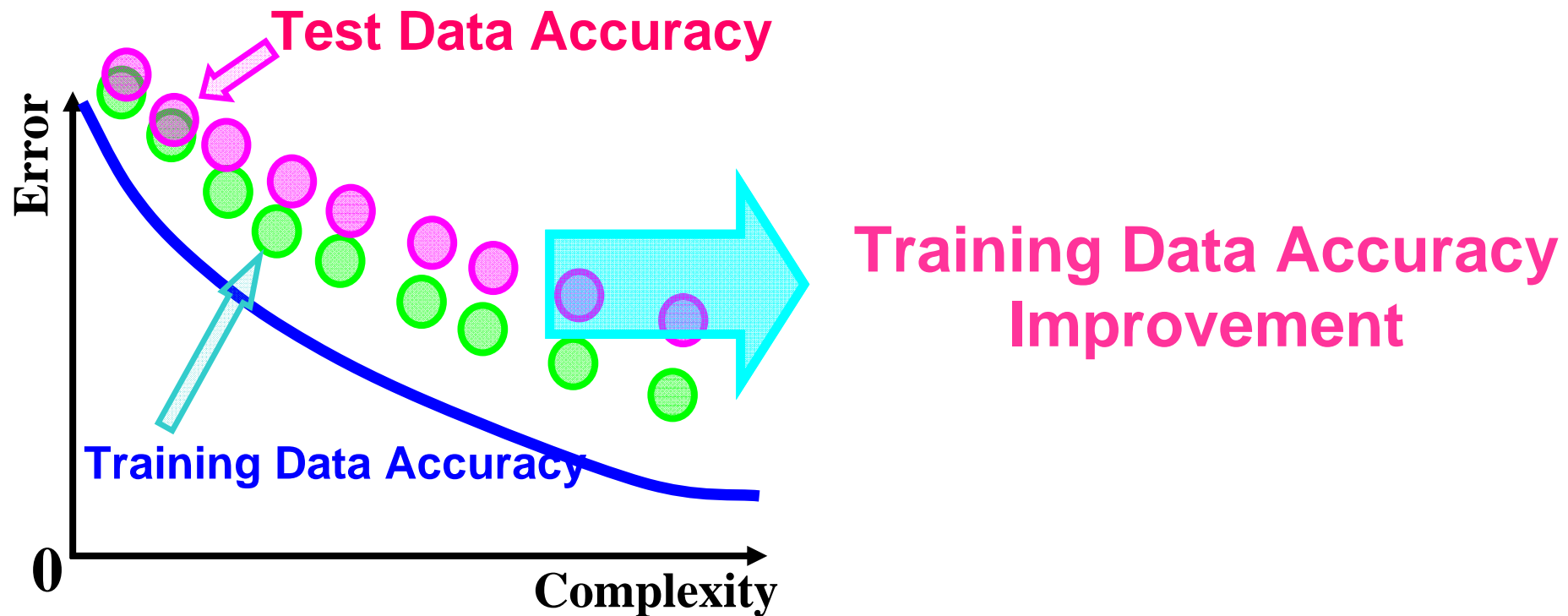
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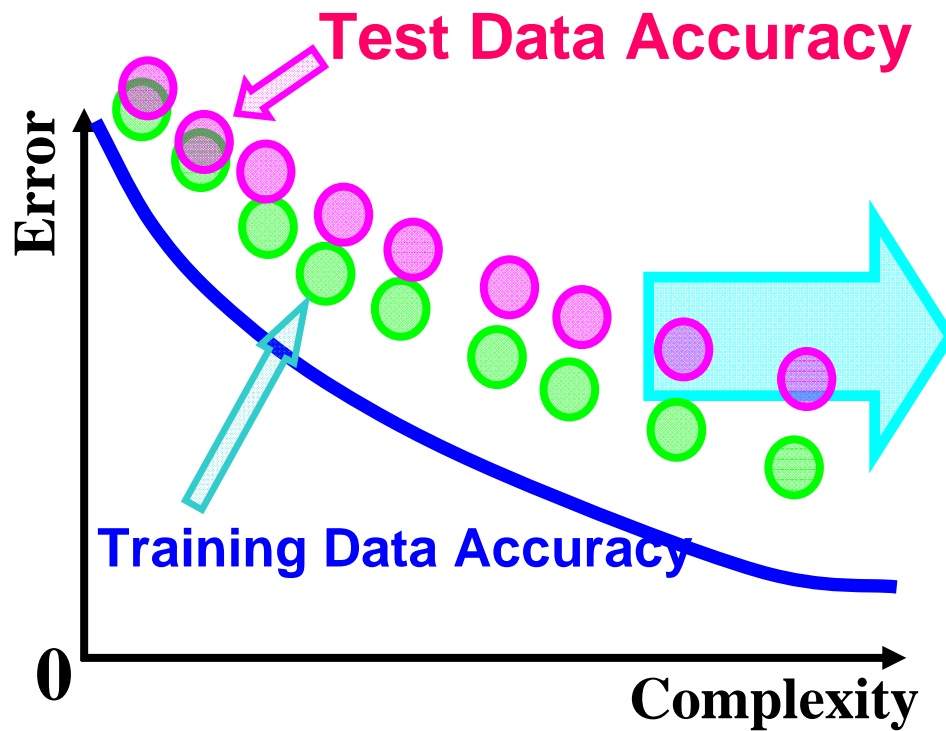


Poor Convergence

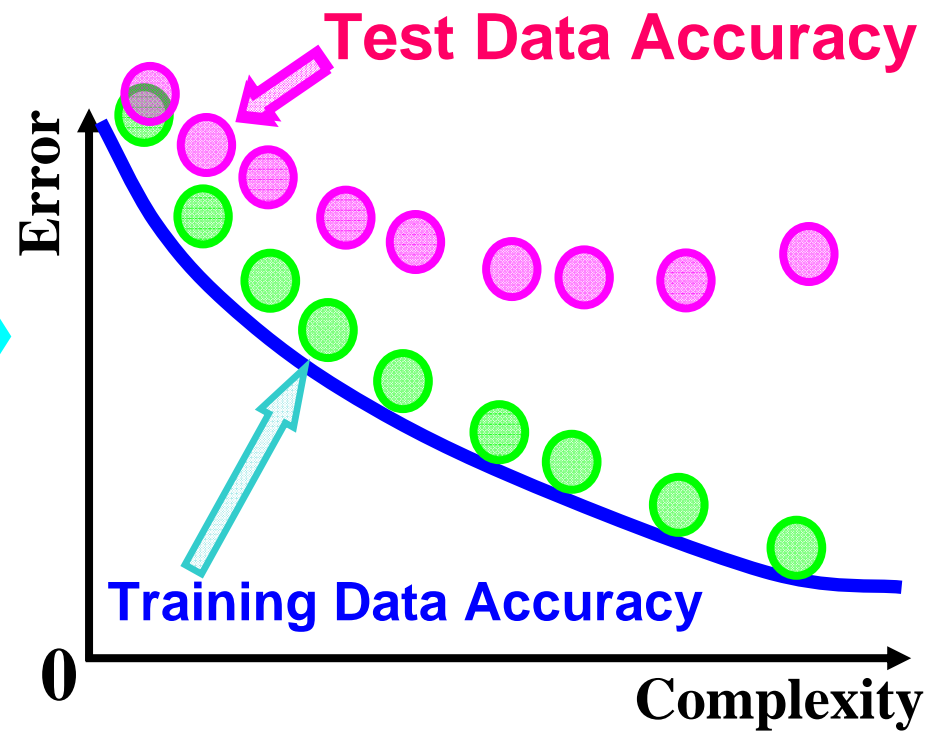
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2. Possibility of Over-Fitting: Difficulty in Learning

The improvement in the training data accuracy does not always mean the improvement in the test data accuracy.



Poor Convergence

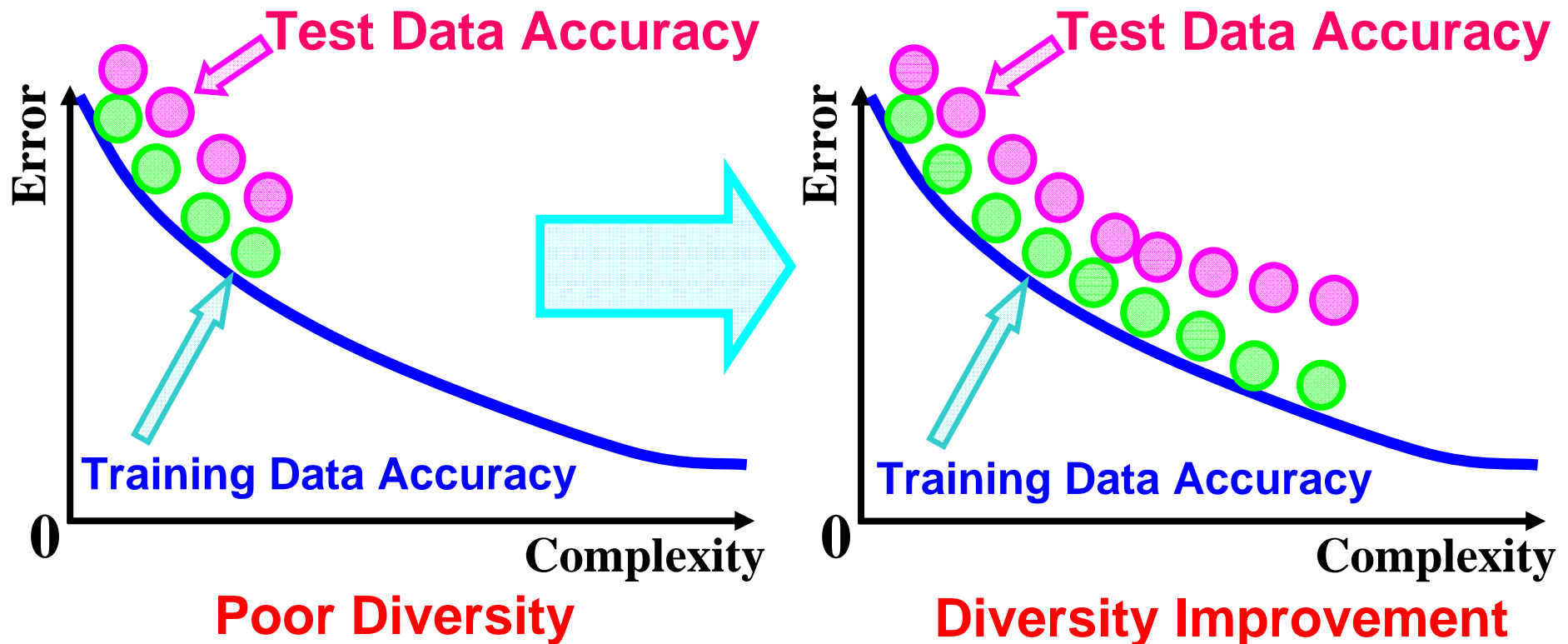


Convergence Improvement

Recent Studies: Improvement in training data accuracy leads to Improvement in test data accuracy.

Gacto MJ, Alcalá R, Herrera F (2009) **Adaptation and application of multi-objective evolutionary algorithms for rule reduction and parameter tuning of fuzzy rule-based systems**, *Soft Computing* 13 (5): 419-436

Ishibuchi H, Nakashima Y, Nojima Y (2009) **Performance evaluation of evolutionary multiobjective optimization algorithms for multiobjective fuzzy genetics-based machine learning**, *Proc. of FUZZ-IEEE 2009*.



Our Experimental Results

MoFGBML Algorithm (Framework: NSGA-II)

Multi-Objective Fuzzy Genetics-Based Machine Learning

[Analysis of interpretability-accuracy tradeoff of fuzzy systems by multiobjective fu](#)

[H Ishibuchi, Y Nojima - International Journal of Approximate Reasoning, 2007 - Elsevier](#)

This paper examines the interpretability-accuracy tradeoff in fuzzy rule-based classifiers using a multiobjective fuzzy genetics-based machine learning (GBML)

algorithm. Our GBML algorithm is a hybrid version of Michigan and ... [Google Scholar](#)

[Cited by 63](#) - [Related articles](#) - [All 6 versions](#) **Ishibuchi & Nojima, IJAR 2007**

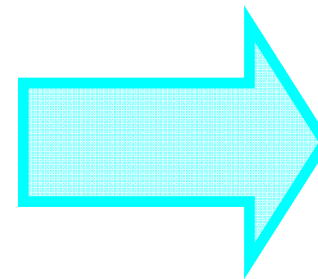
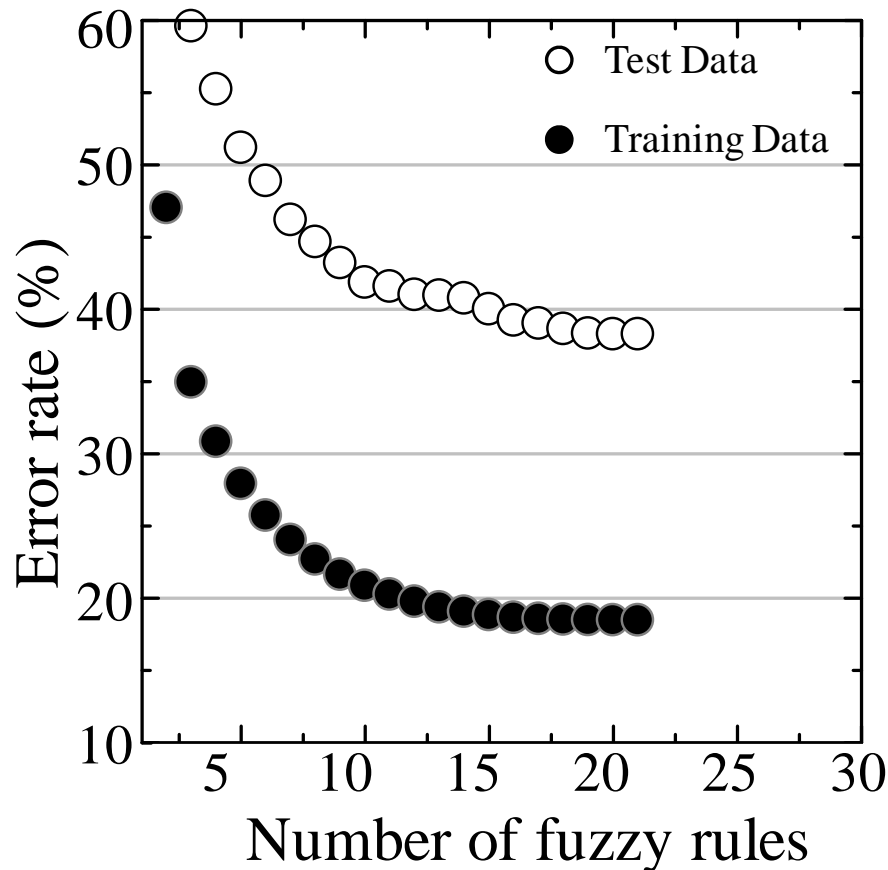
NSGA-II Basic Setting

- Population size: 200 individuals
- Termination Condition: 2000 generations
- Multiple Fuzzy Partitions: Granularities 1-5

Three Variants of MoFGBML Setting

- Diversity Improvement Method (Mating, EJOR 2008)
- Termination Condition: 20000 generations
- Multiple Fuzzy Partitions: Granularities 7

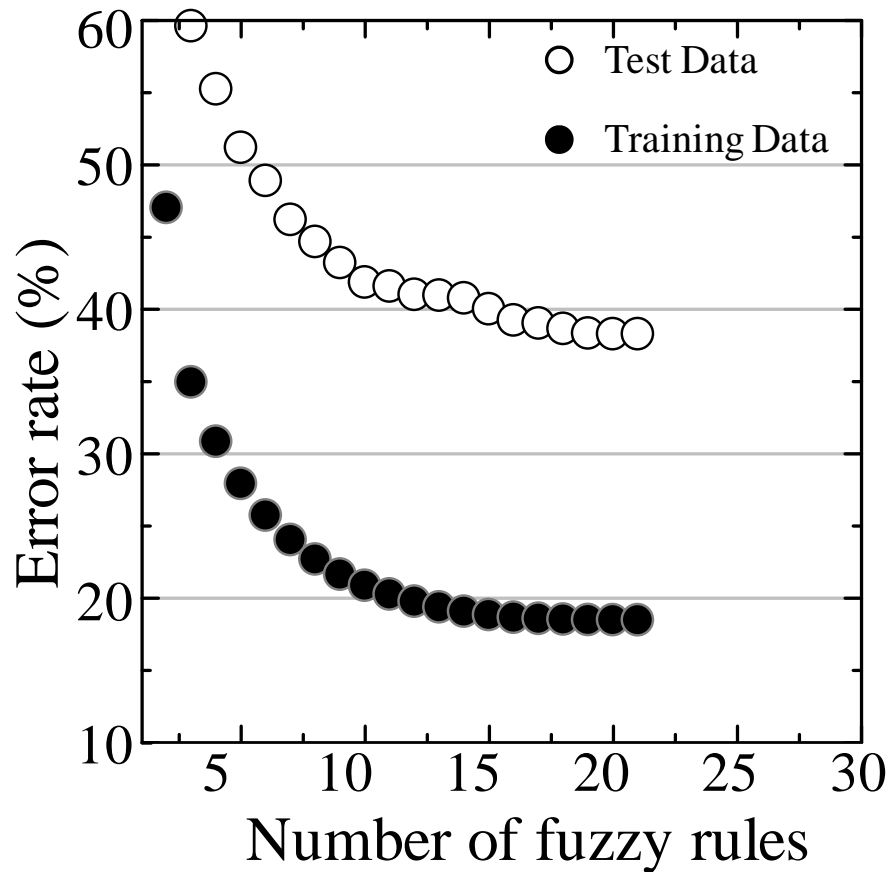
Experimental Results (Glass)



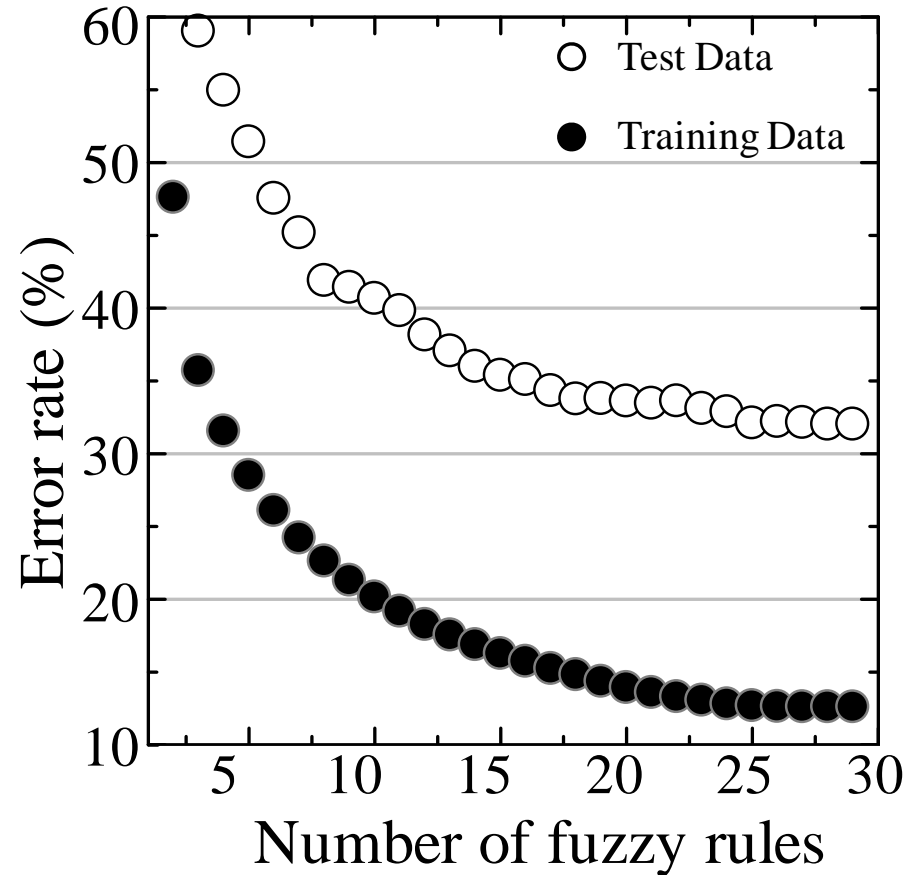
**Training Data
Accuracy
Improvement**

NSGA-II Basic Setting

Experimental Results (Glass)

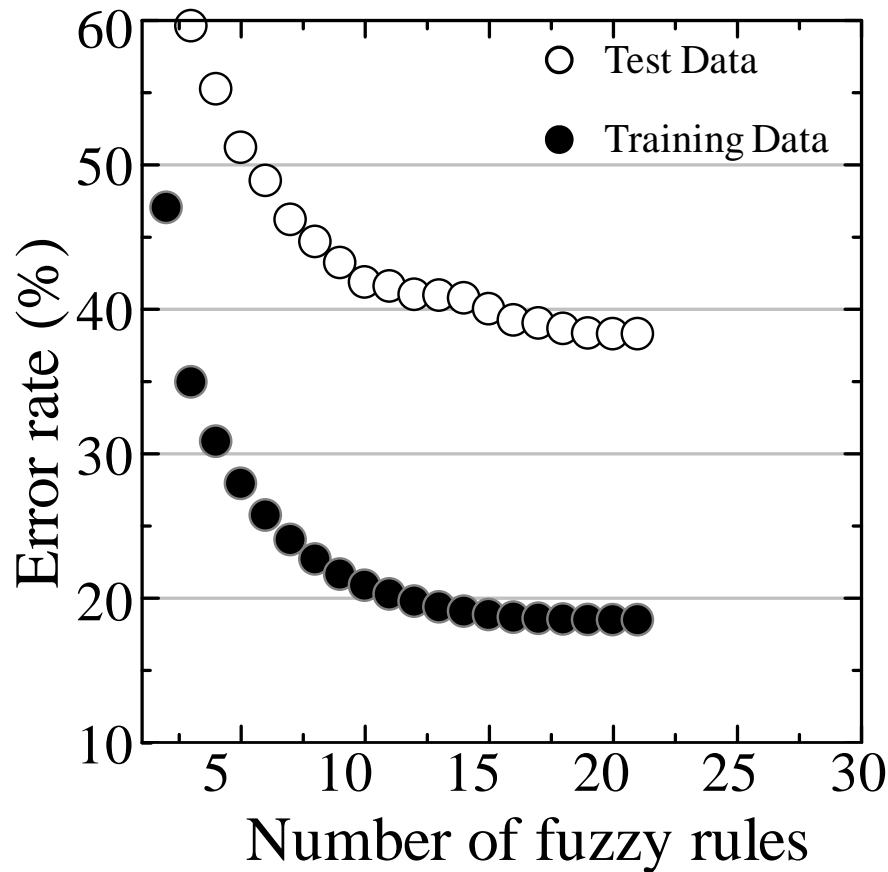


NSGA-II Basic Setting

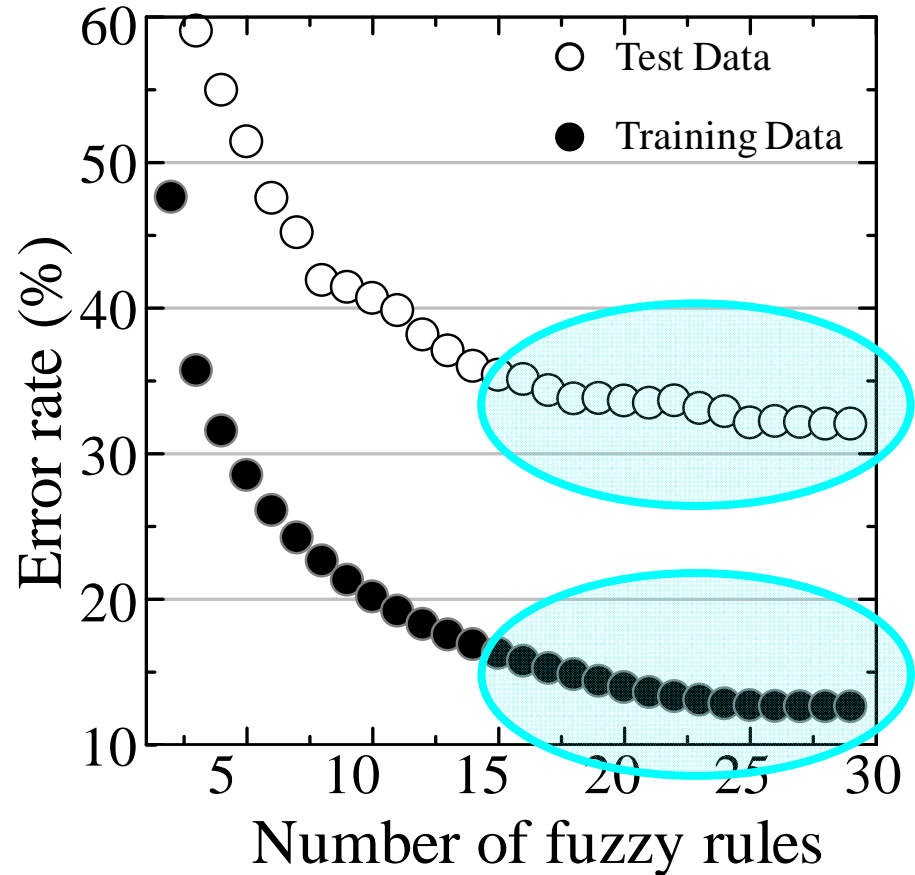


NSGA-II with Mating

Experimental Results (Glass)

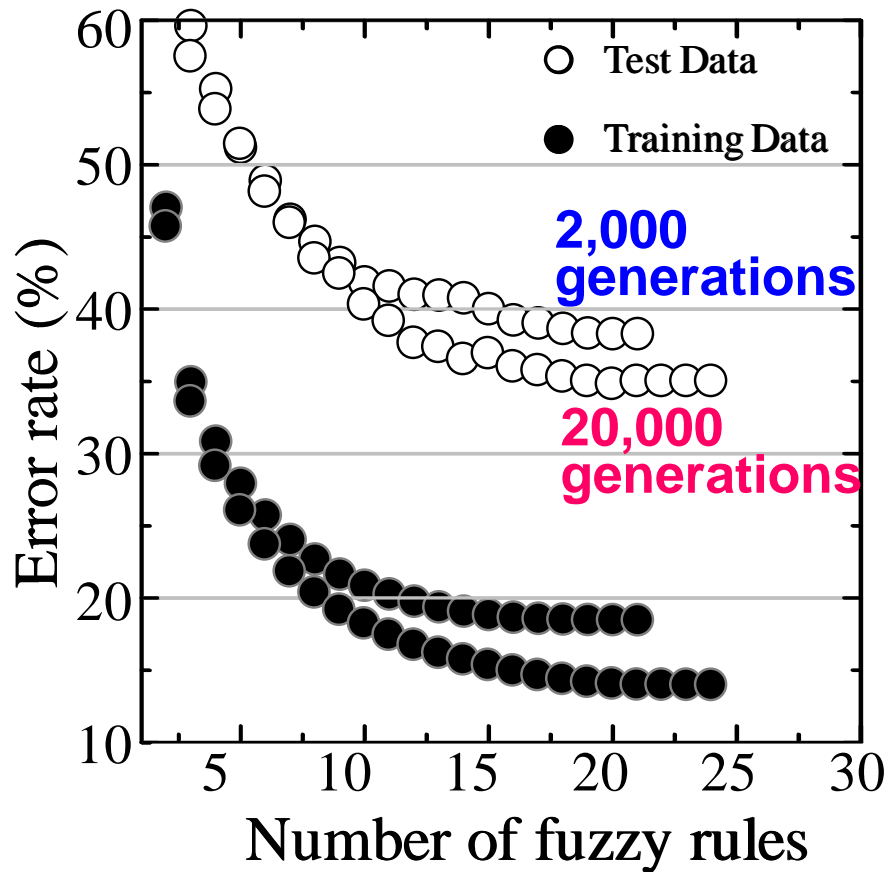


NSGA-II Basic Setting

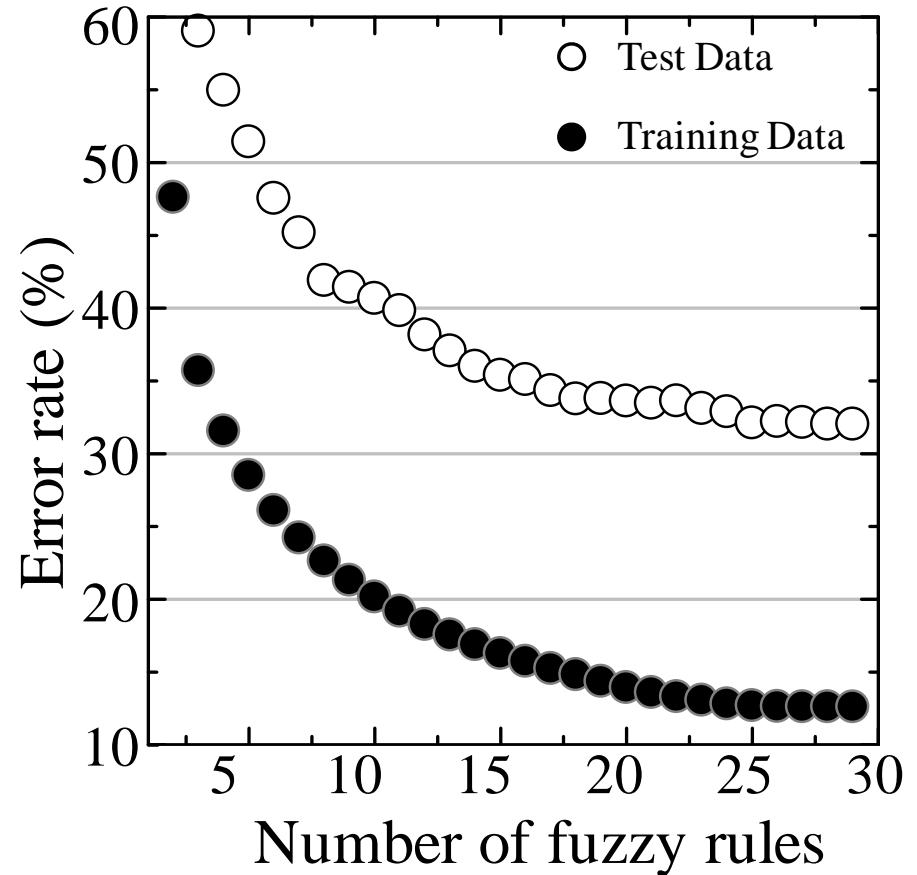


NSGA-II with Mating

Experimental Results (Glass)

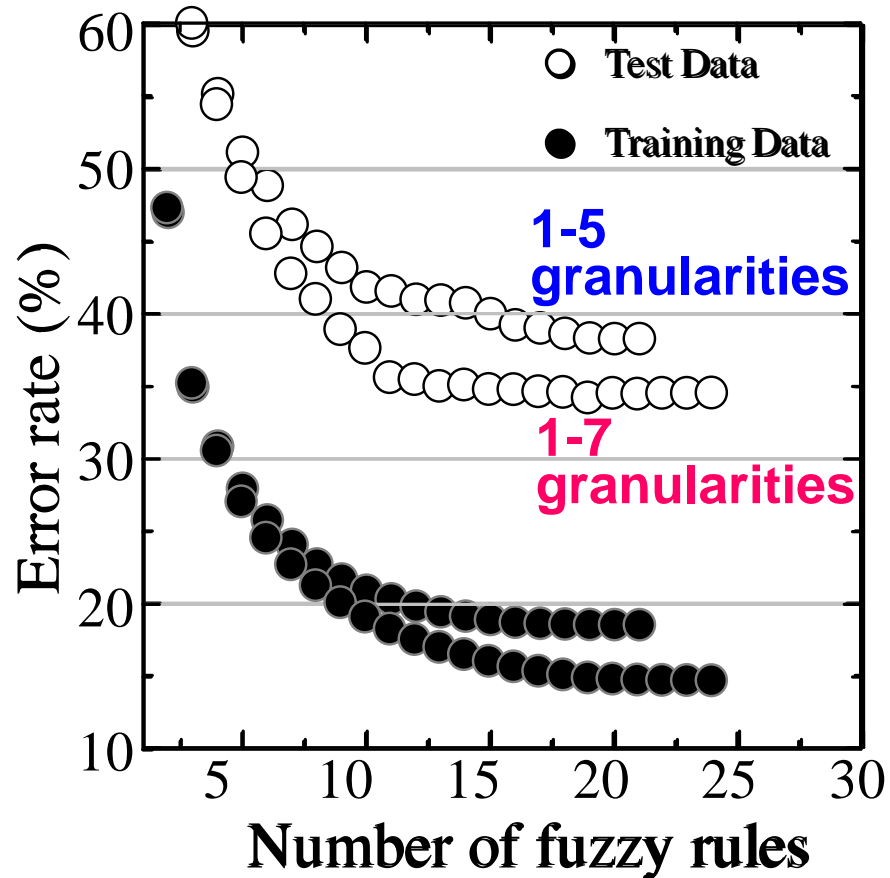


NSGA-II 20,000 Generations

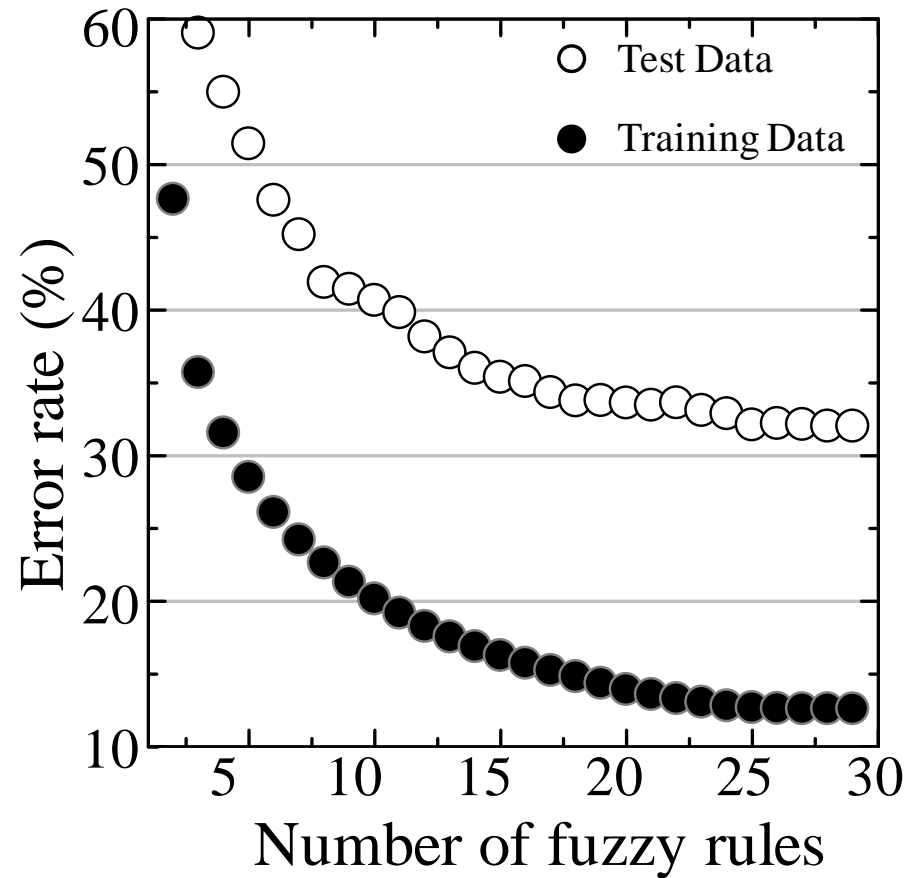


NSGA-II with Mating

Experimental Results (Glass)



NSGA-II Granularities 1-7



NSGA-II with Mating

Our Experimental Results

Simple Changes of Objectives (GEFS 2010, Spain)

Original Formulation

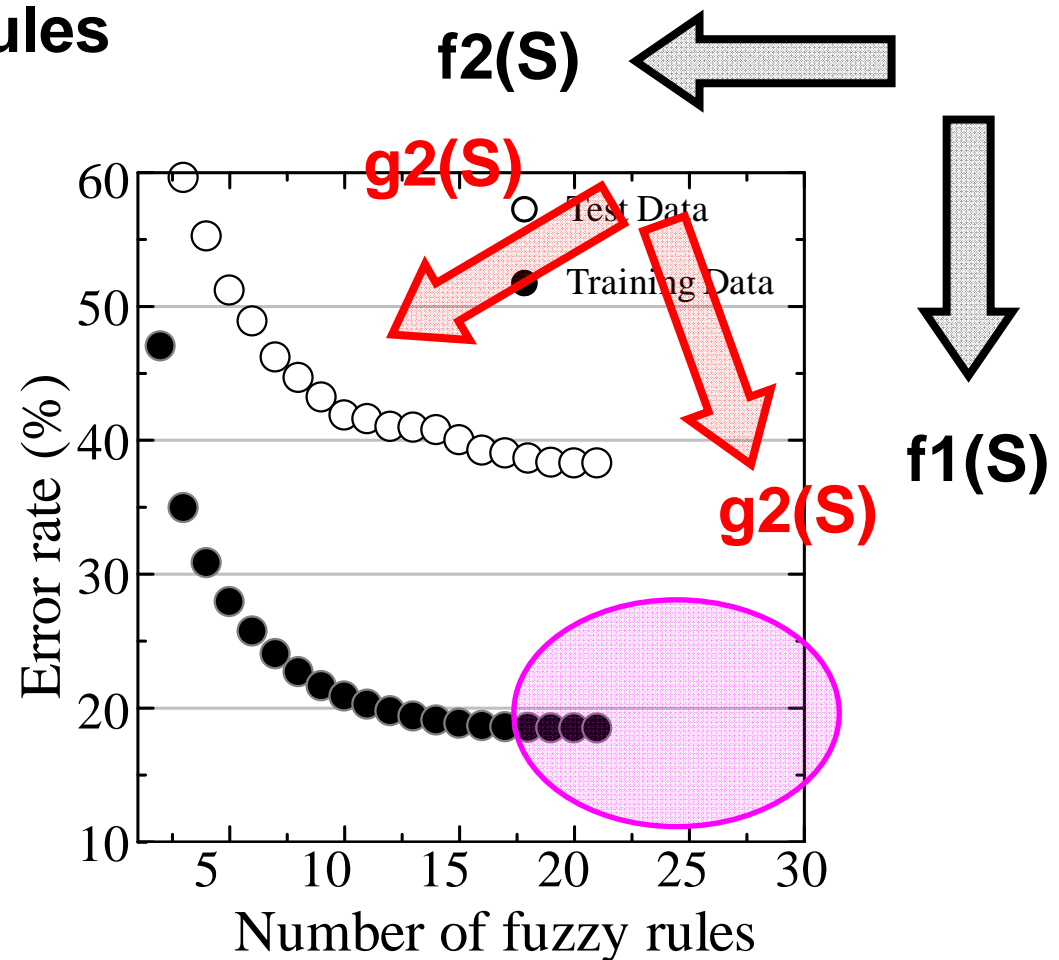
$f1(S)$: Error Rate (%)

$f2(S)$: Number of Fuzzy Rules

Simple Modification

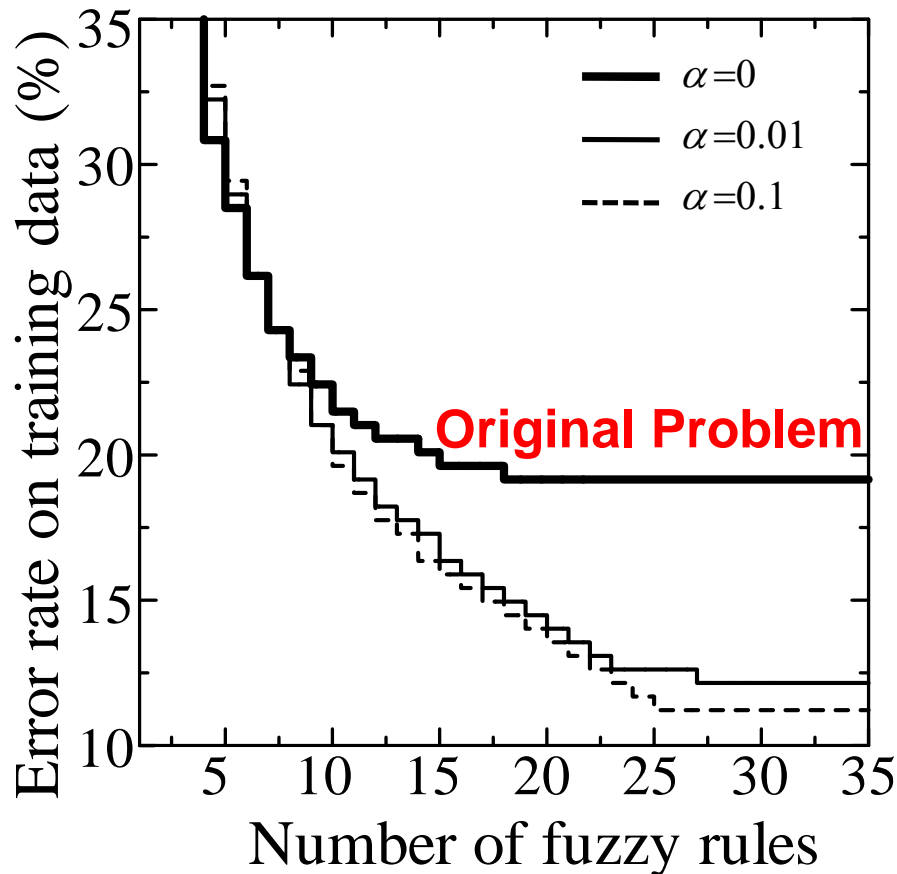
$$g1(S) = f1(S) - \alpha f2(S)$$

$$g2(S) = f2(S) + \alpha f1(S)$$

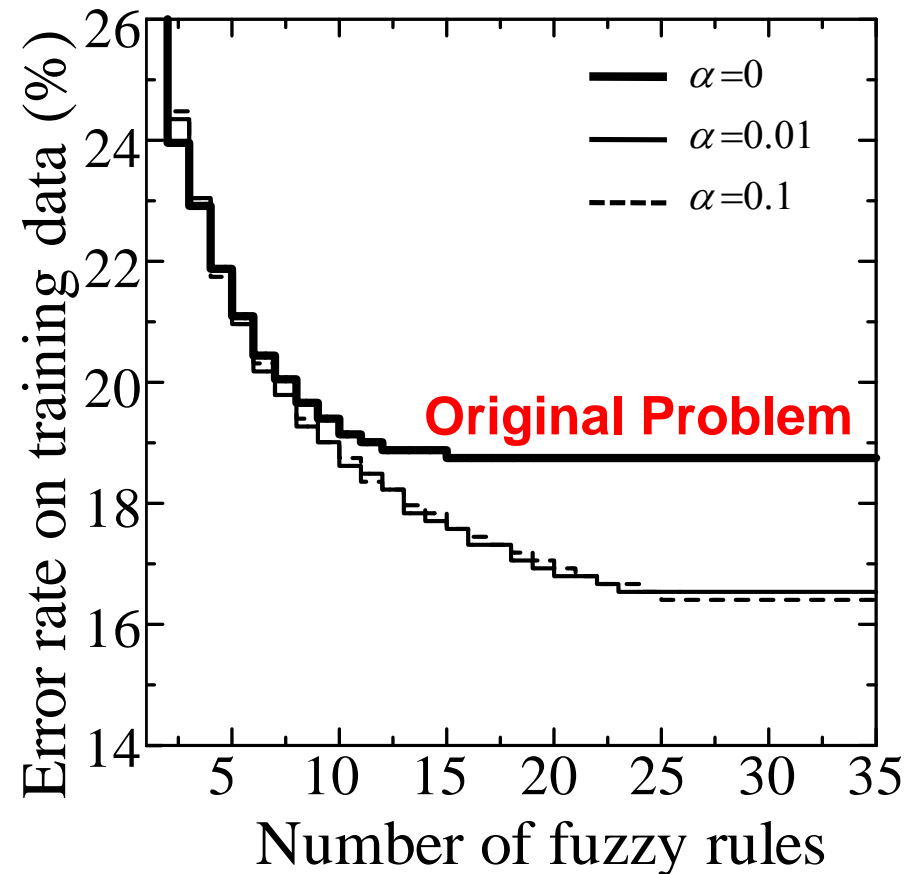


Our Experimental Results

Simple Changes of Objectives (GEFS 2010, Spain)



(a) Glass data.



(b) Diabetes data.

Our Experimental Results

Simple Changes of Objectives (GEFS 2010, Spain)

Original Formulation

$f1(S)$: Error Rate (%)

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Simple Modification

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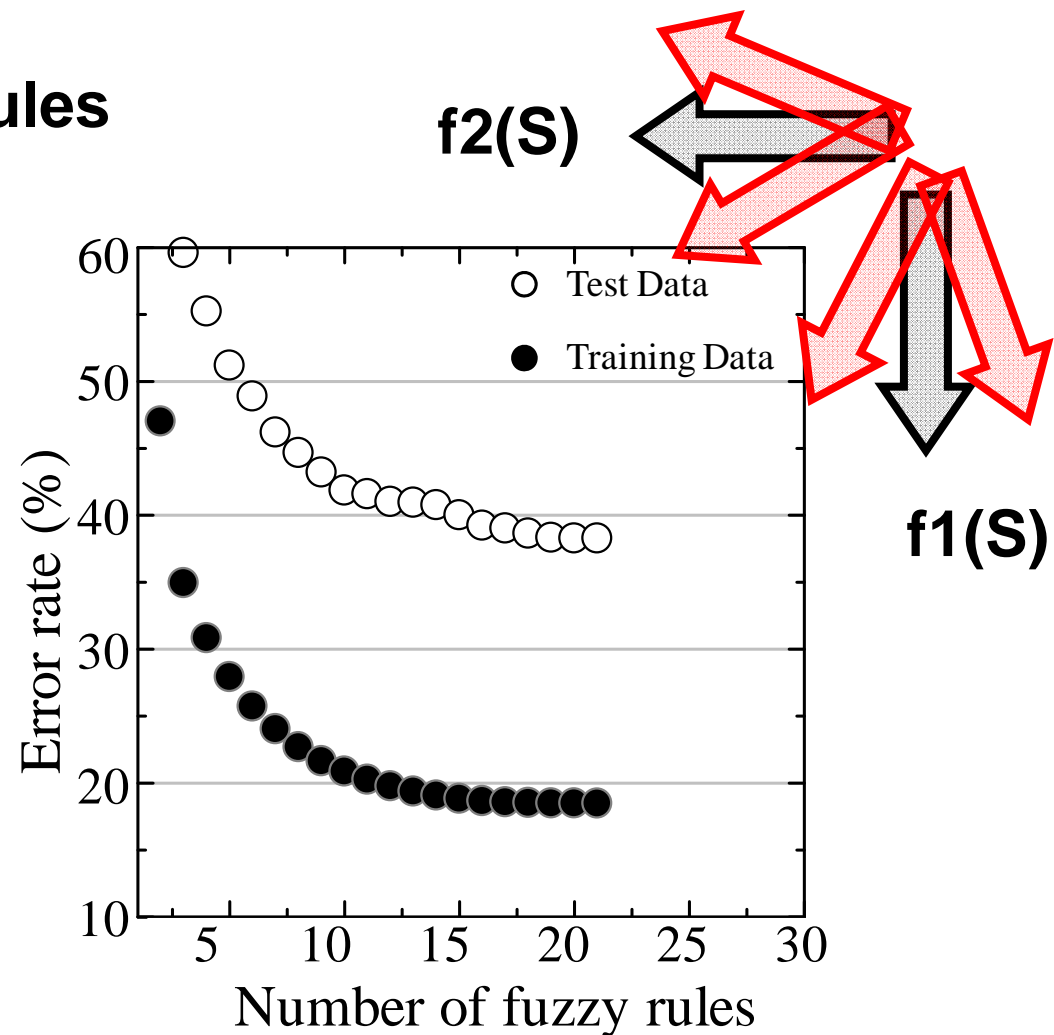
Four-Objective

$g1(S) = f1(S) - \alpha f2(S)$

$g2(S) = f1(S) + \alpha f2(S)$

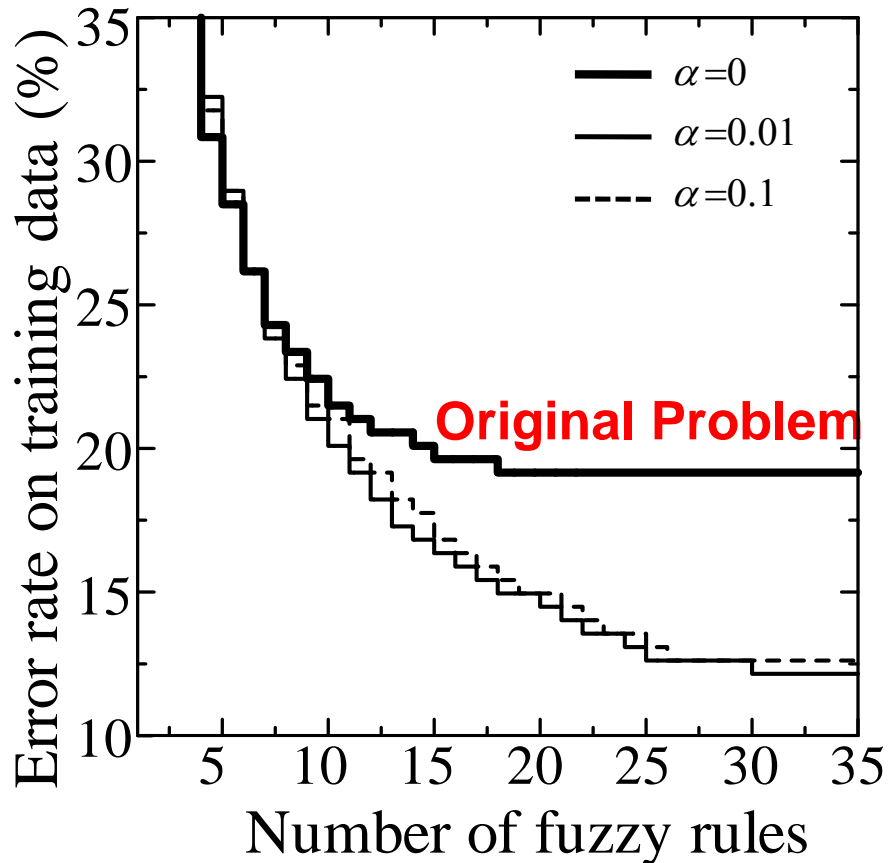
$g3(S) = f2(S) - \alpha f1(S)$

$g4(S) = f2(S) + \alpha f1(S)$

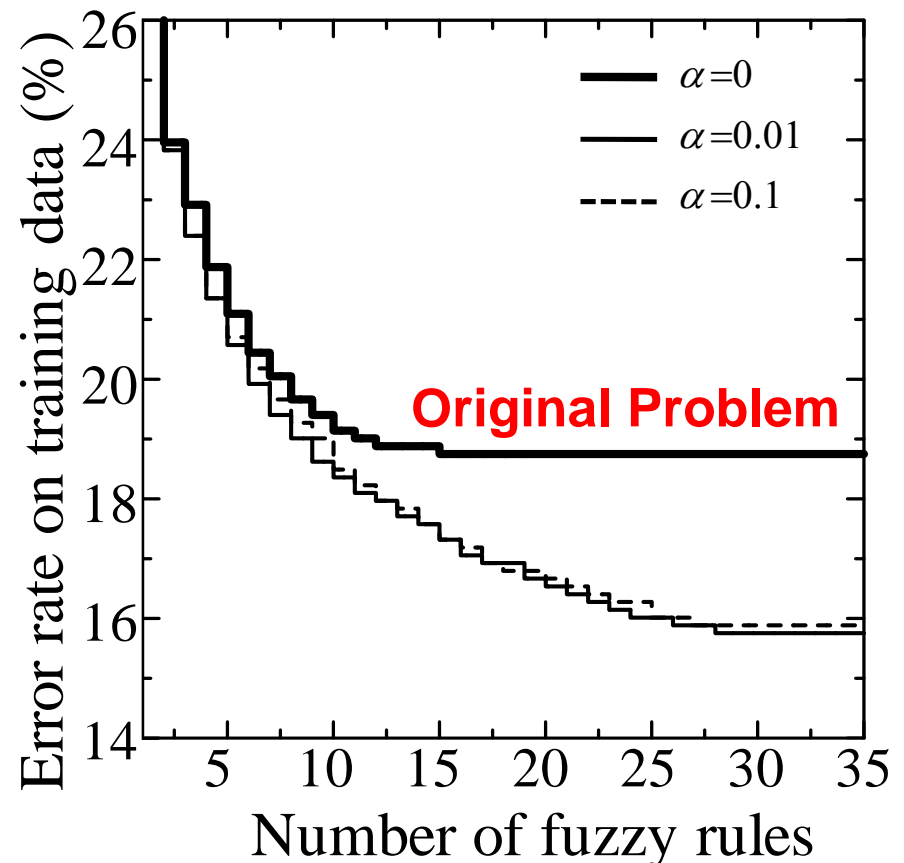


Our Experimental Results

Four-Objective Formulation (GEFS 2010, Spain)



(a) Glass data.



(b) Diabetes data.

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Handling of Interpretability in Our Former Studies

- [1] H. Ishibuchi et al. (1995) Selecting fuzzy if-then rules for classification problems using genetic algorithms, IEEE TFS.
ISI Web of Knowledge Citations: 273 times
- [2] H. Ishibuchi et al. (1997) Single-objective and two-objective genetic algorithms for selecting linguistic rules for pattern classification problems. Fuzzy Sets & Systems.
ISI Web of Knowledge Citations: 107 times
- [3] H. Ishibuchi et al. (2001) Three-objective genetics-based machine learning for linguistic rule extraction. Information Sciences.
ISI Web of Knowledge Citations: 94 times

Interpretability Maximization = Complexity Minimization

- **Minimization of the number of fuzzy rules**
- **Minimization of the number of antecedent conditions**

Interpretability of Fuzzy Systems

Ishibuchi et al. (1995, 1997, 2001)

Interpretability Maximization = Complexity Minimization

- Minimization of **the number of fuzzy rules**
- Minimization of **the number of antecedent conditions**

Interpretability of Fuzzy Systems

Ishibuchi et al. (1995, 1997, 2001)

Interpretability Maximization = Complexity Minimization

- Minimization of **the number of fuzzy rules**
- Minimization of **the number of antecedent conditions**

Many other factors are related to the interpretability

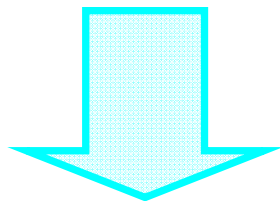
Interpretability of Fuzzy Systems

Ishibuchi et al. (1995, 1997, 2001)

Interpretability Maximization = Complexity Minimization

- Minimization of **the number of fuzzy rules**
- Minimization of **the number of antecedent conditions**

Many other factors are related to the interpretability



Special Sessions and Many Related Papers

- **IFSA 2009 Conference**
- **ISDA 2009 Conference (4 Papers with Interpretability in Their Titles)**
- **FUZZ-IEEE 2010 Conference**

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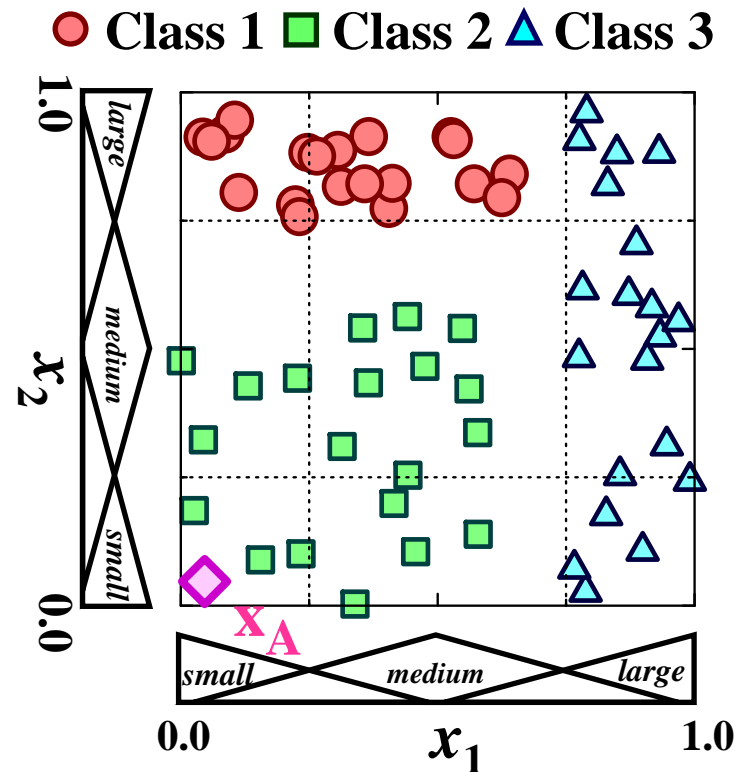
Another Issue in Interpretability

Explanation of Classification Results

Explanation Ability

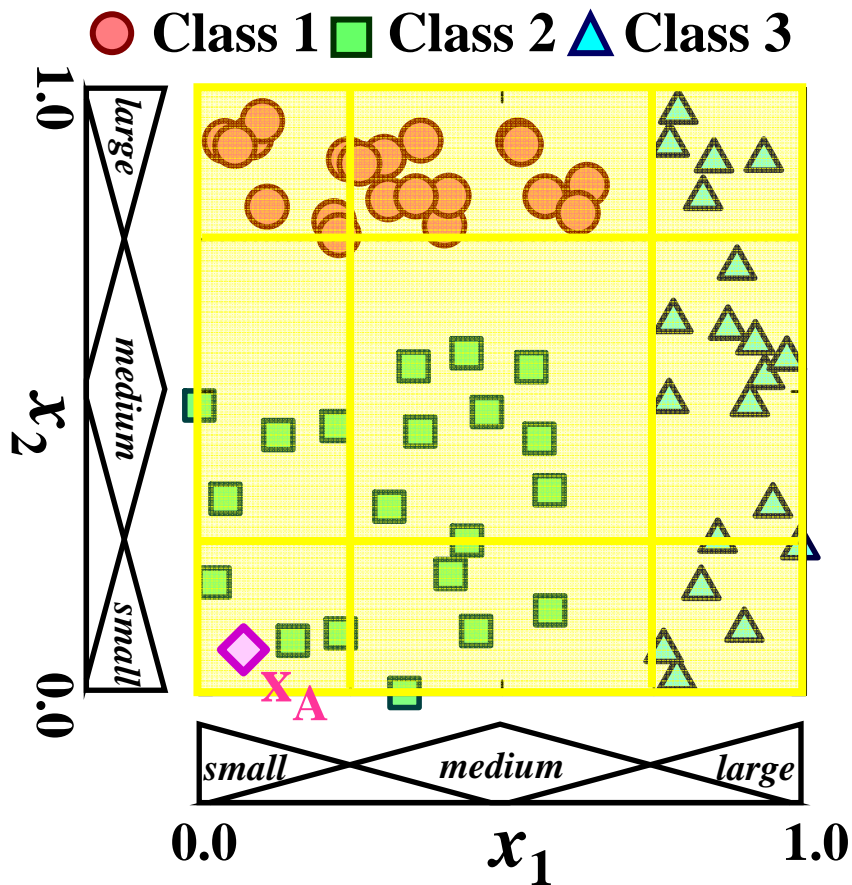
The ability of fuzzy rule-based systems to explain why a new pattern is classified as a particular class.

Example: Classification of a pattern \diamond : $\mathbf{x}_A = (0.05, 0.05)$

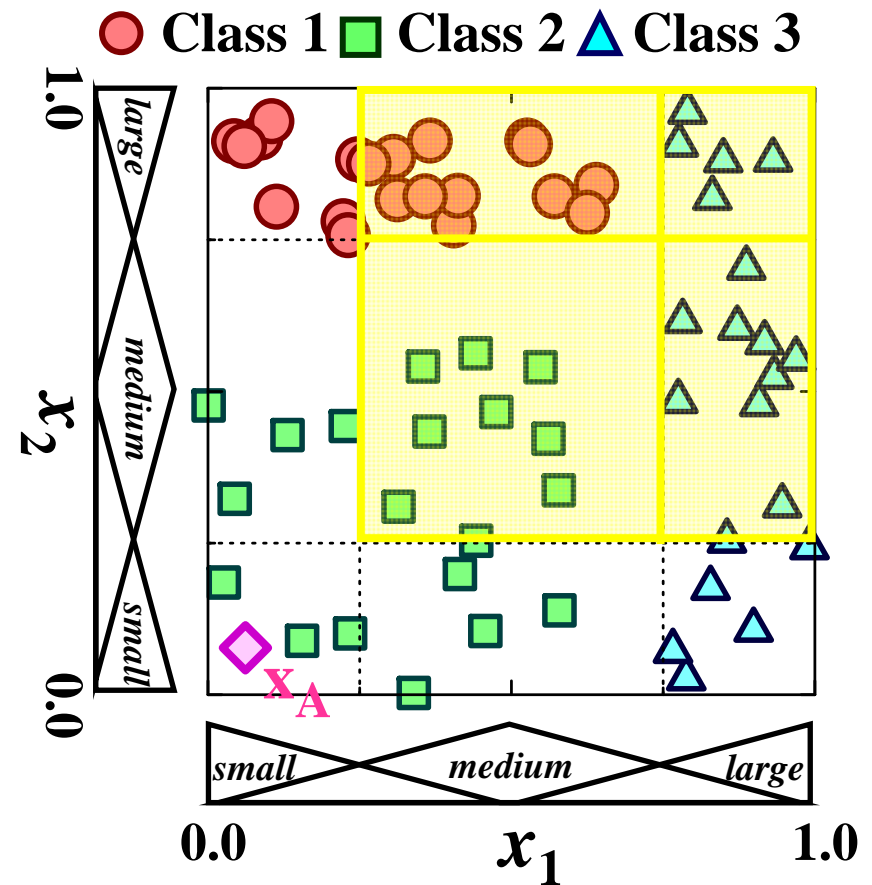


Comparison between Rule Sets 1 and 2

Classification of \diamond : $\mathbf{x}_A = (0.05, 0.05)$

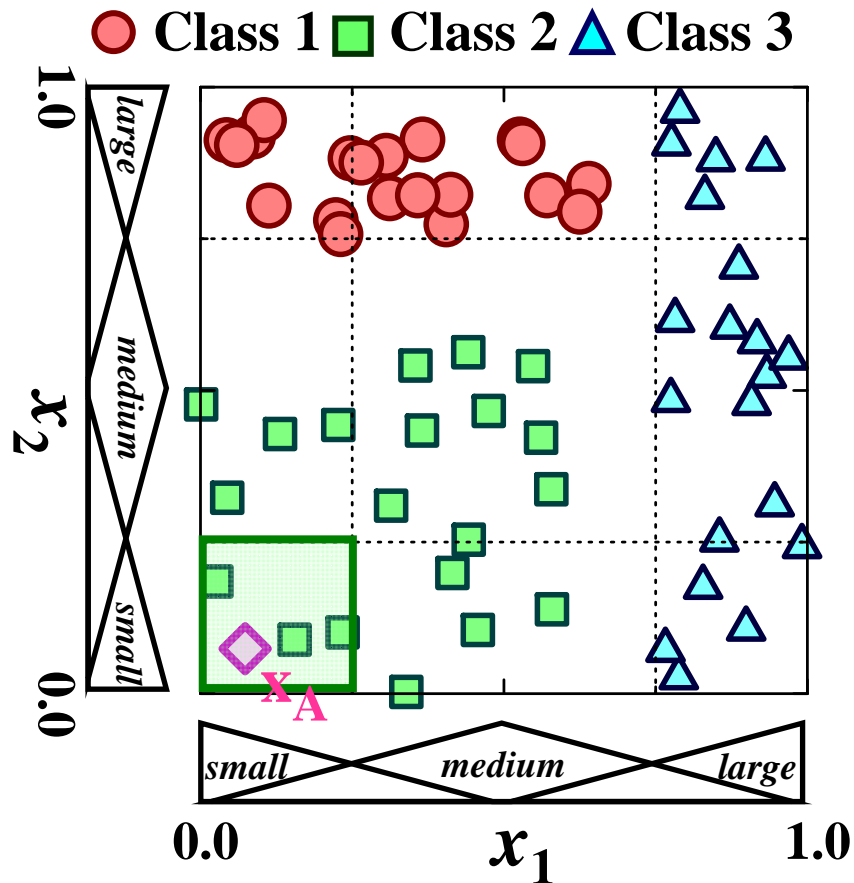


(1) Rule Set 1: Nine Rules

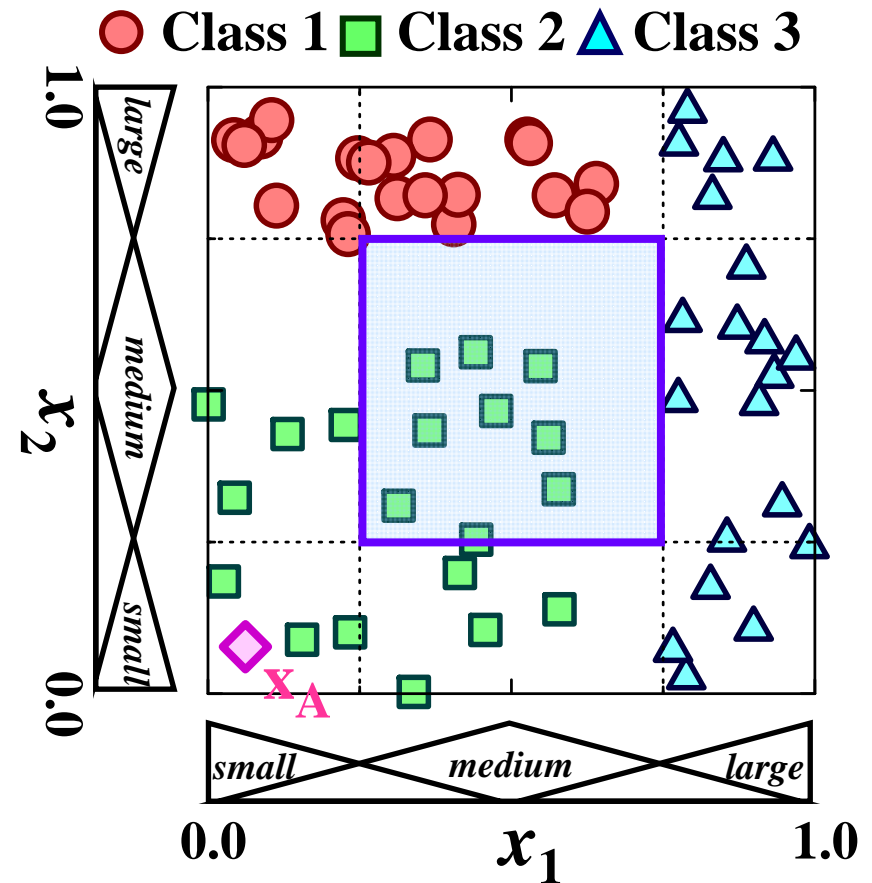


(2) Rule Set 2: Four Rules

Comparison in Explanation Capability Responsible Rules for Classification



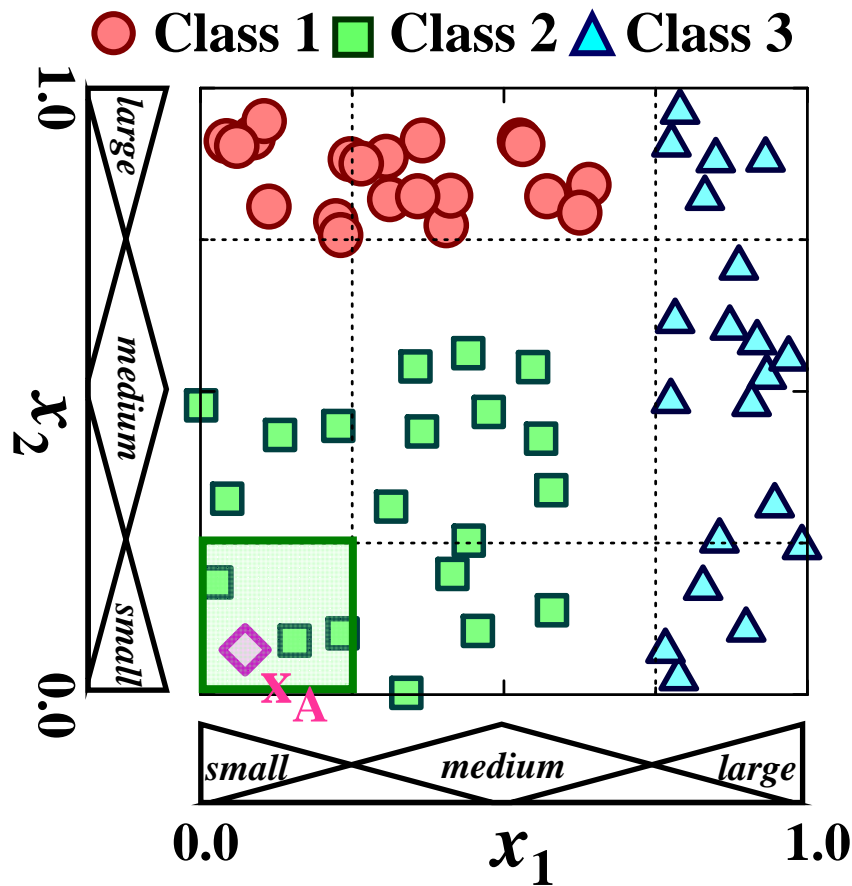
R_1 : If x_1 is *small* and x_2 is *small* then Class 2



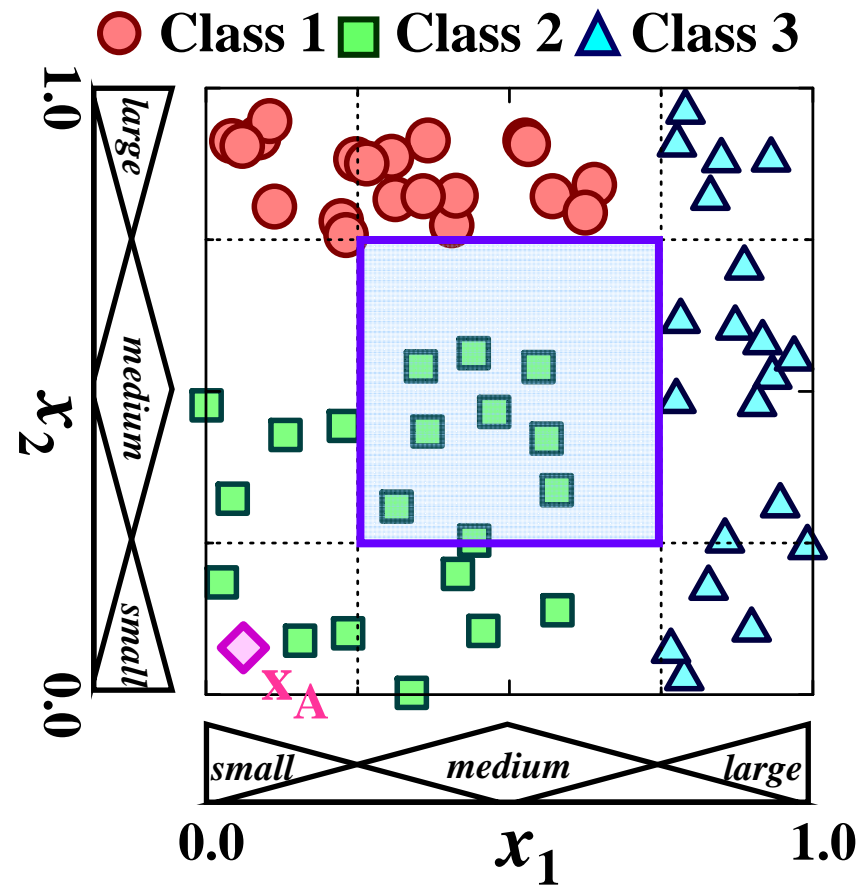
R_5 : If x_1 is *medium* and x_2 is *medium* then Class 2

Comparison in Explanation Capability Responsible Rules for Classification

R_1 seems to be a better explanation for the classification of x_A .



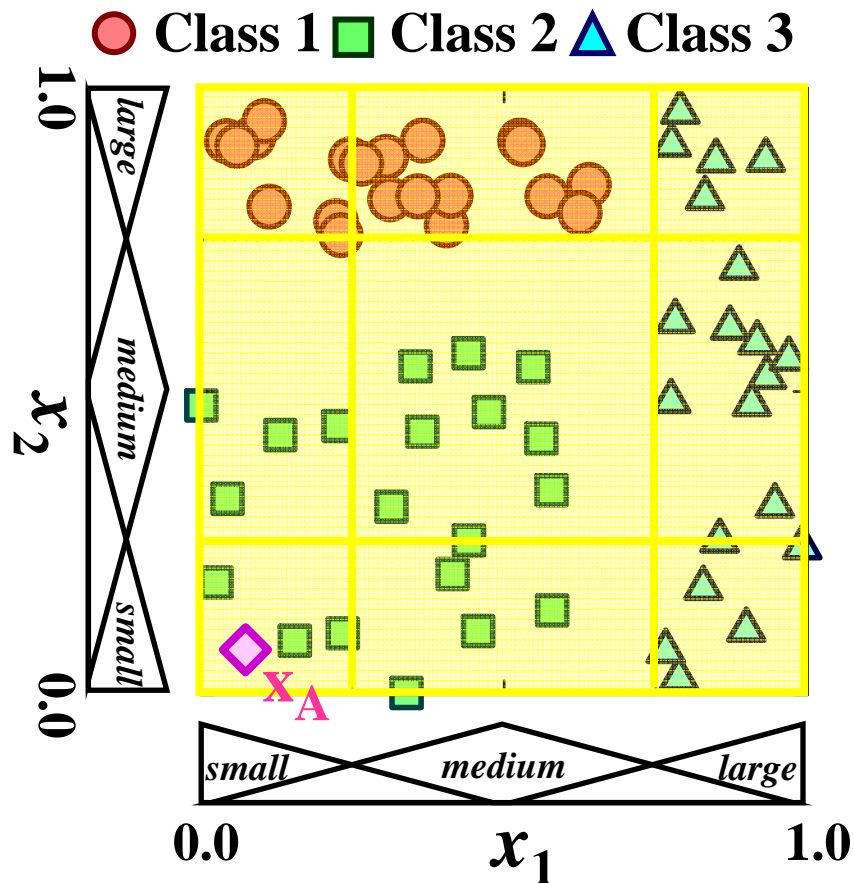
R_1 : If x_1 is *small* and x_2 is *small* then Class 2



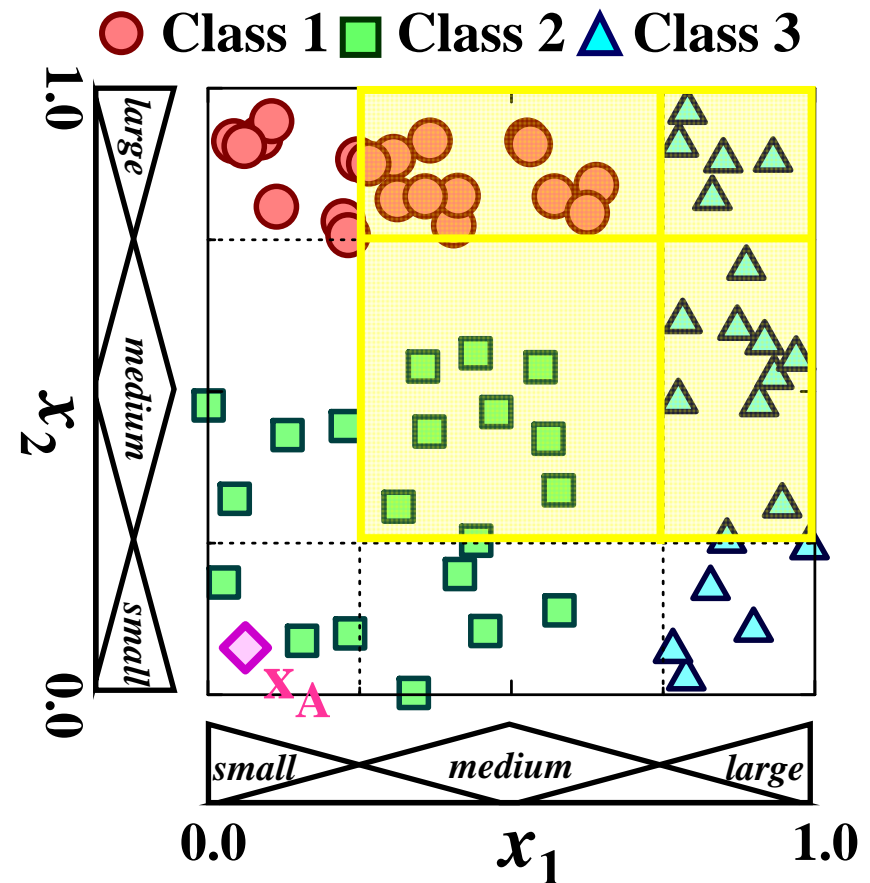
R_5 : If x_1 is *medium* and x_2 is *medium* then Class 2

Comparison between Rule Sets 1 and 2

Rule Set 1 seems to have higher explanation ability while Rule Set 2 is simpler than Rule Set 1.



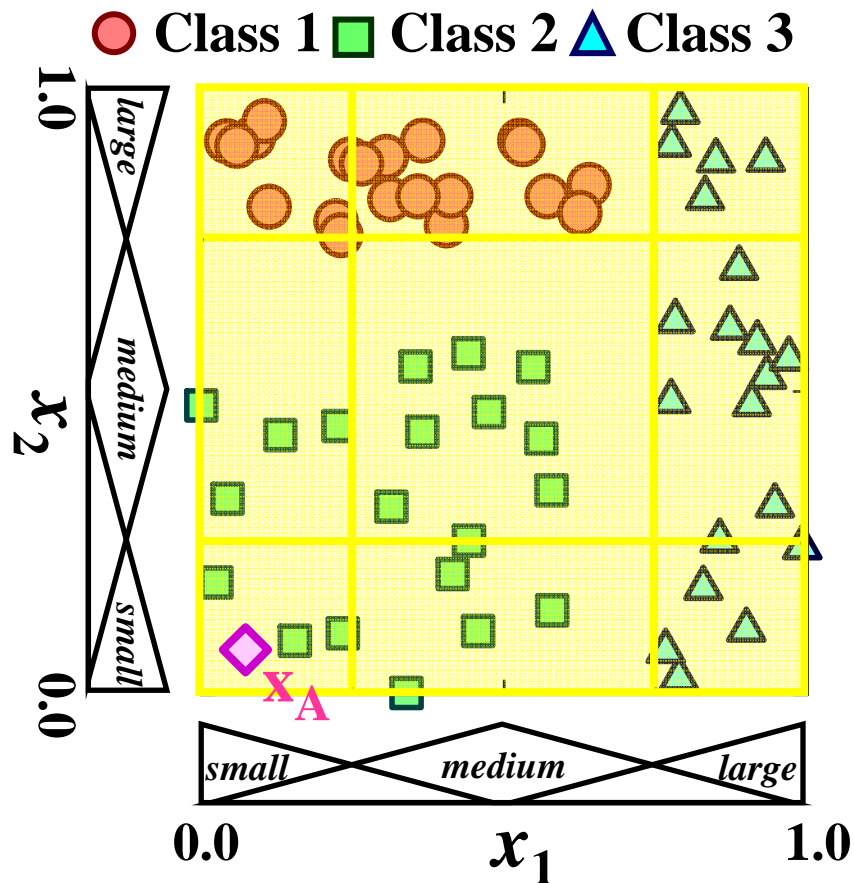
(1) Rule Set 1: Nine Rules



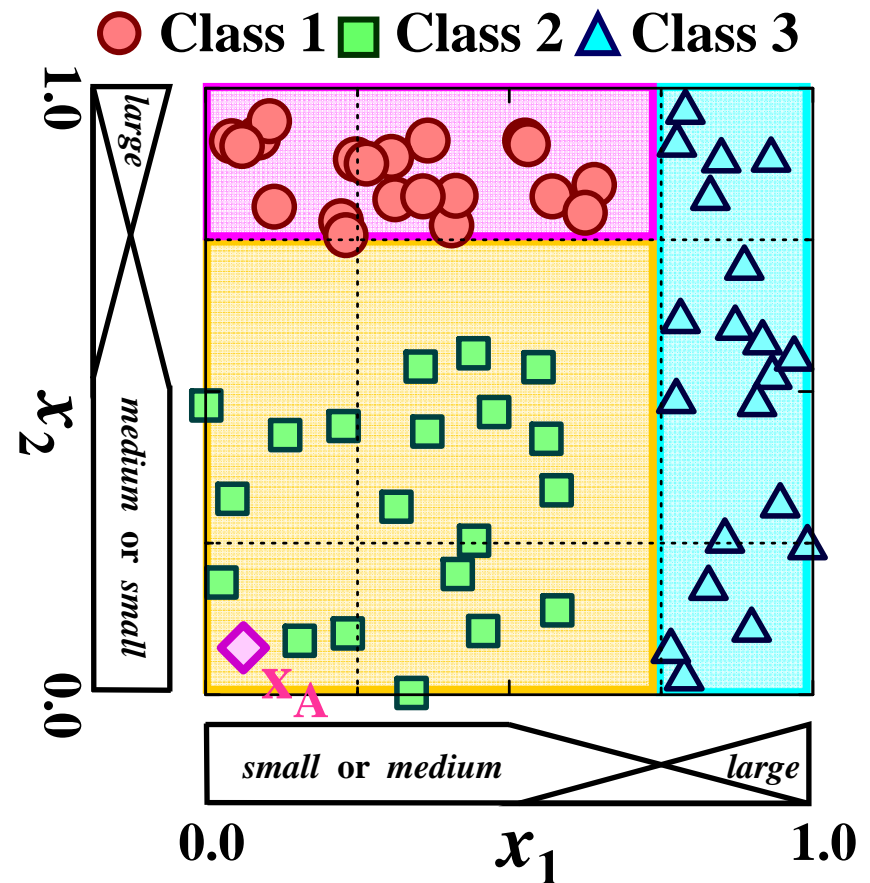
(2) Rule Set 2: Four Rules

Comparison between Rule Sets 1 and 4

Classification of \diamond : $\mathbf{x}_A = (0.05, 0.05)$

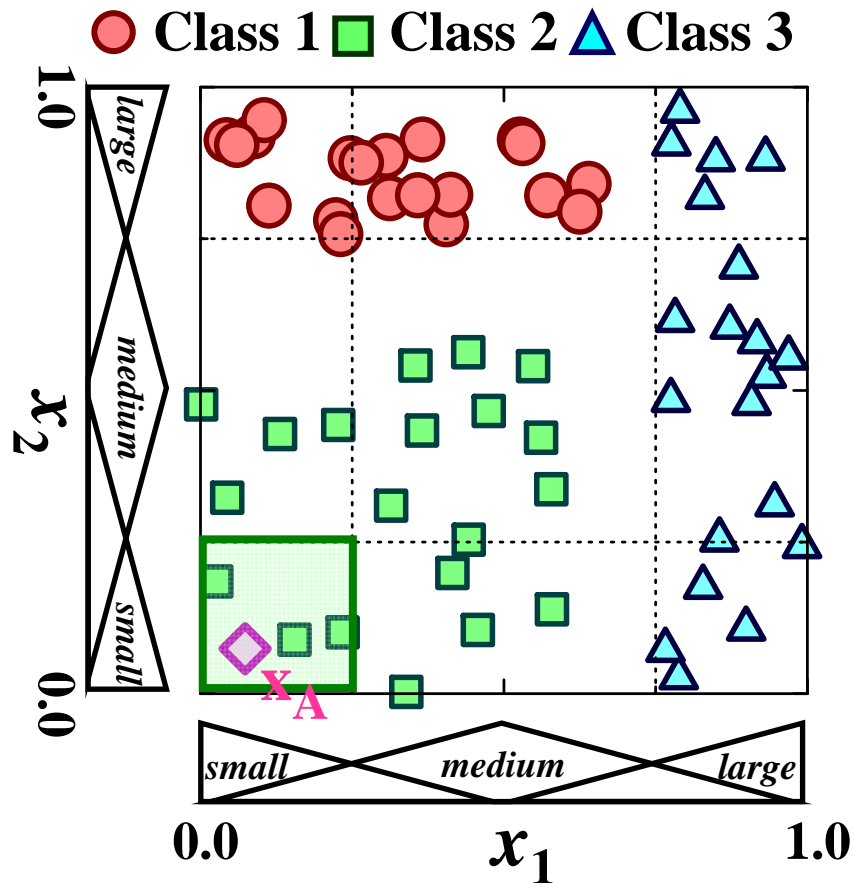


(1) Rule Set 1: Nine Rules

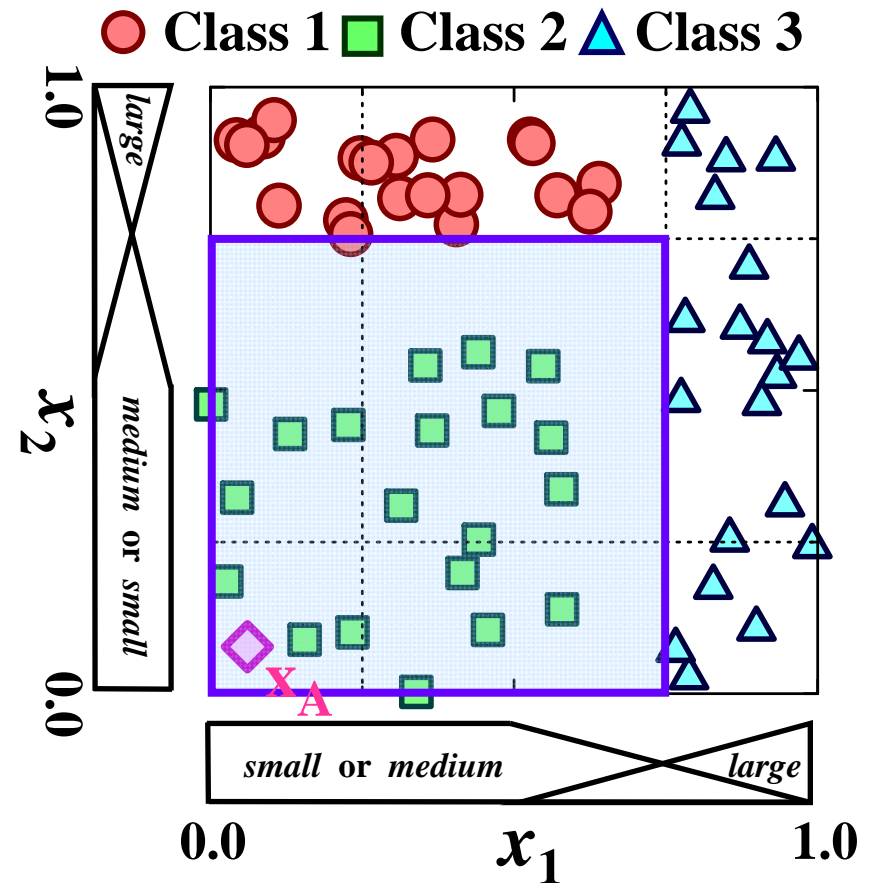


(4) Rule Set 4: Three Rules

Comparison in Explanation Capability Responsible Rules for Classification



R_1 : If x_1 is *small* and x_2 is *small*
then Class 2

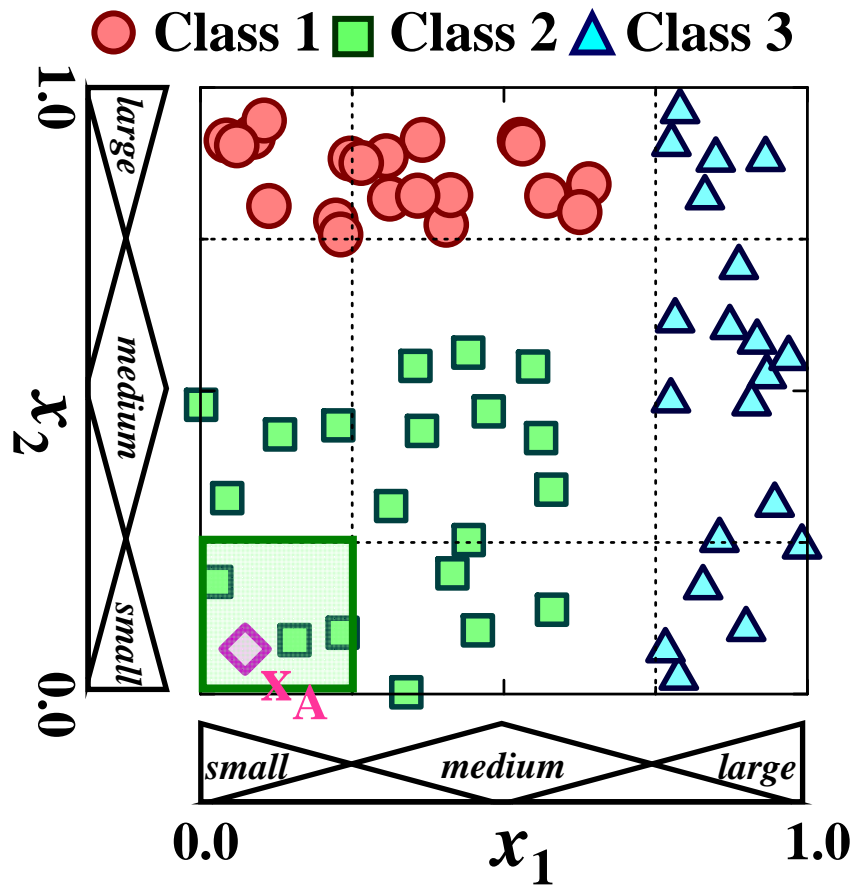


R_{1234} : If x_1 is *small or medium*
and x_2 is *small or medium*
then Class 2

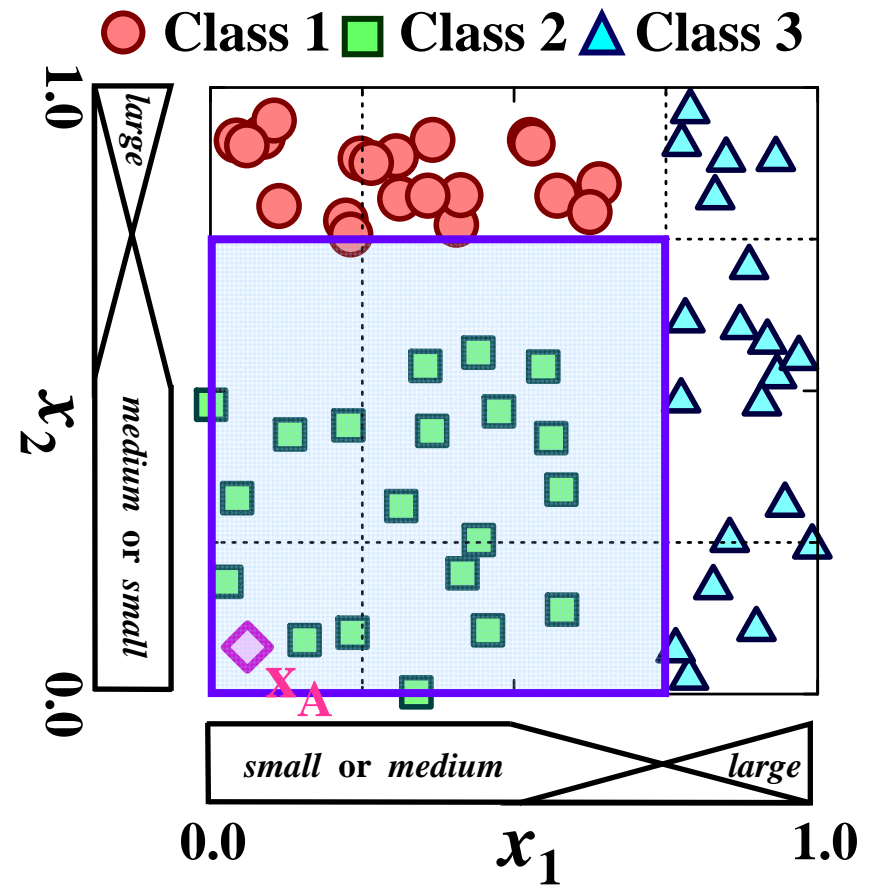
Comparison in Explanation Capability

Responsible Rules for Classification

R_1 seems to be a better explanation for the classification of x_A .



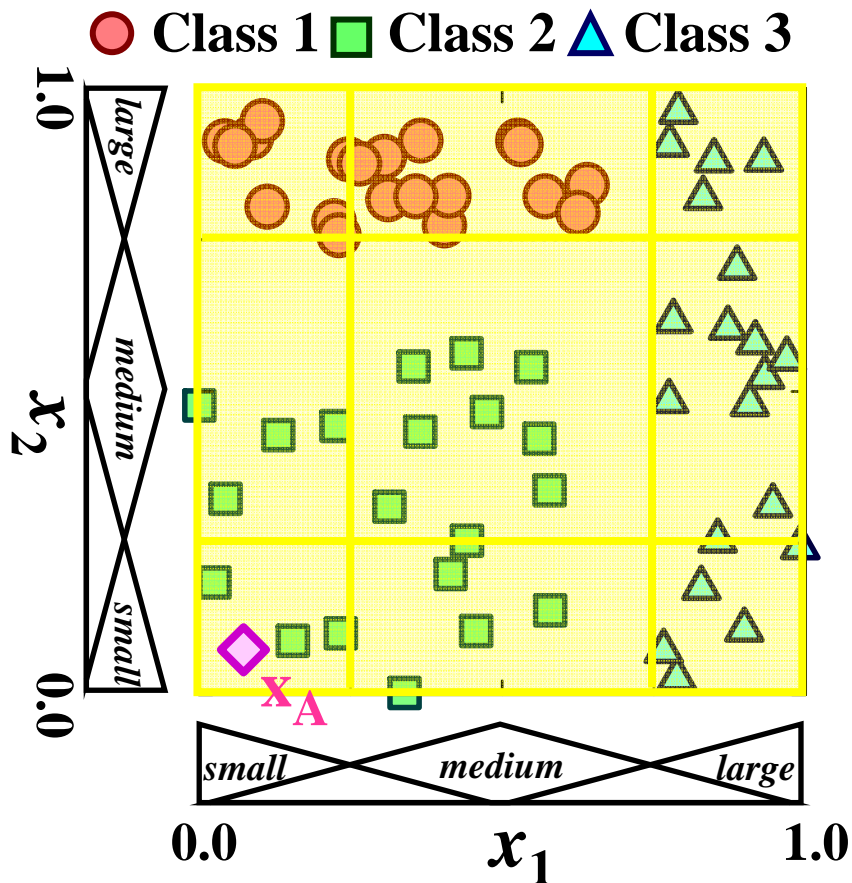
R_1 : If x_1 is *small* and x_2 is *small* then Class 2



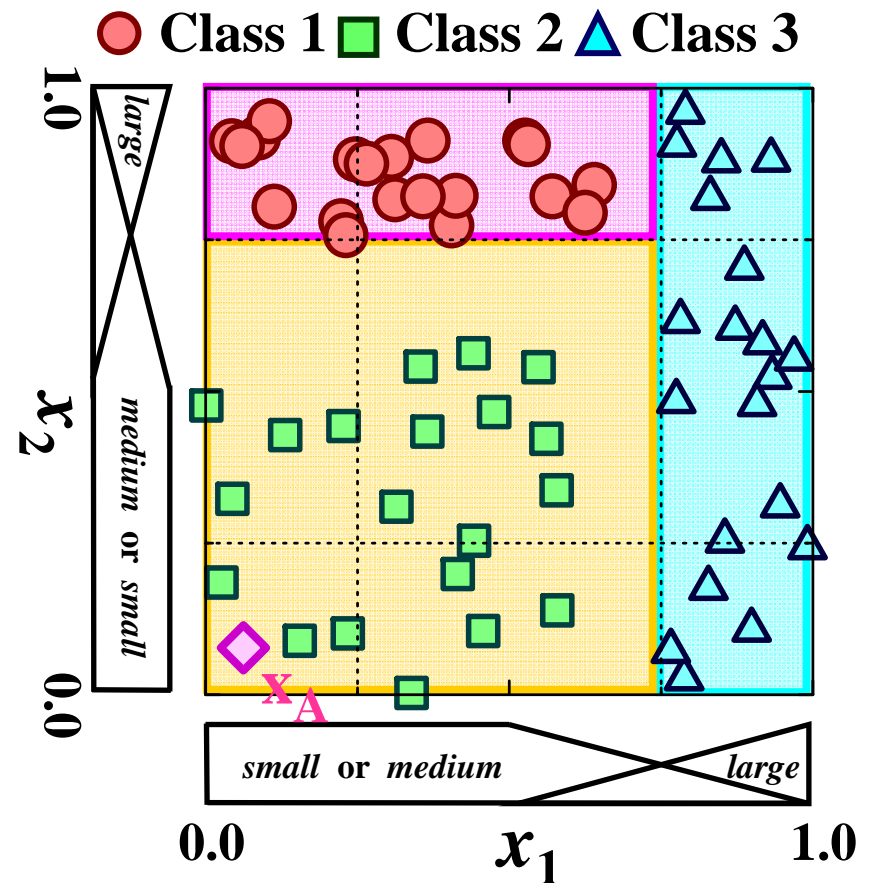
R_{1234} : If x_1 is *small or medium* and x_2 is *small or medium* then Class 2

Comparison between Rule Sets (1) and (4)

Rule Set 1 seems to have higher explanation ability while Rule Set 4 is simpler than Rule Set 1.



(1) Rule Set 1: Nine Rules

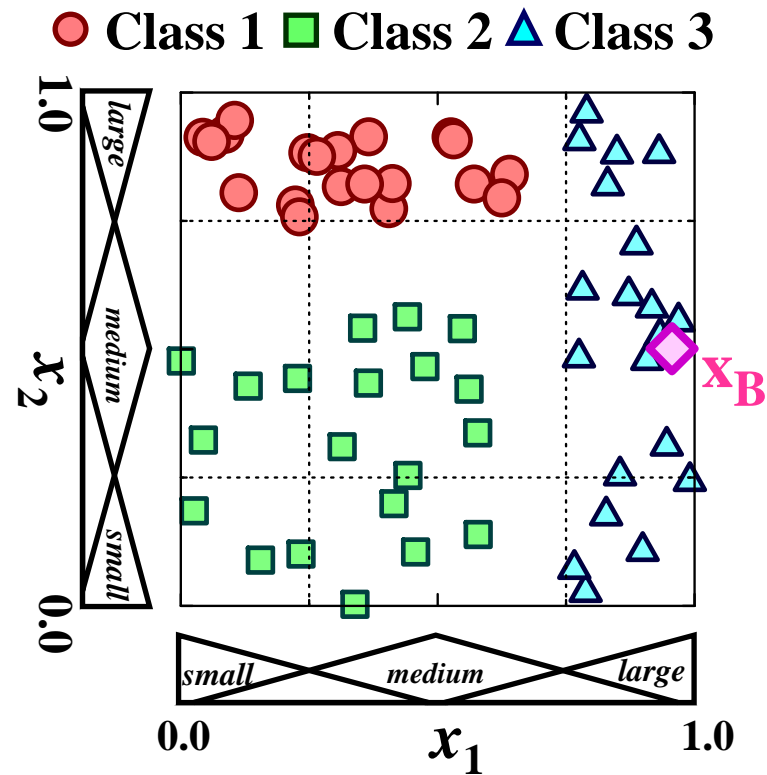


(4) Rule Set 4: Three Rules

Classification Capability

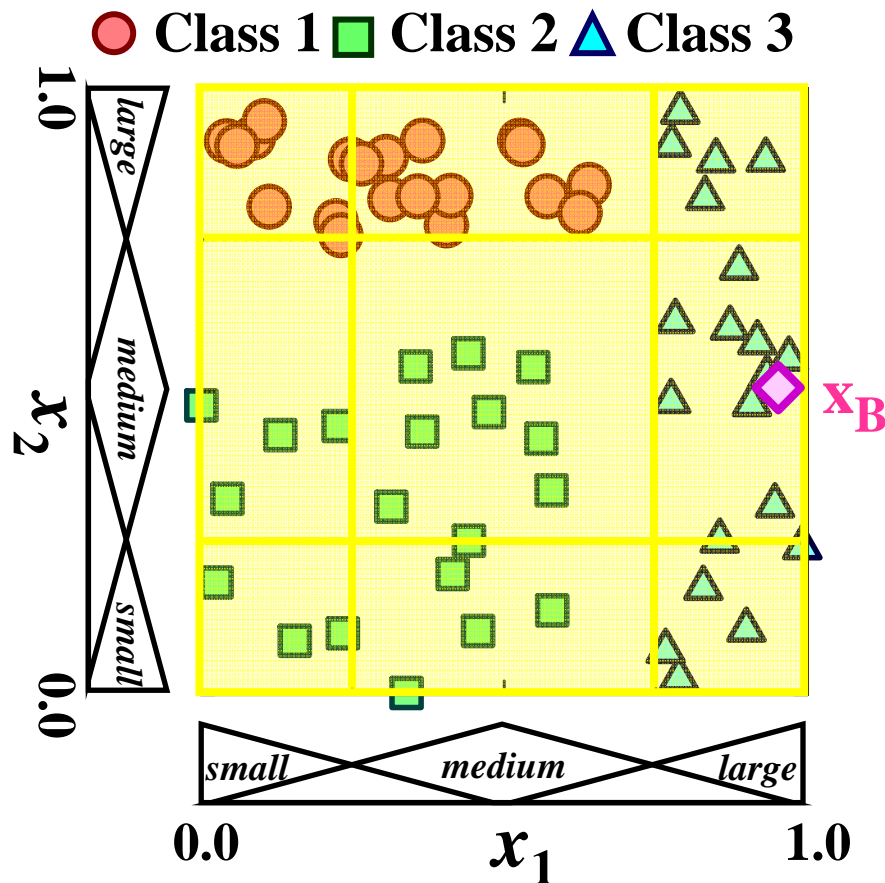
Another Example $\mathbf{x}_B = (0.95, 0.50)$

Classification of \diamond : $\mathbf{x}_B = (0.95, 0.50)$

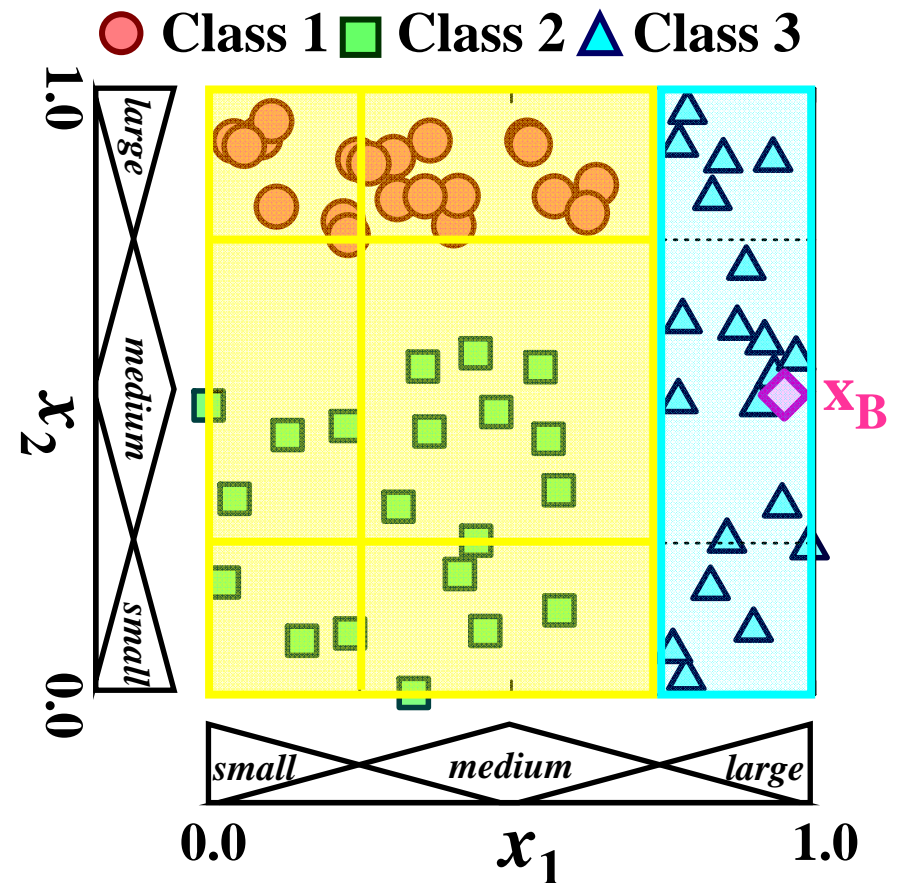


Comparison between Rule Sets 1 and 3

Classification of \diamond : $\mathbf{x}_B = (0.95, 0.50)$



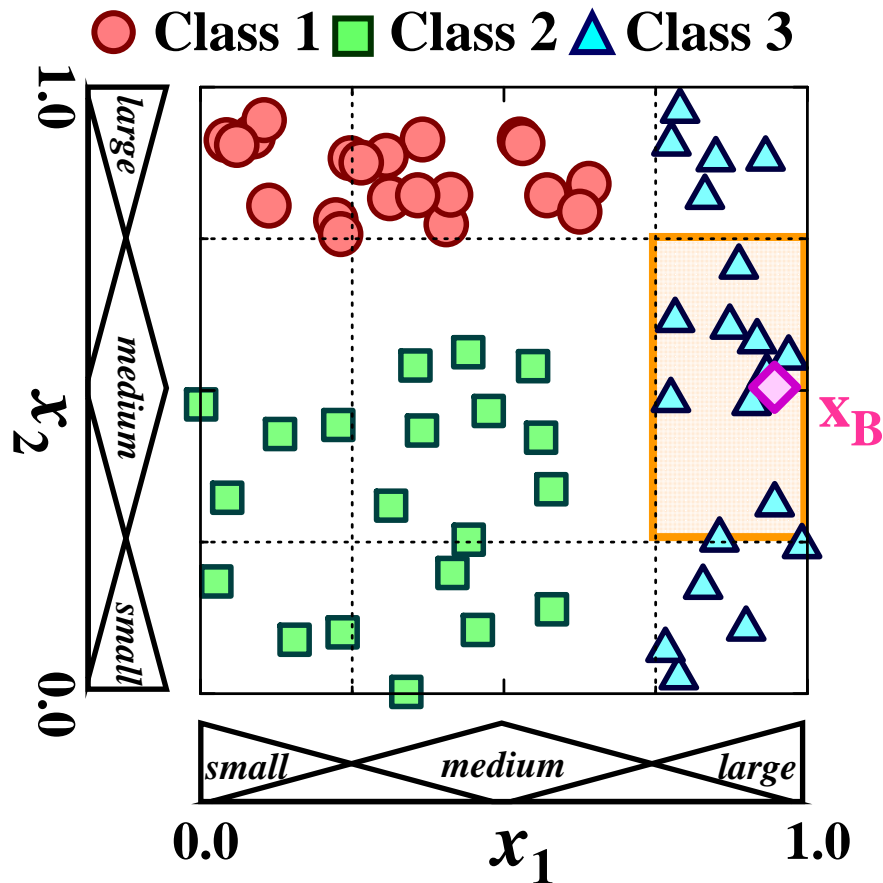
(1) Rule Set 1: Nine Rules



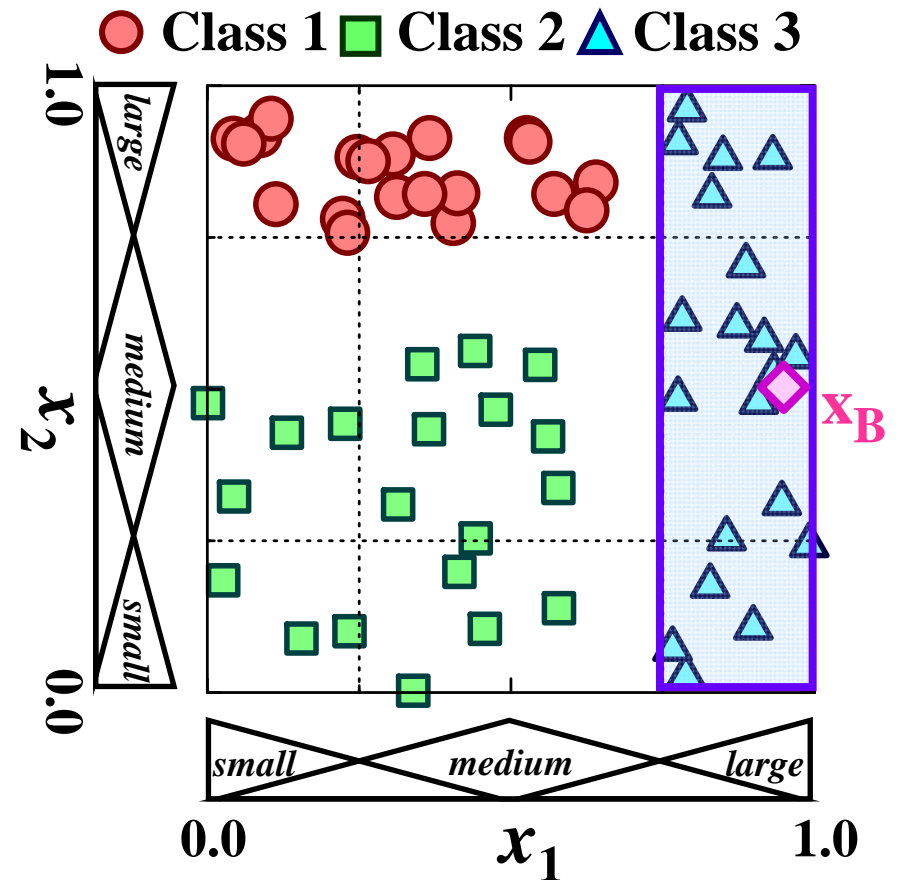
(3) Rule Set 3: Seven Rules

Comparison in Explanation Capability

Responsible Rules for Classification



R_8 : If x_1 is *large* and x_2 is *medium* then Class 3

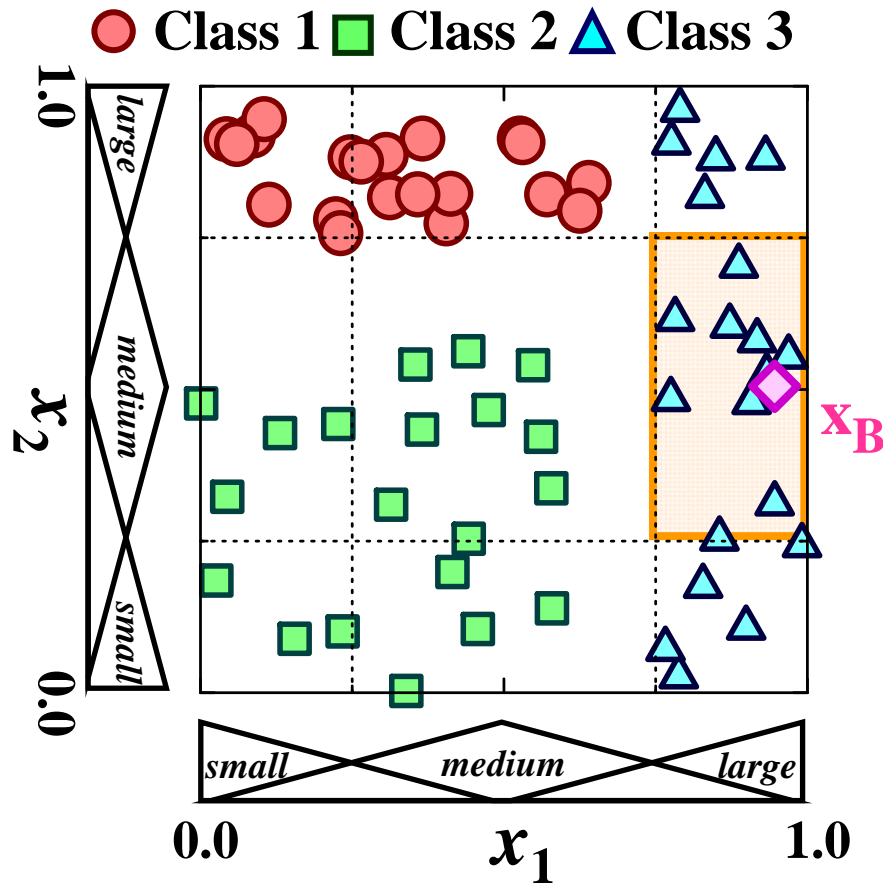


R_{789} : If x_1 is *large* then Class 3

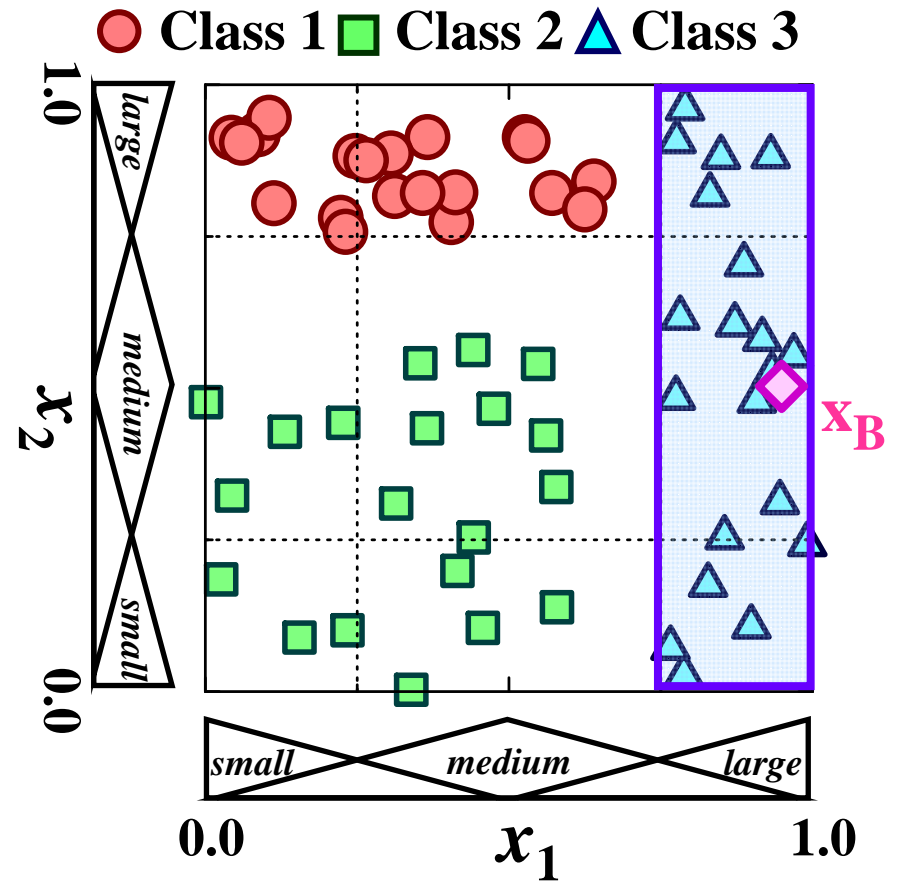
Comparison in Explanation Capability

Responsible Rules for Classification

Which is a better explanation for the classification of x_B between R_8 and R_{789} ?



R_8 : If x_1 is *large* and x_2 is *medium* then Class 3

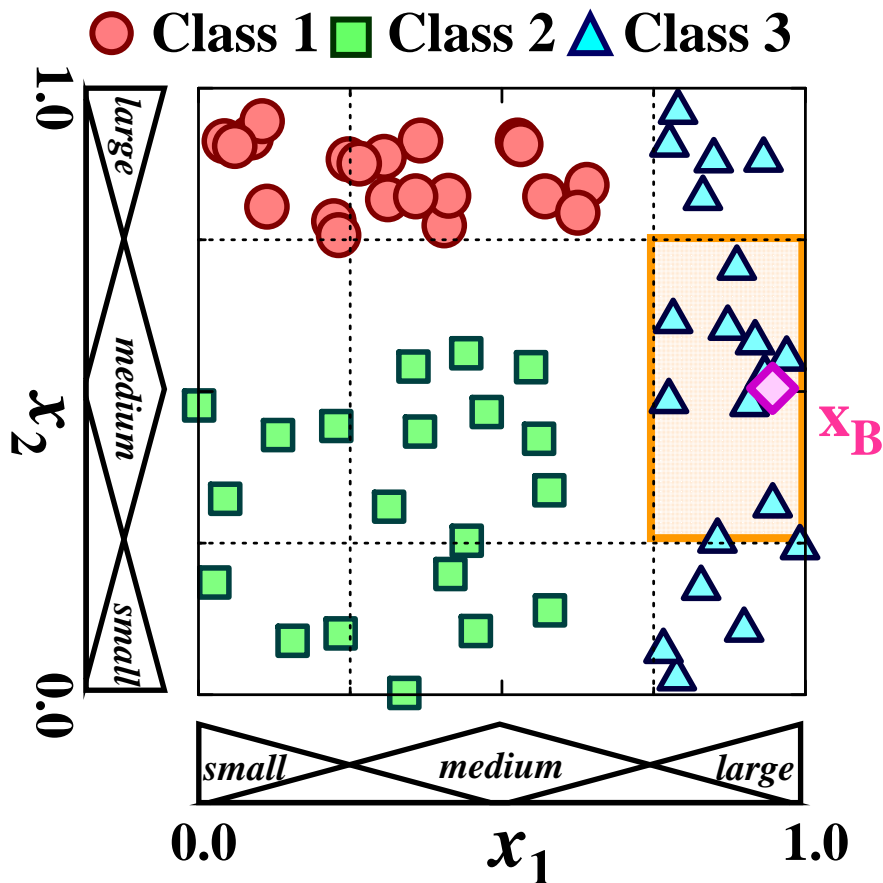


R_{789} : If x_1 is *large* then Class 3

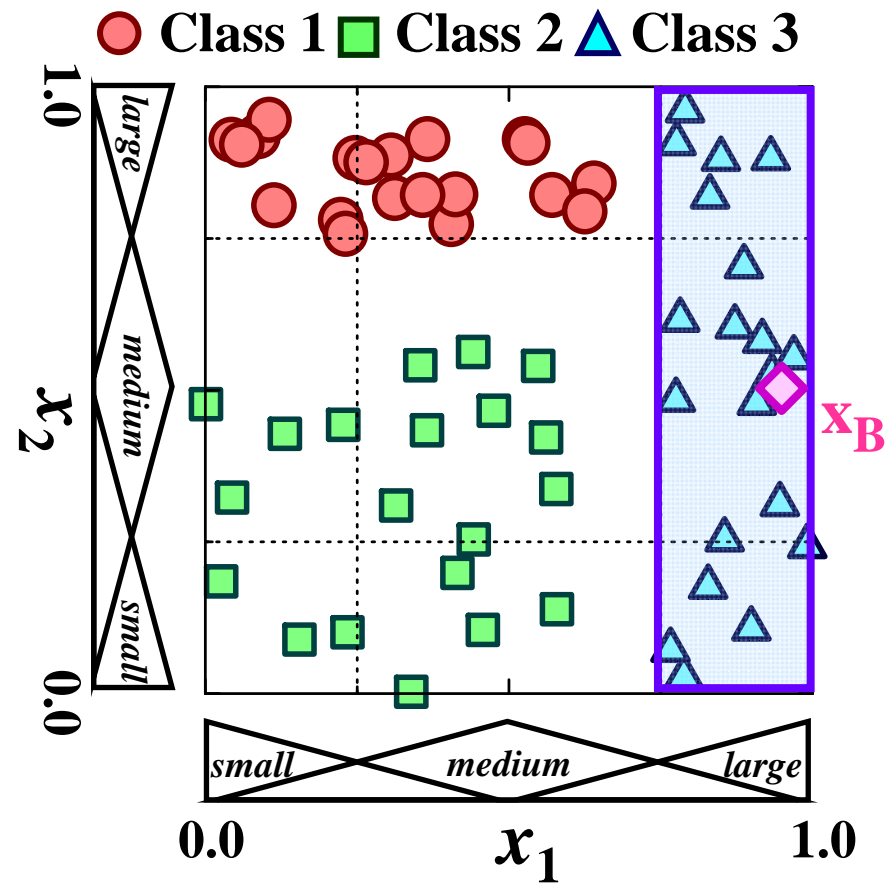
Comparison in Explanation Capability

Responsible Rules for Classification

Which is a better explanation for the classification of x_B between R_8 and R_{789} ? It is a very difficult question for me to answer.



R_8 : If x_1 is *large* and x_2 is *medium* then Class 3



R_{789} : If x_1 is *large* then Class 3

Contents of This Presentation

1. Introduction to Fuzzy Rule-Based Classification

- Is Fuzzy Rule-Based Classification a Popular Research Area?

2. Fuzzy Rule-Based Classifier Design

- Accuracy Improvement
- Scalability to High-Dimensional Problems
- Complexity Minimization

3. Multiobjective Fuzzy Rule-Based Classifier Design

- Formulation of Multi-objective Problems
- Accuracy-Complexity Tradeoff Analysis
- Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions

- Search Ability of EMO for Fuzzy System Design
- Definition of Interpretability of Fuzzy Systems
- Explanation Ability of Fuzzy Rule-Based Systems
- Various Classification Problems: Imbalanced, Online, ...

Fuzzy Classifiers on Various Problems

We have a lot of different types of classification problems where fuzzy rule-based classifiers have not been well-utilized and have a large potential usefulness:

1. Imbalanced Data
2. Semi-Supervised Learning
3. Active Learning
4. On-Line Learning
5. . . .
6. . . .

Fuzzy Classifiers on Various Problems

IFSA 2009 Invited Talk



[Fuzzy Logic in Machine Learning](#)

Eyke Hüllermeier

Department of Mathematics and Computer Science
Philipps-Universität Marburg, Germany

Eyke HÜLLERMEIER is with the Department of Mathematics and Computer Science at Marburg University (Germany), where he holds an appointment as a Full Professor and heads the Knowledge Engineering & Bioinformatics Lab. He holds M.Sc. degrees in mathematics and business computing, a Ph.D. in computer science, and a Habilitation degree, all from the University of Paderborn (Germany). His research interests are focused on machine learning and data mining, fuzzy set theory, uncertainty and approximate reasoning, and applications in bioinformatics. He has published numerous research papers on these topics in leading journals and major international conferences. He is on the editorial board of several journals, including *Fuzzy Sets and Systems*, *Soft Computing*, and the *International Journal of Data Mining, Modeling and Management*. Moreover, he is a board member of the European Society for Fuzzy Logic and Technology (EUSFLAT), a co-ordinator of the EUSFLAT working group on Learning and Data Mining, and head of the IEEE CIS Task Force on Machine Learning.

Fuzzy Classifiers on Various Problems

IFSA 2009 Invited Talk

Fuzzy Logic in Machine Learning


Eyke Hüllermeier

Department of Mathematics and Computer Science
Philipps-Universität Marburg, Germany

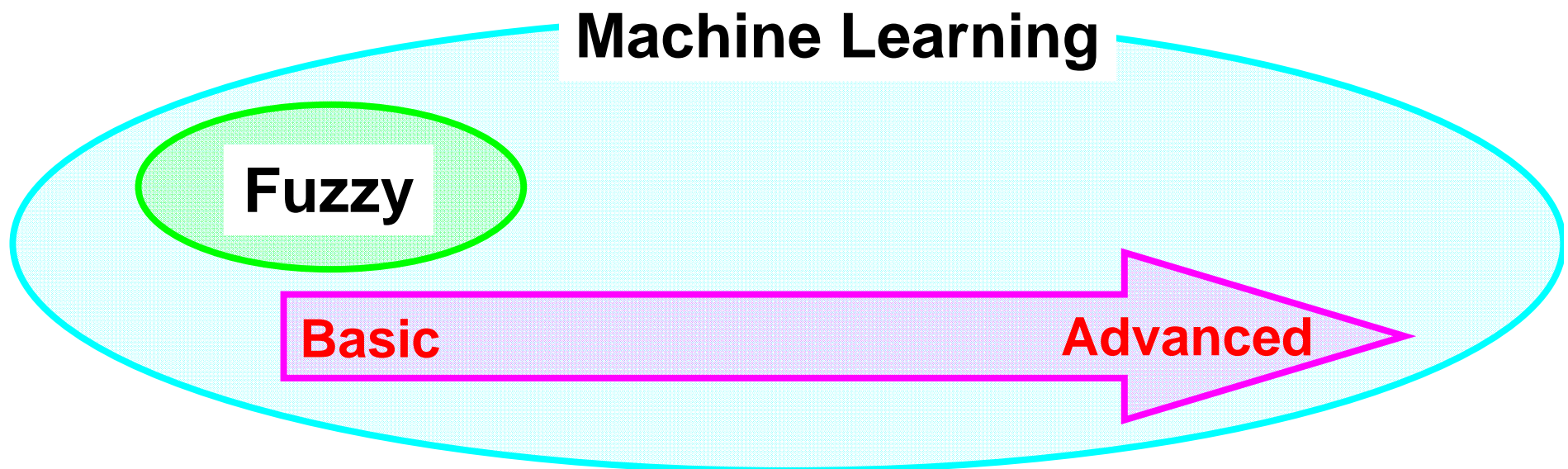
The purpose of this talk is twofold. First, it is intended to convey an idea of the state-of-the-art in fuzzy logic-based machine learning, to be understood as the application of formal concepts, methods, and techniques from fuzzy set theory and fuzzy logic in the field of machine learning and related research areas, such as data mining and knowledge discovery. In this regard, potential contributions that fuzzy logic can make to machine learning will be especially highlighted, though some deficiencies of this line of research will also be pointed out. Second, some promising directions of future research in this field shall be sketched and promoted, including problems of ranking and preference learning, the representation of uncertainty in model induction and prediction, and the use of fuzzy modeling techniques for feature generation.

Fuzzy Classifiers on Various Problems

IFSA 2009 Invited Talk



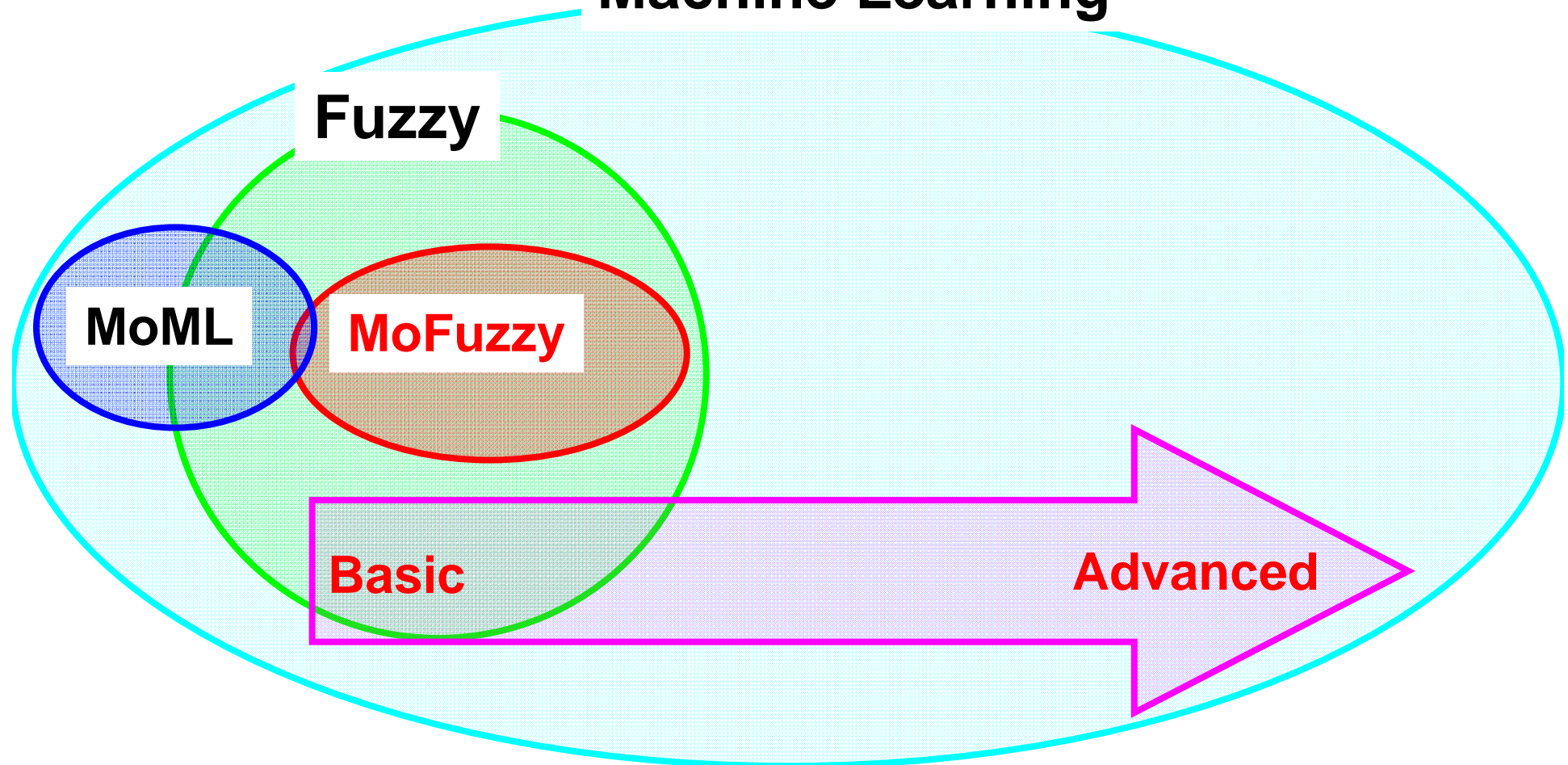
[Fuzzy Logic in Machine Learning](#)
Eyke Hüllermeier
Department of Mathematics and Computer Science
Philipps-Universität Marburg, Germany



Fuzzy Classifiers on Various Problems

ISDA 2009 Invited Talk by Hisao Ishibuchi

Machine Learning



Conclusions

1. Introduction to Fuzzy Rule-Based Classification

- Is Fuzzy Rule-Based Classification a Popular Research Area?

Yes !

2. Fuzzy Rule-Based Classifier Design

- Accuracy Improvement
- Scalability to High-Dimensional Problems
- Complexity Minimization

3. Multiobjective Fuzzy Rule-Based Classifier Design

- Formulation of Multi-objective Problems
- Accuracy-Complexity Tradeoff Analysis
- Maximization of Generalization Ability

4. Current Hot Issues and Future Research Directions

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- Definition of Interpretability of Fuzzy Systems
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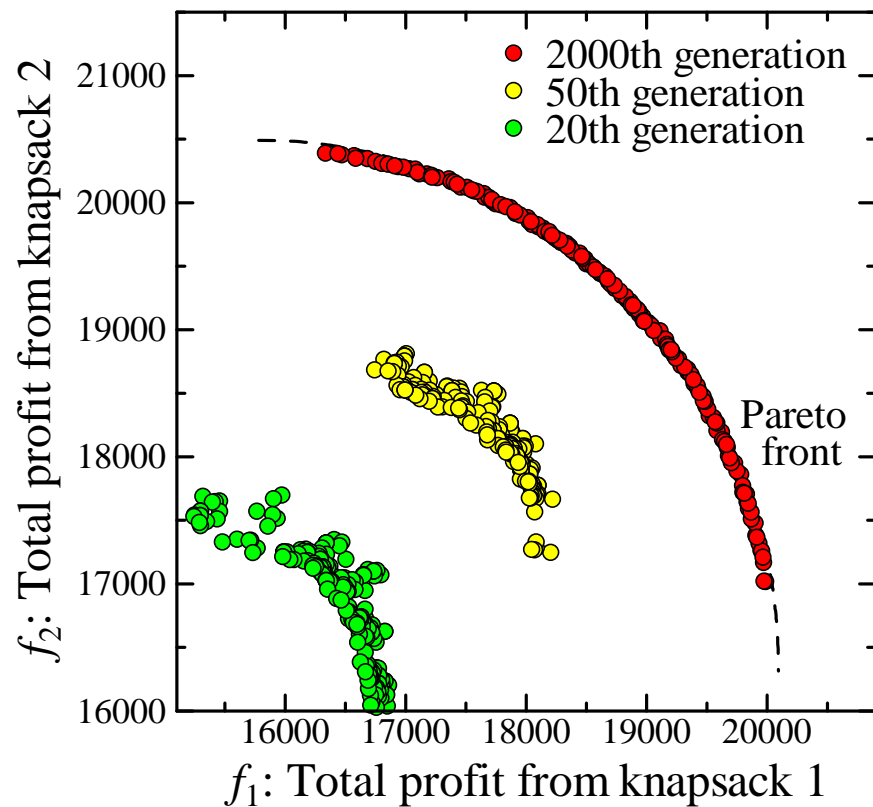
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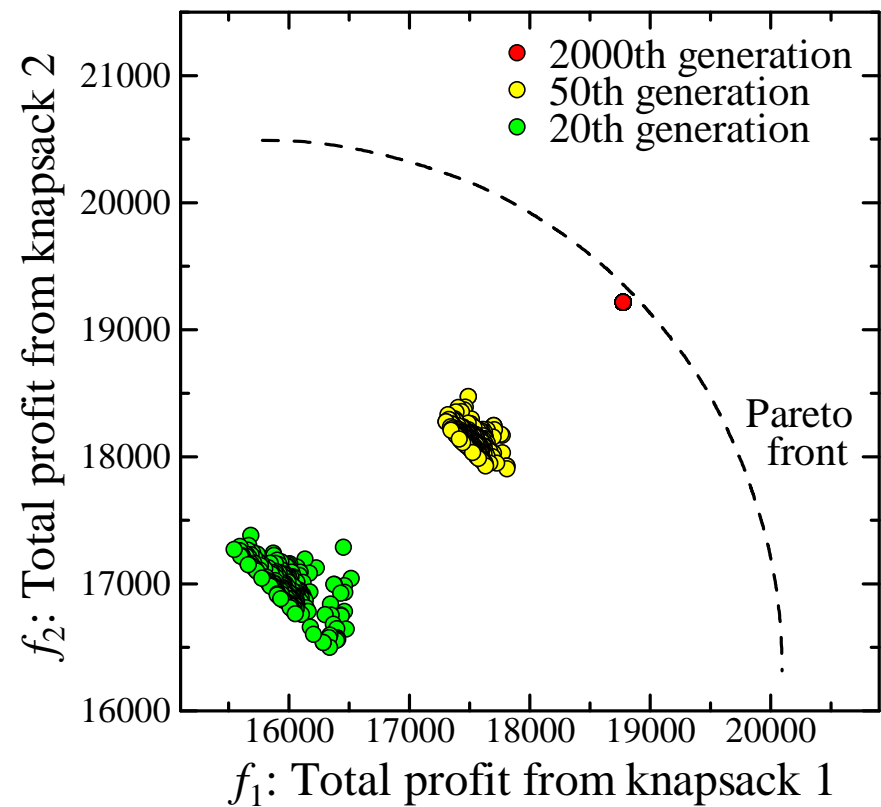
We have still a lot of interesting research issues.

Appendix: Comparison of the Two Approaches

Two-objective maximization problem



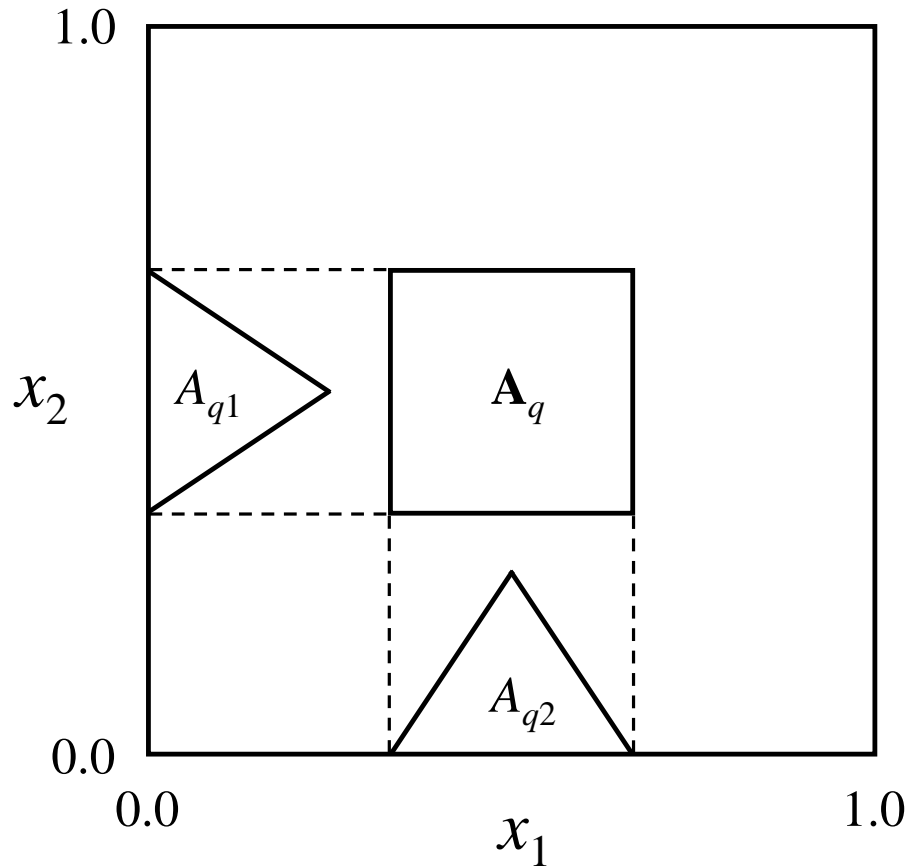
EMO Approach



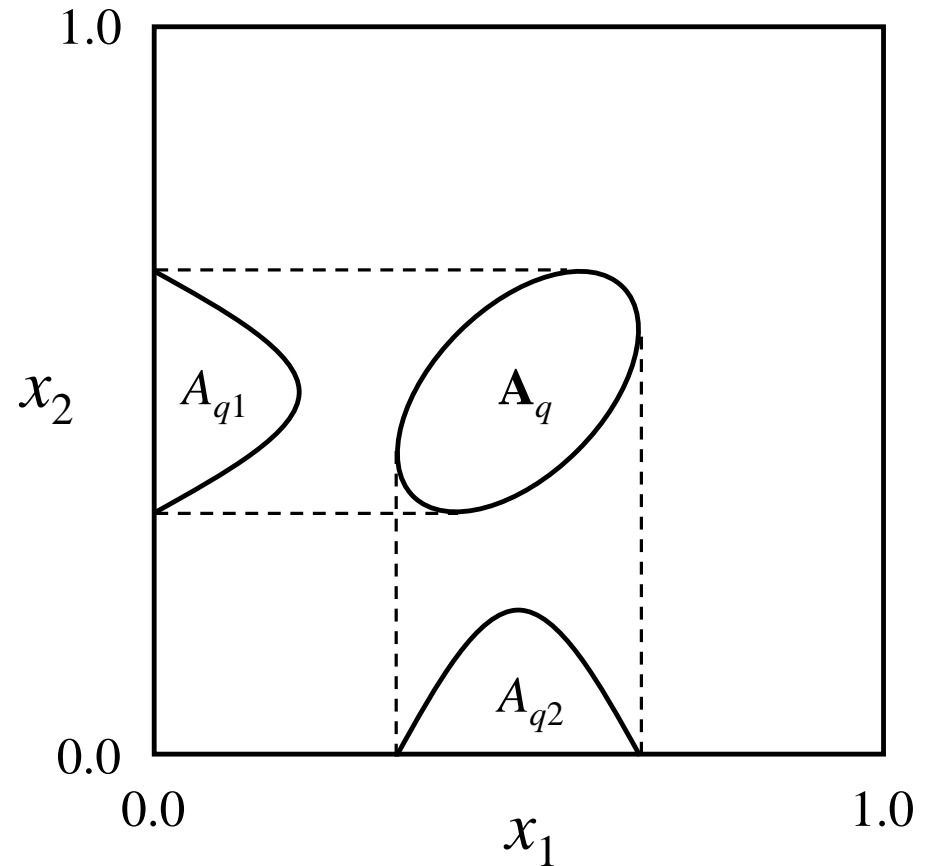
Weighted Sum Approach

Experimental results of a single run of each approach

Appendix: Two-Dimensional Antecedent Fuzzy Sets

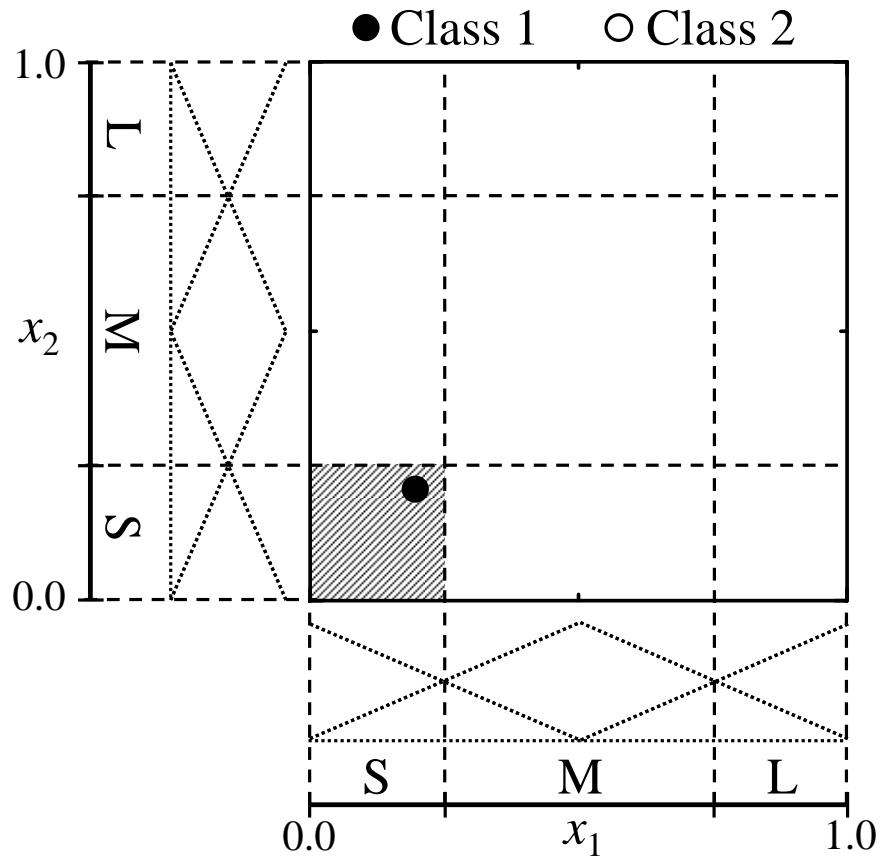


(a) A two-dimensional fuzzy vector.

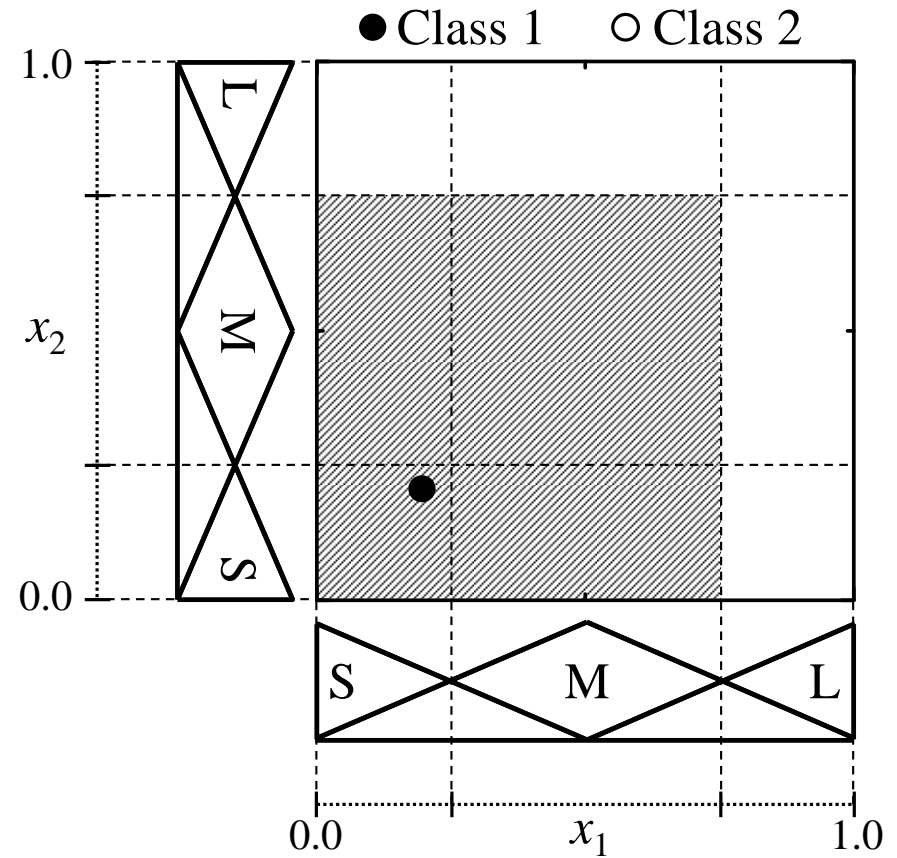


(b) An ellipsoidal antecedent fuzzy set.

Appendix: Interval Rules vs Fuzzy Rules

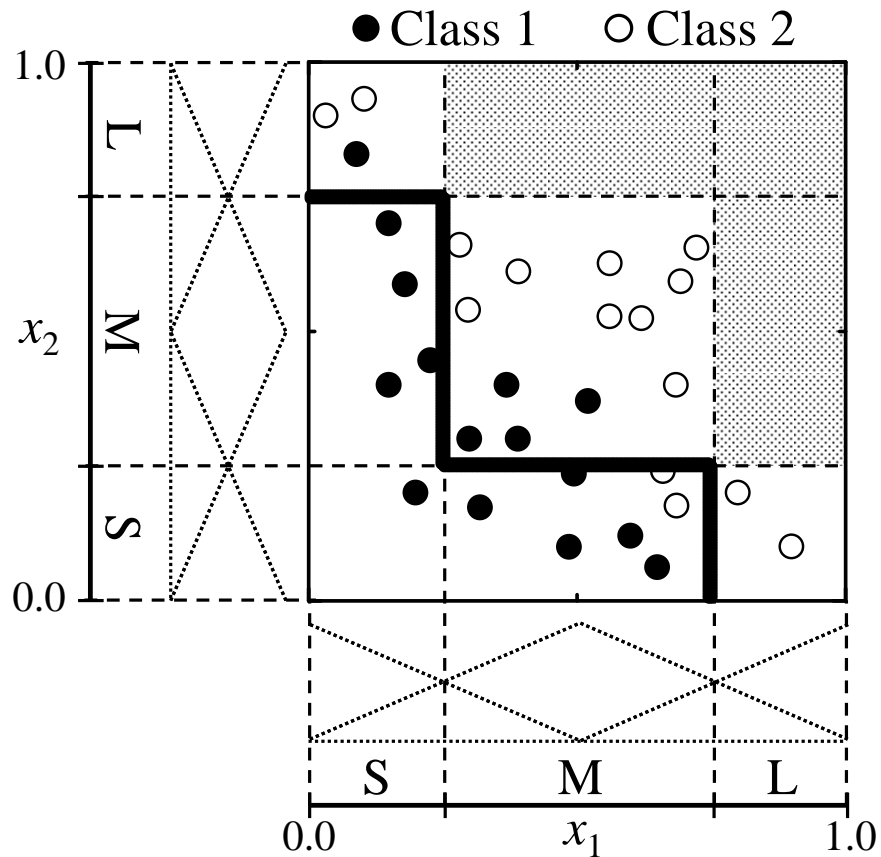


(a) Interval Rules

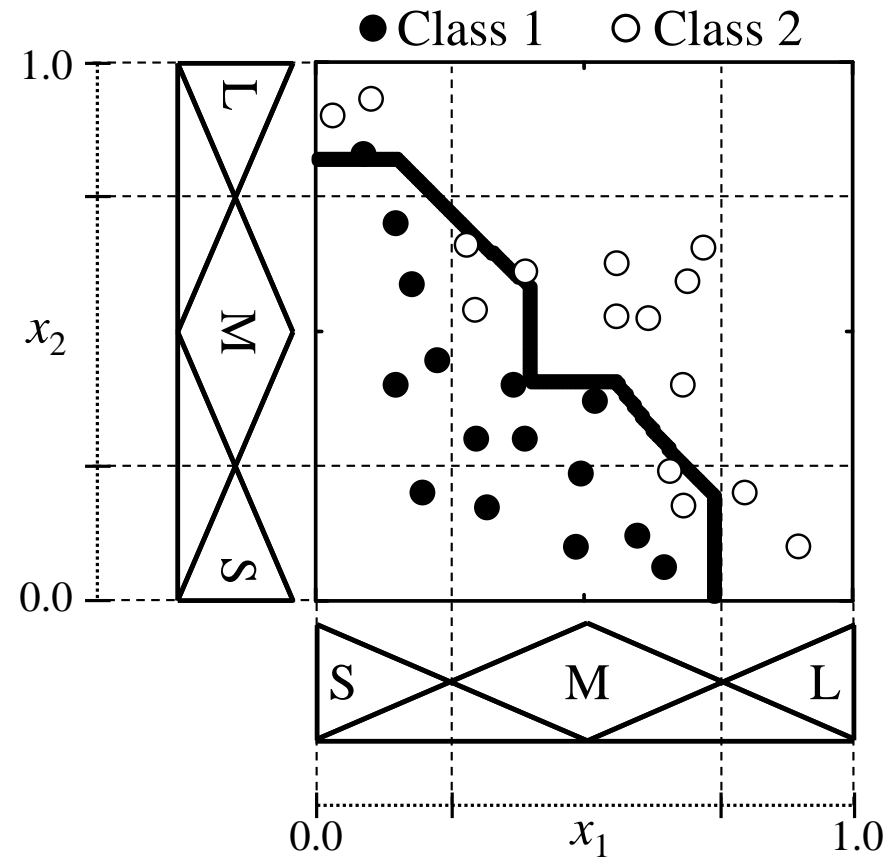


(b) Fuzzy Rules

Appendix: Interval Rules vs Fuzzy Rules



(a) Interval Rules



(b) Fuzzy Rules